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A NUMERICAL APPROACH TO INVESTIGATE SOUND RADIATION FROM INFINITE CYLINDRICAL SHELLS

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ABSTRACT

Understanding the vibroacoustic behaviour of cylindrical structures and evaluating sound radiation are crucial for reducing unwanted noise in industrial environments and controlling breakout noise from ducts in buildings. However, analysing sound radiation through cylindrical shells is more complex than in plate-like structures due to additional modes arising from surface curvature and the coupling between structural and acoustic waveguides. This paper presents preliminary results of a numerical approach aimed at modelling sound radiation from infinite cylindrical shells to the external environment. The differences in sound radiation between planar and cylindrical structures are examined through numerical vibroacoustic simulations, analysing single and double-layer infinitely extended cylindrical shells. These preliminary results serve as a foundational basis for optimizing cylindrical acoustic mitigations for industrial applications.

Keywords: *Sound radiation, infinite cylindrical shell, vibroacoustic simulations, finite element method*

1. INTRODUCTION

The dynamic and vibroacoustic behaviour of cylindrical shells present a higher degree of complexity compared to planar structures and have been subjects of significant interest over the past four decades. Fuller and Fahy investigated the dispersion behaviour in fluid-filled cylindrical shells [1]. Manconi and Mace proposed a finite element approach to characterize wave propagation in cylindrical shells [2]. Additionally, Fuller analytically examined sound radiation from infinite cylindrical shells based on the Donnell-Mushtari shell theory [3]. Wang et al. investigated the radiation efficiency of a cylindrical shell of finite length under mechanical excitation [4]. Numerous studies [5, 6] have focused on sound transmission through cylindrical shells, particularly in relation to noise control applications for aircraft and vessels, where acoustic plane waves impinge externally on the shell surface. Vibroacoustic problems involving outward radiation due to internal acoustic sources or mechanical excitation of the shell's wall, such as breakout noise in ventilation ducts or internal-to-external sound transmission loss, are often studied with simplifying assumptions. These assumptions may include considering only plane wave propagation or assuming a thin shell [7, 8]. Cummings provided a comprehensive review of sound transmission and radiation from ducts in 2001 [9]. To overcome the limitations of analytical approaches, numerical methods have been used to analyse sound radiation from cylindrical shell structures [10, 11]. This paper proposes a finite element analysis (FEA) to investigate sound radiation in infinite cylindrical shell structures, considering both acoustically thin and thick shells, as described in Section 2. Single and double-leaf infinitely extended cylindrical shells are analysed, considering both mechanical excitation through a point force applied to the shell wall and airborne excitation due to a random acoustic field within the internal fluid medium. Preliminary results are presented in Section 3, which serve as a preparatory step for modelling and optimizing cylindrical acoustic mitigations in industrial applications, to be developed in the continuation of this study.

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2. MATERIAL AND METHODS

The numerical analysis was conducted using the commercial finite element method software COMSOL Multiphysics®. A model of an air-filled steel cylindrical shell immersed in a semi-infinite air medium was implemented for a fully coupled structural-acoustic simulation. The cylindrical shell was modelled as a 3D elastic solid with an internal radius r and thickness h. The external fluid domain, with a radius $R = 2r$, was approximated as a semi-infinite fluid using a cylindrical perfectly matched layer (PML). A 2D model was employed to simulate an infinitely extended system along the shell's axis, assuming the plane strain approximation. A triangular mesh was utilised for both the acoustic and elastic domains, ensuring that the maximum element size was set to guarantee at least 6 elements for the shortest acoustic wavelength considered. To evaluate the influence of the forcing condition on the radiated sound, two scenarios were considered: mechanical excitation, applied as a point force on the shell wall (a line force along the shell's generator), and airborne excitation, generated by a random sound field in the fluid inside the shell, modelled through plane waves with random phase and heading angle. Two different systems were modelled, considering both a thin shell with $f_c/f_r > 1$ and a thick shell with $f_c/f_r < 1$, where f_c is the critical frequency of a flat plate and f_r is the shell's ring frequency. The geometrical characteristics of these systems are provided in Table 1.

Table 1. Geometrical characteristics of the modelled systems.

System	r [m]	h [m]	f_c [Hz]	f_r [Hz]
Thick	0.300	0.015	817	2678
Thin	0.500	0.003	4086	1607

Multilayer structures, whether planar or cylindrical, are widely used to enhance sound insulation. In this study, double-layer cylindrical systems were considered by modelling the inner shell with the same characteristics as those described in Table 1. The external shell had the same thickness as the internal one and a radius $r' = r + h + d$, where d represents the air gap between the two elastic structures. Two different air-cavity depths were investigated: $d_1 = 0.1$ m and $d_2 = 0.005$ m. For all the analysed structures, the average normal vibration velocity was evaluated over the surface of the external shell. Additionally, the sound power radiated under free-field conditions was determined by integrating the sound intensity in the external fluid domain over a circular boundary concentric with the shell.

3. RESULTS

The results of the numerical simulations are presented and discussed in this section, focusing initially on the single-layer cylindrical shells. Figure 1a) shows the computed sound power levels for a thin cylindrical shell, while Figure 1 b) shows the spectra for a thick cylindrical shell. It is evident that in thin-shell systems, the radiated sound power exhibits a significant increase above the ring frequency (f_r) under both types of excitation. Below the ring frequency, a greater number of peaks are observed with airborne excitation, indicating that the random acoustic field can excite more coupled modes within the elasto-acoustic system compared to a mechanical force applied to the shell wall. The additional peaks observed exclusively in the radiated sound power spectrum for airborne excitation at lower frequencies are close to the eigenfrequencies of rigid-wall duct modes, characterised by distinct acoustic pressure distributions across the cross-section (e.g., $f_{01} \approx 200$ Hz, $f_{02} \approx 333$ Hz, $f_{10} \approx 459$ Hz, ...). In the case of thick-shell systems, lower radiated sound power levels are observed for both airborne and structure-borne excitations due to the higher mass per unit length and stiffness of the structure. When the shell is excited by a mechanical force, a few peaks can be observed in the sound power spectra approximately at the eigenfrequencies of the in-vacuum structural modes of a cylindrical shell (e.g., $f_2 \approx 105$ Hz, $f_4 \approx 563$ Hz, ...). Similar to thin shells, an internal random acoustic sound field can excite a greater number of coupled modes propagating within the elasto-acoustic system compared to a mechanical force. Peaks in the radiated sound power can be observed at frequencies near both rigid duct and in-vacuum shell modes (e.g., $f_2 \approx 105$ Hz, $f_3 \approx 295$ Hz, $f_{01} \approx 335$ Hz, $f_{02} \approx 556$ Hz, $f_4 \approx 563$ Hz, ...).

It may be useful to analyse the vibroacoustic behaviour of the investigated structures in relation to their critical frequency f_c in terms of radiation efficiency σ. In Figure 2, the radiation index L_σ computed for the investigated structures is compared with that of an infinite flat plate of the same thickness, considering structure-borne excitation. It is observed that both thin and thick shells can efficiently radiate sound even below the critical fre-

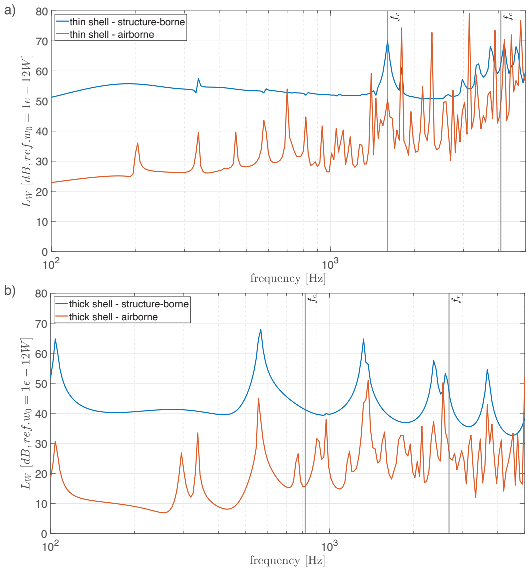

Figure 1. Sound power levels radiated by the cylindrical shell for airborne and mechanical excitations: a) thin shell; b) thick shell.

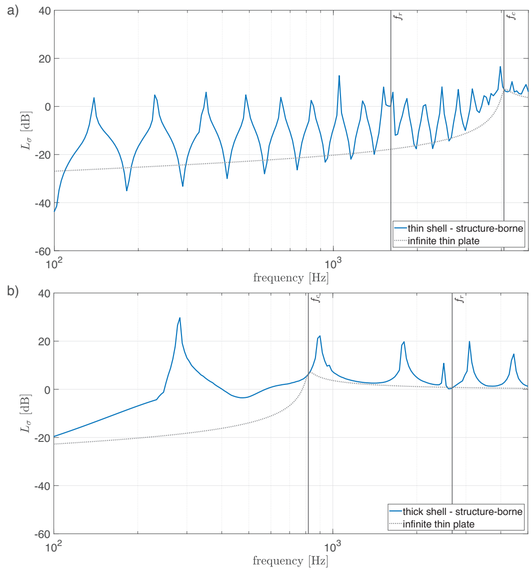

Figure 2. Comparison between the radiation index of the a cylindrical shell and an infinite flat plate: a) thin shell; b) thick shell.

quency, as indicated by peaks where $L_\sigma > 0$. In cylindrical shells, sub- and supersonic propagating waves coexist below the critical frequency, and shell modes that can couple with the acoustic wavenumber in both the internal and external fluids exhibit efficient radiation. Figure 2 a) shows that above the critical frequency, a thin shell has a high modal density and behaves similarly to a flat plate in terms of sound radiation. However, for thick shells, as shown in Figure 2b), the modal contribution remains significant even above the critical frequency due to the lower modal density. Moreover, between the peaks, the radiation index approaches unity, as expected for a flat plate. These results are consistent with findings from other studies that used a different numerical approach to analyse sound radiation in cylindrical shells [10], which verifies the reliability of the implemented model.

The vibro-acoustic behaviour of double-layer cylindrical shells with an air cavity was analysed in terms of insertion loss (IL) with respect to a single-layer cylindrical shell, rather than sound transmission loss, to avoid making assumptions about the diffusivity of the sound field incident on the inner surface of the shell. The results for airborne excitation are shown in Figure 3. Two cavity depths were investigated: $d_1 = 0.100$ m and $d_2 = 0.005$ m. A consistent trend can be observed for all the considered systems. At lower frequencies, the IL increases with frequency after a sharp dip associated with the mass-spring-mass resonance. For comparison purposes, the mass-spring-mass resonance frequency of a flat double-leaf partition is indicated in Figure 3, which does not accurately approximate the resonant frequency of the double-cylindrical shell systems (only the value for a cavity with a depth of 0.005 m is shown because the resonances for the larger cavity fall below the investigated frequency range). As the frequency approaches the ring frequency, a reduction in the IL is observed for both thick and thin shells. However, in the case of thin shells, the IL starts to increase again above the critical frequency.

4. CONCLUSIONS

This paper presents a 2D finite element analysis (FEA) that investigates sound radiation in infinite cylindrical shell structures. The analysis considers both acoustically thin and thick shells, excited either by a mechanical point force or by an airborne random sound field in the internal fluid medium. The preliminary analysis focuses on the vibroacoustic behaviour of cylindrical shells, which exhibit a higher degree of complexity compared to planar struc-

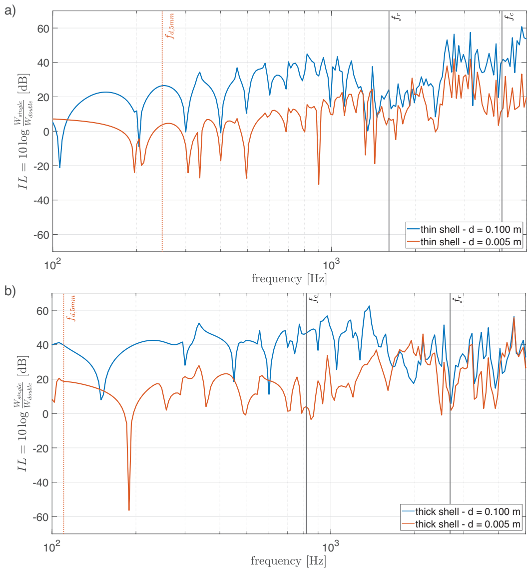

Figure 3. Insertion loss obtained with a double-layer cylindrical shell structure with respect to the single-layer system: a) thin shell; b) thick shell.

tures. The results indicate that the sound power radiated by a thin-shell systems significantly increases above the ring frequency, while the increase in sound power levels radiated by an acoustically thick shell starts from the critical frequency. The thick-shell system shows lower amplitude of the radiated sound power due to higher mass and stiffness. In both systems, the presence of an internal random acoustic field results in the excitation of more coupled modes compared to a mechanical force. The evaluation of the main characteristic of the insertion loss of double-cylindrical shells highlights their higher involvement compared to a flat double-leaf partition. The characteristic mass-spring-mass resonant frequency of the double-cylindrical shell does not solely depend on the mass per unit of length of the two cylindrical shells and the air cavity depth. Thus, further investigation is required for a better understanding of the vibroacoustic behaviour of cylindrical shell structures. However, the presented results, consistent with the findings of other studies, verify the reliability of the implemented model. The follow-up of this study will further develop and validate this modelling approach to obtain more advanced techniques for the optimization of cylindrical acoustic treatments for noise mitigation in industrial applications.

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