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GEOMETRIC OPTIMIZATION OF A MULTIPLE COILED-UP RESONATORS FOR A BROAD BAND ACOUSTIC ABSORPTION

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ABSTRACT

Several analysis have been performed to optimize the geometry of a metamaterial that consists of a series of quarter-wave coiled-up resonators coupled in parallel. With the aim of achieving a broadband resonator, the minimum resonant frequency of absorption was defined equal to 100 Hz, which will define the second harmonic of 300 Hz (it is proportional to $3/4\lambda$). This frequency range is then divided into different resonators so that the envelope of the high-harmonics is such as to obtain a high broadband absorption. The optimization process requires different analysis tools: first of all an analytical model [1] was used to obtain the best geometry, in terms of length of resonator ducts and inlet section of the holes; than a 3D FE model was used to consider the more complex 3D geometry and the interaction of the different resonators. Both models have been validated by a series of experimental measurements conducted in the laboratories of the University of Ferrara: they can well predict the acoustic behavior of the investigated system, both with single resonators and in parallel, in order to calculate the absorption coefficient at normal incidence.

Keywords: *Acoustic metamaterial, quarter-wave coiled-up resonators, analytical model, finite element method, broadband sound absorption.*

1. INTRODUCTION

Nowadays, low-frequency sound absorption is a problem of great interest, since it is difficult to achieve using common absorption mechanisms [2]. Sound absorbing materials require high thicknesses to be acoustically efficient at low frequencies, while resonators provide high but too selective absorption.

The study of the present metamaterial fits into this context: coupling several coiled-up quarter-wave resonators in parallel, we want to obtain a system with a good absorption at low frequencies and limited thickness, since with these resonators it is possible to reduce the thickness thanks to the possibility of "rolling up" the channels in which the sound field develops [3]. However, being resonant elements, they are characterized by a very selective acoustic absorption at their own resonance frequencies: to overcome this problem, ducts of different lengths can be coupled in parallel so as to obtain an efficient system in broadband acoustic absorption.

Starting from these points, this study proposes an optimization method for metamaterials that consist of coiled-up quarter-wave multi-resonators. The first stage of the process involves the use of an analytical model which can optimize the geometry of the system: the sound absorption coefficient is maximized through the study of the porosity (which is the ratio between the areas of the holes of the in parallel channels and the frontal area of the total metamaterial); at the same time the lengths of the channels are optimized, in order to define the peak frequencies in such a way as to obtain good broadband absorption, starting from a minimum frequency of 100 Hz.

The second phase involves the creation of the optimized 3D geometry, using CAD software, which is then simulated using a FE model in order to consider some effects that are neglected by the analytical model: the latter considers the rectilinear geometry of the ducts and not the real rolled-up one. Both the analytical and the numerical models have been validated through a series of experimental measurements that have been carried out in the standing wave tube: some prototypes obtained through 3D printing were tested to evaluate any critical issues, first of all in the printing phase and then in the coupling of the single resonators and of the total system [1].

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2. ANALYTICAL OPTIMIZATION

To calculate the normal incidence sound absorption coefficient of the coiled-up metamaterial, an analytical model is used [1], which define the losses inside the duct as follows: in the inlet section it uses the Johnson-Champoux-Allard (JCA) model to quantify the dissipative losses due to the variation of impedance of the section narrowing; along the channel path it uses the Narrow Region model, to quantify the thermo-viscous losses due to friction of the air particles against the walls. Once the impedances of the individual channels have been determined, the total acoustic surface impedance of the system with the in parallel resonators is obtained in analogy to the electric circuits theory.

The geometric optimization procedure (figure 1) concerns a metamaterial consisting of 16 resonators in parallel: the longest duct has a resonance frequency of 100 Hz, while the shortest duct has a frequency of 300 Hz , arranged in four rows and four columns of a sample with a front surface equal to 20x20 cm (*l_tot*).

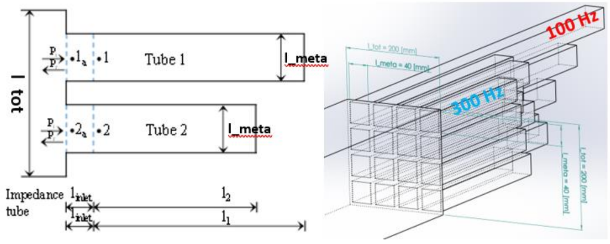

Figure 1. Metamaterial with 16 quarter-wave resonators in parallel.

In the first stage the section of each duct was optimized (*l_meta*) to maximize the amplitude of the absorption peak: as the porosity of the system increase ($\varphi = l\_meta^2 / l\_tot^2$), the peak of absorption of the single duct changes inside a range of values: the optimization shows a maximum value of absorption by setting a *l_meta* = 30.0 mm for all the ducts.

In the second stage, the lengths of each individual duct was analyzed to obtain a high broadband absorption. After fixed the minimum absorption frequency (100 Hz) and its first harmonic (300 Hz), this range of frequencies must be divided in such a way that the envelope of the 16 resonators returns an absorption trend without "dips". This is done by acting on the lengths of the ducts which are related to the frequency peaks through the following equation (1):

$$f_{res} \cong \frac{c_0(2n-1)}{4l} \qquad (1)$$

where c_0 is the speed of sound in the fluid inside the channel (air), n=1,2,3,… is the index of the considered harmonic and l the length of the channel.

3. EXPERIMENTAL MEASUREMENTS SET-UP

The analytical model, an efficient tool both in terms of possible optimizations and timing of simulation, has some limits: it cannot consider complex 3D geometries, such as for example a coiled-up resonator or the acoustic interactions between the various channels. For these reasons, the 3D optimized geometry was created with CAD software and then simulated with finite elements model by implementing the experimental set-up of UNI EN ISO 10534-2 [4]. The two curves in figure 2 show good agreement, with some differences in terms of amplitude and frequency shift: this is due to the construction of the 3D geometry, in which the ducts, even if they have an average path equal to the length defined by the analytical model, could present some geometric deviations. Above 2000 Hz there is a collapse of the FE absorption as the cut-off frequency of the simulated tube occurs.

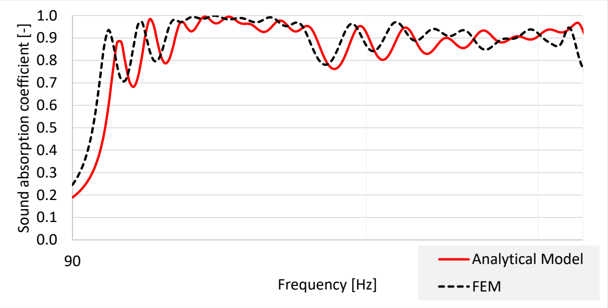

Figure 2. Normal incidence sound absorption coefficient – comparison between analytical and FEM.

Defined the optimized geometry using the analytical model and then confirmed by the 3D FEM simulation, the next step will consist in the experimental validation of the metamaterial.

The metamaterial was designed by printing each single resonator, using the 3D printing technique in PLA material: the resonators are placed inside a frontal mask (figure 3b), which orders them in parallel and respects the optimized porosity. The positioning of the resonators was also designed to minimize the overall size of the system, both in terms of thickness and section: in fact, the resonators are contained within the 20x20 cm front section of the metamaterial.

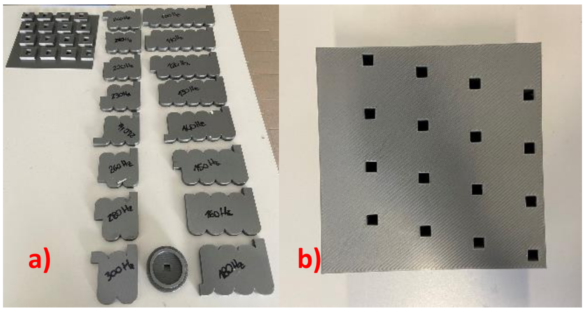

Figure 3. Single resonators (a) and frontal mask (b).

4. CONCLUSIONS

The present study proposes a complete geometry optimization procedure of a metamaterial, which consists of 16 coiled-up resonators coupled in parallel. The main goal is to obtain an high absorption broadband system, starting from a minimum frequency of 100 Hz with a limited thickness.

In the first stage, an analytical model is used to optimize the geometry: the front section of the ducts is optimized with respect to the front section of the metamaterial in order to maximize the absorption amplitude (which means optimizing the porosity of the metamaterial); at the same time, the length of the ducts is optimized to obtain a good broadband sound absorption coefficient (the length of the ducts defines the frequency peaks of the absorption).

The analytical model is also confirmed by the 3D FEM simulation, which reproduces the experimental measurement setup, implementing some conditions that are difficult to consider in the analytical model.

At the moment, the metamaterial is under construction, using the 3D printing technique, in order to perform the acoustic absorption measurements as soon as possible, according to the measurement standard [4] and the setup shown in figure 3.

5. ACKOWLEDGEMENTS

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