DOI: 10.61782/fa.2023.0730

forum acusticum 2023

SHEAR FLOW EFFECTS IN DUCTS: APPLICATION TO DIRECT IMPEDANCE EDUCTION OF ACOUSTIC LINERS

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ABSTRACT

Impedance eduction of acoustic liners is commonly implemented under the hypothesis of uniform mean flow by applying Ingard–Myers boundary condition at the lined wall. However, flows in ducts are intrinsically sheared, and it is reasonable to question the validity of the uniform flow hypothesis, especially when considering large ducts, high-order acoustic modes and flow velocities representative of aircraft nacelles. This paper studies the effects of shear flow in such a framework. A numerical multimodal method computes the acoustic modes and the acoustic pressure field in a 2D lined duct with shear flow. Then, an eduction procedure taking the flow profile into account is introduced. The performance of the latter is numerically compared against results obtained using the uniform flow assumption. A significant gain in robustness is demonstrated, in particular for ducts with large cross-sections, high-speed flows and upstream propagating waves.

Keywords: *shear-flow effect, impedance eduction, acoustic liner, duct acoustics.*

1. INTRODUCTION

In current aircraft engines, the inlet, bypass and exhaust parts of the nacelle are treated with acoustic liners to reduce noise emissions. The investigation of the behaviour of liners under realistic conditions is an essential part of their development and design. To that end, it is mandatory to be able to educe correctly the impedance of the acoustic treatments in well controlled experiments.

Impedance eduction can be classified into two main categories, inverse and direct methods, the latter being the subject of the present study. Direct impedance eduction consists in the calculation of the liner impedance using the axial wavenumber of one of the propagating modes in the lined section. To that end, most direct methods rely on the assumption of a uniform mean flow by using Ingard–Myers boundary condition [1], whereas others assume a sheared mean flow and use Pridmore Brown equation [2]. Recently, some studies have suggested and shown that the assumption of uniform flow may introduce errors into the impedance results, either in case of large flow speed [3], 3D computations [4] or large ducts [5]. Nevertheless, most of previous research focused on small cross-section ducts, while extended studies on the effect of shear flow in large ducts are lacking.

This paper reports a numerical investigation in order to prepare for a subsequent experimental campaign to adapt this methodology to the MAINE Flow facility [6] which has a duct with a large cross-section. The numerical investigation concerns the influence of the flow profile on direct impedance eduction of liners for large ducts and of flow velocities representative of aircraft nacelles. First, we introduce the numerical methods used to compute the acoustic field given the impedance, and the impedance given the acoustic field. Then, comparisons between eduction methods that take or not into account the flow profile are displayed.

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2. NUMERICAL METHODOLOGY

2.1 Acoustic field computation in 2D with shear flow

In the whole study, a 2D duct composed of three sections is considered (see Fig. 1). The first and third sections have hard walls, while the middle section is treated with the same locally reacting liner on both walls. The axial and vertical coordinates are denoted respectively x and y. A single incident duct mode is propagating toward the liner from the left-hand side. The duct exit on the right-hand side is considered anechoic.

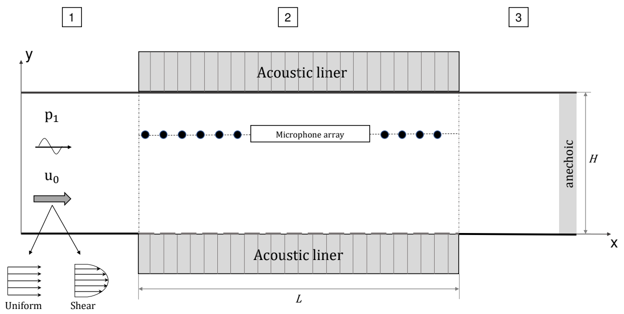

Figure 1. Schematic of the 2D duct with a lined section.

To apply the present analysis to a realistic configuration, we consider the MAINE Flow facility [6]. The height of the duct is $H = 0.28$ m and the length of the lined section is $L = 0.8$ m. A linear array of 40 virtual microphones spaced by $\Delta x = 0.02$ m is placed at 2/3 of the height of the duct, following the configuration proposed by NASA [2]. The first and last positions are 0.01 m away from the two interfaces.

Sound propagation in a duct with a parallel shear flow can be computed using the Linearised Euler Equations (LEEs), written here for the vertical acoustic displacement ξ and the acoustic pressure p:

$$\frac{\partial p}{\partial y} + \rho_0 \frac{D_0^2 \xi}{Dt^2} = 0 \quad , \tag{1a}$$

$$\rho_0 c_0^2 \frac{D_0^2}{Dt^2}\frac{\partial \xi}{\partial y} - c_0^2 \frac{\partial^2 p}{\partial x^2} + \frac{D_0^2 p}{Dt^2} = 0 \quad . \tag{1b}$$

The impedance boundary condition can be directly written $\xi = -p/(\mathrm{i}\omega\rho_0 c_0 Z)$ and $\xi = p/(\mathrm{i}\omega\rho_0 c_0 Z)$ for the lower and upper walls, respectively. Here, Z is the specific surface impedance (i.e. normalised by $\rho_0 c_0$).

In each section, assuming that pressure p and displacement ξ are modal sums with an axial dependence given by $\mathrm{e}^{-\mathrm{i}k_x x}$ yields

$$\frac{\partial p}{\partial y} - \rho_0(\omega - u_0 k_x)^2 \xi = 0 \tag{2a}$$

$$-\rho_0 c_0^2(\omega - u_0 k_x)^2 \frac{\partial \xi}{\partial y} + c_0^2 k_x^2 p - (\omega - u_0 k_x)^2 p = 0 \tag{2b}$$

In each section of the duct, the acoustic pressure can be expressed as a sum of modes propagating in both directions. The associated eigenvalue problem for the axial wavenumbers and mode shape functions is solved using a pseudo-spectral method [7], with N Chebyshev polynomials to represent p and ξ. The linear system is closed by the two boundary conditions, and the sound field in the whole duct is calculated using a mode-matching method based on the conservation of mass and momentum [8].

2.2 From the wavenumbers to the impedance

The principle of direct method for impedance eduction is to calculate the liner impedance after having measured an acoustic wavenumber in the lined section. Usually, this is done under a uniform mean flow assumption using Ingard–Myers boundary condition:

$$\frac{\partial p}{\partial n} = -\frac{D_0^2}{Dt^2}\frac{p}{\mathrm{i}\omega c_0 Z}. \tag{3}$$

Having both walls lined with the same liner implies applying this condition at both walls. A quadratic equation is then obtained, and the solutions are

$$Z = \frac{(k_0 - M k_x)^2 \sin(k_y H)}{\mathrm{i}k_0 k_y[\cos(k_y H) \pm 1]}. \tag{4}$$

This expression contains two types of solution distinguished by "±" in the denominator, in which "−" corresponds to symmetric modes and "+" to antisymmetric modes. The choice is completely controlled by the (anti)symmetry of the incident mode. When the upper and lower walls are lined with the same material, the total sound field will retain the symmetry of the incident mode. For instance, if the incident mode is antisymmetric, then one should use the sign "−" in the expression above.

The originality of this paper is to go beyond the uniform flow assumption by considering the flow profile in the link between the wavenumber and the impedance. To that end, a collocation method is used to solve Eq. (2) with p and ξ unknown. Further details about this fast way of impedance computing are given in [9].

3. NUMERICAL EDUCTION EXPERIMENTS

3.1 Set up and method

The numerical set-up is the one previously described and shown in Fig. 1. The considered liner is a model SDOF (Single Degree of Freedom) liner defined by its impedance $Z = 1 - \mathrm{i}\cot(k_0 h_{cav})$ where h_{cav} mimics the presence of a cavity of height $h_{cav} = 0.03$ m. To provide synthetic data for testing the eduction procedure, the acoustic field is calculated from 300 to 3000 Hz by means of the method introduced previously. To that end, we use an existing mean flow profile measured in the MAINE Flow facility during a previous study at Mach 0.63. A theoretical inverse-law profile is fitted to this data and is used in the numerical model.

To estimate the axial wavenumbers from the signals computed at the (virtual) microphone positions, a HTLS (Harmonic retrieval via Total Least Squares solution) technique [10] is implemented, which is based on a shift-invariance property of matrix in Vandermonde decomposition and on the singular value decomposition.

3.2 Results

To validate the proposed eduction configuration and the associated methods, numerical eduction for flows with Mach number between 0.1 and 0.7 are finally performed and analysed.

Fig. 2 compares the educed impedance obtained by the uniform and shear direct methods, with the plane mode as an incident mode propagating downstream. It is found that the results obtained by the shear flow assumption are generally in excellent agreement with the imposed impedance. In addition, results obtained taking the flow profile into account lead to much better results than the ones obtained under uniform flow assumption using Ingard–Myers boundary condition. Fig. 3 compares also both direct impedance eduction methods, but for waves propagating against the flow. In this case, using the uniform flow assumption induces even larger discrepancies compared to the reference curve. These discrepancies are also greater than the ones observed in previous studies. This is due to the size of the duct which induces a complex sound field with many propagating modes. One can also note that increasing the flow speed seems to add a tiny bias to the results obtained by the shear flow assumption at high frequencies. Thus, taking the flow profile into account appears to be mandatory in the context of large duct sections. Note also that these are ideal cases without added noise or calibration errors. This is quite striking that, even here, such large differences can be observed between the performance of both eduction methods.

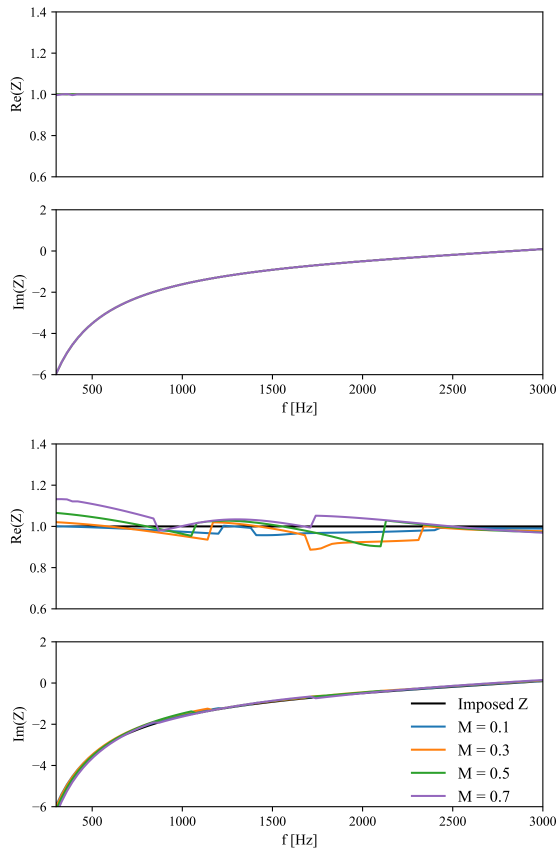

Figure 2. Impedance eduction results without noise using the uniform (top) and shear (bottom) flow assumption, for different flow velocity. The waves and the flow are in the same direction. Black solid line: imposed impedance.

4. CONCLUSION

In this work, the effects of shear flow on direct impedance eduction was studied numerically in a 2D duct. It has been shown that different results are obtained if the flow profile is considered or not. These discrepancies become significant for large ducts with many modes propagating, and even more if waves are against the flow. In these cases, it means that impedance calculated with Ingard-Myers

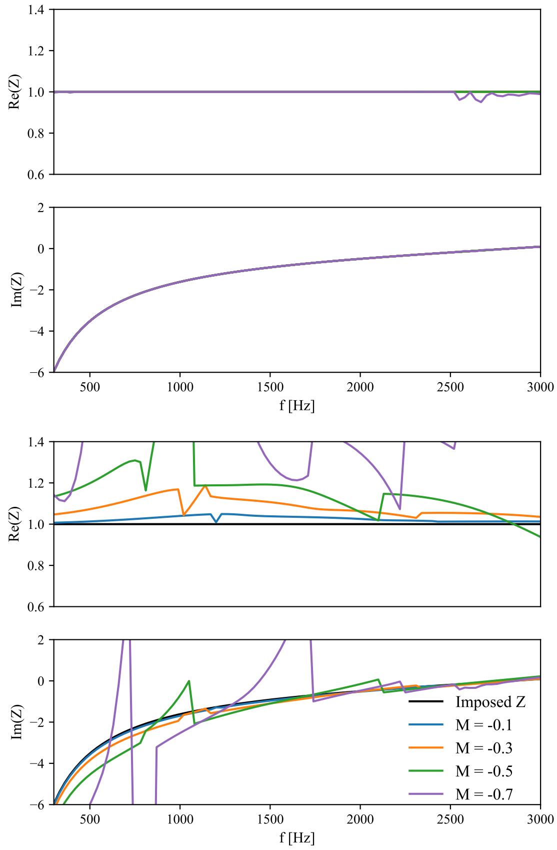

Figure 3. Impedance eduction results without noise using the uniform (top) and shear (bottom) flow assumption, for different flow velocity. The waves and the flow are in opposite direction. Black solid line: imposed impedance.

boundary condition in an experiment will deviate from the actual value of the liner impedance. For future works, the method described here will be applied to MAINE Flow for multiple liners and considering different incident modes and propagation directions.

5. ACKNOWLEDGEMENT

The authors acknowledge the financial support from the ANR industrial chair ASTRAL (ANR-20-CHIN-0005) and from Safran Aircraft Engines.

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