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DEVELOPING A FINITE ELEMENT MODEL FOR THE STUDY OF AN EXPERIMENTAL BATCH OF VIOLINS

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ABSTRACT

During the Bilbao project, six violins were carefully built in order to investigate the influence of the plate thickness on the dynamics, sound and playing characteristics of the complete instruments. To this end, three violins with medium sized backs, each paired with a thin, normal, or thick top and three with medium tops, each paired with a thin, medium, or thick back. Despite careful control and reduction of the influence of handwork by CNC cutting the outside of the plates, there are various sources of variations among the parts, in particular in the wood properties. In order to separate the effect of the intentional thickness variations from irreducible natural variability in wood properties and geometric tolerance, a complete finite element model is being developed using COMSOL software. This model takes into account the geometry of the Bilbao project violins while the wood properties are obtained by optimising the numerical vibratory behavior to match the experimental modal data of the free plates, just after they were CNC routed. This model will then be used to quantify the respective effects of random variations on the vibratory behavior, which could be very useful for violin makers who work by hand with a highly variable material.

Keywords: *FEM, Violin, Wood properties.*

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1. INTRODUCTION

Exploring the links between the sound and playing characteristics of finished violins and the properties of their constituents has been the focus of many studies led by violin makers and scientists since the pioneering work of Carleen Hutchins and Frederik Sanders in the 1950s [1]. Finding correlations between a maker choice and the response of the instrument is particularly challenging because violins are traditionally made by hand in a very variable material (spruce for the top, maple for the back), and assembled with glue.

In such context, the Bilbao project aimed at exploring the influence of the thickness of the plates (top and back) only by using automated carving process with CNC router and controlling as carefully as possible the properties of wood used (from the same tree), in order to minimize the various sources of variations [2]. Six violins were built: three instruments with medium backs, each paired with a considered thin, medium, or thick top; similarly, three violins with medium tops, each paired with considered thin, medium, or thick backs.

However, despite this careful control there are still many sources of variations among the violins, in particular in the wood properties and assembly process. In order to separate the effect of the desired thickness variations from other undesired variations, a complete finite element model (FEM) is being developed using the COMSOL software. FEM has been more and more used in the recent years to investigate the effects of different design choices on the violin's acoustic and vibrational properties [3–6], but usually on a simplified violin or on a specific instrument which is not fully characterized. For the Bilbao project violins, the outside geometry is very accurately controlled and repeated (as being CNC routed) and modal analysis has been conducted on the plates, body and full violins at different stages during the construction process, which thus offers an unprecedented opportunity to compare the results of FEM simulations with measurements. This paper presents the first step of building the FE model of the Bilbao violin: a FE model of the top plate, just after it was cut by the CNC (without major maker intervention).

2. MATERIAL AND METHODS

2.1 Model description: geometry

A computer-controlled router was used to replicate the arching design of the Huberman violin, an esteemed instrument crafted by Antonio Stradivari in 1713, on the outside (see Fig. 1) and the plate was cut with a uniform thickness of 3.5 mm. The advantage is that we could import the geometry file used for CNC routing the plates in COMSOL and thus ensure that geometry of the simulated violin is identical to the geometry of the real violins within the accuracy of a CNC router.

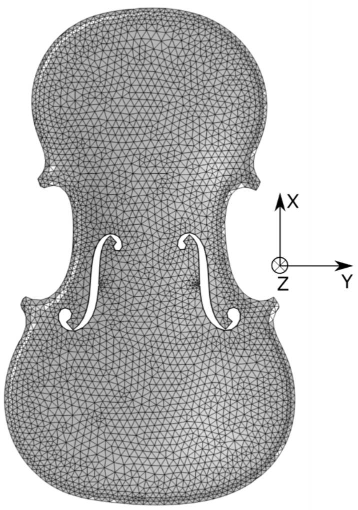

Figure 1: Bilbao project's violin top plate meshed with tetrahedral elements using COMSOL meshing tool.

2.2 Wood properties

It is challenging to determine the properties of wood without destructively testing it [7]. Furthermore, analyzing the properties of violin wood presents difficulties due to its orthotropic nature, which results in varying behaviors along different axes and evolution of properties following hygrometry. In addition, the material can exhibit local variations in density and rigidities, following annual rings (for temperate species), fiber orientation and ordering of the different cells (mainly tracheids for the wood used for top plates). The Bilbao team conducted two types of measurements to determine the density of the six spruce, *Picea abies*, quarters that were used to make the six plates. The first measurement was geometric, by measuring the dimensions of the wedge (and thus estimating the volume) and the weight after the wedge was kept in a climatic chamber for a long time to ensure stability. The second measurement was done by the so-called water displacement method. Both methods gave similar results, which are provided in Tab. 1 for the six tops, in the "measured" column. However, these values deviate largely from the densities that we estimated by dividing the weight of the newly cut plates (as measured by the Bilbao team) by the volume of the object defined in the geometry file, with a mean relative variation of 8.3 %.

Table 1: Measured, estimated and relative variation of the densities of the six top plates.

Top	Measured ρ [kg.m^{-3}]	Estimated ρ [kg.m^{-3}]	Rel. variation [-] (%)
A	380	409	7.6
B	390	417	6.9
C	370	415	12.2
D	370	404	9.2
E	360	388	7.8
F	370	392	5.9
Mean	373	404	8.3

The Bilbao team measured as well the sound velocity in both the longitudinal (along the X axis) and radial (along the Y axis) directions of the cut plates using a deviced called Lucchimeter [8]. However, we assume that these measurements are not very accurate and the relationship between the sound velocities and the Young's moduli in the longitudinal ($E_L = E_X$) and radial ($E_R = E_Y$) directions is not straightforward for an arched thin plate.

Therefore, while the geometry of the plates is accurately known, the wood properties are largely unknown and we propose to estimate them by optimizing the first six simulated eigenfrequencies so they match as closely as possible the measured ones.

3. COST FUNCTION

Figure 2 (a) shows the target of the optimisation: the first six eigenfrequencies, along with the modal shapes measured by the Bilbao team on top A.

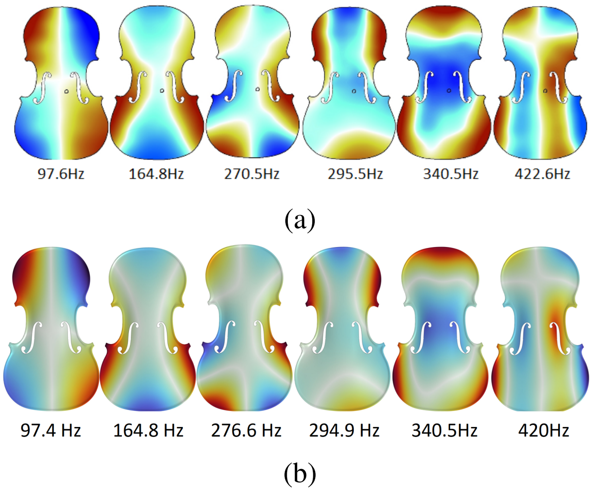

Figure 2: (a) The first six modes and corresponding frequencies measured by the Bilbao team on top A. (b) The first six simulated modes and associated frequencies obtained for top A after optimisation (see section 4.3). The displacement along the Z axis is displayed.

The optimisation process aims to minimise the error, as defined in Eqn. (1), by adjusting the material parameters. In this equation, the error represents the average relative discrepancy between the simulated frequencies (F_{sim}) and the experimental frequencies (F_{exp}) over the first $n = 6$ modes.It quantifies how closely the simulated frequencies match the experimental ones.

$$\text{error} = \frac{100}{n} \sum_{i=1}^{n} \frac{|f_{\text{exp},i} - f_{\text{sim},i}|}{f_{\text{exp},i}} \quad (1)$$

The modal shapes were used to check that there is no mode switch, thus ensuring that the frequencies (simulated and measured) that are compared correspond to the same modes.
Experimentally, modes up to 3000Hz could be identified but we only focus on the first six ones as they allow the identification of most elastics constants (see section 4.1 and Fig. 3) without making the optimisation too cumbersome (with an increase of the risk of mode inversions and more local minima).

4. DETERMINATION OF THE MATERIAL PARAMETERS

4.1 Sensitivity analysis

Only the wood properties that are influential on the first eigenfrequencies can be obtained by optimisation in a reliable way. Therefore, a sensitivity analysis has been conducted to determine the influence of each of the ten wood parameters on the six first frequencies. This was done by varying each parameter by 1% and determining, for each mode, the induced relative variation of the eigenfrequency. Then, for each mode, these 10 relative variations were normalised so their sum equalled 1. The resulting values are shown in Fig. 3.
Based on this analysis, we can see that the influence of E_T, G_{TL} and the three Poisson's ratios is very weak, and thus considered as non influential. Therefore, they will not be included in the optimisation. The three Poisson's ratios and E_T will be given nominal values of: $\nu_L = \nu_R = \nu_T = 0.3$ and $E_T = 520$ MPa [9]. As G_{TL} does not vary independently of the density ρ, it will be estimated when the density is fixed (see section 4.2) using the law proposed by Guitard [10] and corrected by Viala [6] for spruce tonewood:

$$G_{TL} = 840 + 1.93(\rho - \rho_0) \quad [\text{MPa}], \quad (2)$$

where $\rho_0 = 450$ kg.m^{-3}. (The constant in front of the brackets has therefore units of MPa.m^3.kg^{-1}.) This law and the following (see Eqns. (3)-(6)) were specifically derived for spruce tonewood as previous laws were more general for softwoods and did not take into account the specificity of spruce species and the fact they are selected for instrument making (lack of defects and resin pockets, relatively low angle of the crystalline cellulose microfibrils in the S2 layer of the secondary wall).

4.2 Density evaluation

While, among all wood parameters, the density has the largest influence on the vibratory response of the plate, the

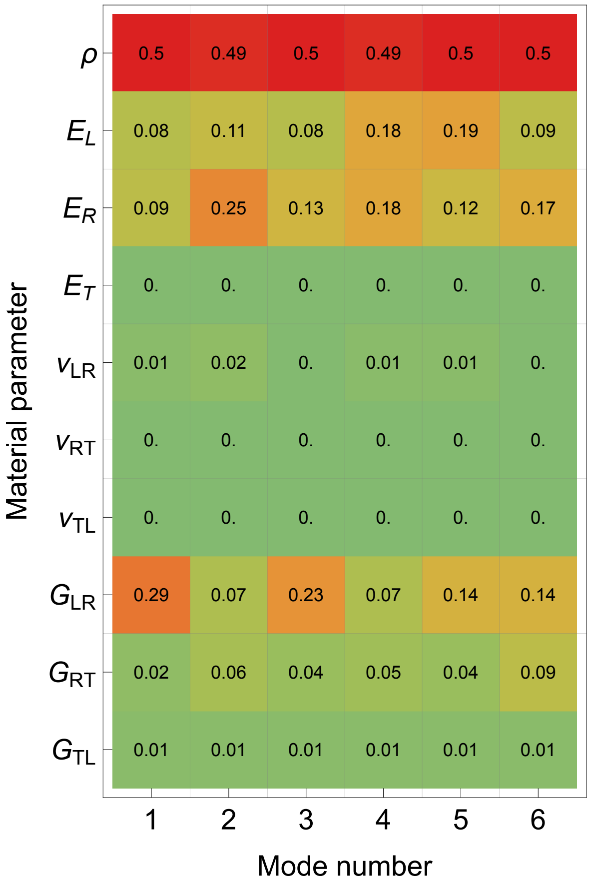

Figure 3: Sensitivity analysis matrix for a 1% variation on each of the parameters (density and nine elastic constants). Results for top C. The values are normalized for each mode. A high value is associated with a large influence.

system is underconstrained: a change in density can easily be compensated by a change in elastic parameters and so a solution can be found by optimisation for any density with a small error. Therefore, the density needs to be fixed *a priori*. To choose between the measured densities and the estimated ones, simulations were performed for each plate and for each of the two densities. The other material parameters were determined (for each density) by using the other laws proposed in [6], giving the elastic constants in MPa:

$$E_L = 13000 + 45(\rho - \rho_0) \tag{3}$$

$$E_R = 1000 + 5.5(\rho - \rho_0) \tag{4}$$

$$G_{LR} = 840 + 1.32(\rho - \rho_0) \tag{5}$$

$$G_{RT} = 48 + 0.018(\rho - \rho_0) \tag{6}$$

For each density, we ran 10 realisations of the simulation with the four elastic constants uniformly distributed within ±10% around the values given by the laws. The relative error (in percentage) between the first six simulated and measured eigenfrequencies, as defined by Eqn. (1), is provided for each top and each of the two densities in Tab. 2, under the format of an average over the 10 realisations along with the standard deviation. Tab. 2 shows that the simulated eigenfrequencies match more closely the measured ones when using the estimated densities. These densities will therefore be used in the rest of the study.

Table 2: Average relative error and standard deviation (over 10 realisations) as defined by Eqn. (1) for the six tops, for both the measured and estimated densities.

Top	Error – measured ρ	Error – estimated ρ
A	4.6 ±1.6%.	1.8±0.8%.
B	2.1 ±0.6%.	2.7±1.4%.
C	6.0 ±1.4%.	1.4±0.6%.
D	4.3 ±1.3%.	2.6±0.4%.
E	4.7 ±0.8%.	2.7±0.4%.
F	5.7 ±0.6%.	3.3±0.7%.

4.3 Identification of the elastic constants E_L, E_R, G_{LR} and G_{RT}

As E_L, E_R, G_{LR} and G_{RT} were found to be moderately to strongly influential, these four parameters were obtained by optimisation using Finite Element model Updating Approach (FEMU) [11]. The optimisation used Matlab's *fminsearch* function, using the error provided in Eqn. (1). The initial values were set to the values obtained with Eqns. (3)-(6) using each plate's estimated density. The values obtained by optimisation for the four elastic constants are provided in Tab. 3. They fall within the range that has been reported for tonewood spruce: 9000-16200 MPa for E_L, 540-112 MPa for E_R, 590-1500 MPa for G_{LR}, and 30 - 45 MPa for G_{RT} [9].
The results of the optimisation process in terms of eigenfrequencies and mode shapes are shown in Fig. 2 (b) for top A. The comparison with the measured data (Fig. 2

Table 3: Elastic parameters (in MPa) obtained by FEMU optimisation for the four constants E_L, E_R, G_{LR} and G_{RT} for the six tops. The average error post optimisation, as calculated with Eqn. (1), is provided in the last column.

Top	E_L	E_R	G_{LR}	G_{RT}	Error
A	9829	888	832	44	0.4%
B	12875	733	714	44	0.4%
C	12415	878	760	34	0.5%
D	10728	825	617	37	1.0%
E	12081	675	588	50	0.3%
F	12417	922	611	39	0.4%

(b)) reveals a strong similarity in frequencies and mode shapes. The Modal Assurance Criterion (MAC) [12] was used to evaluate the resemblance between the vibration modes, consistently yielding an index greater than 0.95 for each mode. These findings indicate a close match between the measured and simulated data, supporting the accuracy of the analysis.

5. CONCLUSIONS

The abundant data from the Bilbao project provides an opportunity to examine the violin making process and, combined with FEM approach, to explore how the construction parameters influence the final behavior of the instrument. In this article we have focused on estimating, for the six top plates, the material properties by optimising them so the first six simulated mode frequencies match as closely as possible the measured frequencies at the first construction step, i.e. after the plates had just been cut with a CNC machine. The optimised elastic constants fell within the ranges reported in the literature for tonewood spruce. However, despite all the care taken by the Bilbao team in selecting the six wedges as similar as possible, using pieces taken from the same tree at the same height, there is still a significant variation in the elastic constants between the six plates. This had already been reported in the literature [13] and shows the difficulty for instrument makers to replicate instruments while working with such variable material.
In the near future, the same methodology will be applied to the back plates. Once the wood properties of the top and back plates are known, a model of the graduated plates with different thicknesses will be created and compared with the experimental data for the whole construction process, from the free plates to the complete instruments.

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