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AN EFFECTIVE MEDIUM MODEL FOR SOUND INSULATION ANALYSIS OF (IN)FINITE MULTI-MODAL VIBROACOUSTIC METAMATERIALS

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ABSTRACT

Vibroacoustic metamaterials are used to improve the sound insulation of a host panel by periodically attaching local resonators. While most vibroacoustic metamaterials exploit single-mode resonators for narrowband improvements, the use of multi-modal resonators has recently emerged to extend the frequency band of effectiveness. However, significant modelling and computational challenges need to be tackled when designing adequate resonator layouts that exploit multiple modes to influence broadband sound transmission. In this work, we show that analytical effective medium models can accurately predict the performance of vibroacoustic metamaterials, simply requiring the extraction of modal parameters from the analysis of the fixed-based resonator. Effective medium models are used to analyze diffuse field sound transmission loss both for infinite structure dimensions, through analytical formulas, and for finite size, through combination with the hybrid deterministic – Statistical Energy Analysis. The accuracy of the proposed prediction methodologies is validated for two different multi-modal metamaterial layouts, that target the suppression of the broadband insulation dip due to coincidence in orthotropic host panels.

Keywords: *vibroacoustic metamaterials, effective medium model, sound insulation*

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1. INTRODUCTION

Vibracoustic metamaterials (VMs) can be used to improve the sound insulation of a host panel. By distributing small resonators on a subwavelength scale, resonance-based bandgaps and sound insulation improvements are obtained close to the resonance frequency of the local resonators. Most VM panels employ only single-mode translational resonators and provide only narrowband insulation improvements. The potential of rotational and multi-modal resonators in broadening the frequency band in which VMs are effective has been recently demonstrated [1]. However, significant modelling challenges need to be tackled when designing adequate resonator layouts that exploit multiple modes to influence the broadband sound transmission of the panel.

The vibroacoustic properties of VM panels are usually predicted through a numerical approach that involves unit cell (UC) modelling and periodic structure theory. For VM panels with infinite dimensions, the created bandgaps and the sound transmission loss (STL) can be studied e.g. through the Wave and Finite Element Method (WFEM) by modelling one single unit cell through finite elements (FE) and imposing periodicity conditions at its boundaries [2]. For VM panels with finite dimensions, the boundary conditions of the panel can be taken into account by modifying the periodicity conditions such that they are coherent with the applied constraints at the boundaries of the panel [3]. When considering diffuse sound fields surrounding a finite panel, the hybrid deterministic-statistical energy analysis (det-SEA) framework can be employed to compute the diffuse STL, which represents the average insulation performance of the panel across a wide range of situations (e.g. across an ensemble of possible source and receiver rooms in buildings) [4]. In this case, a deterministic model of the (periodic) panel is combined with a diffuse model of the surrounding sound fields. The computational cost of numerical models based on periodic structure theory is related to the dimension of the UC FE model, and can become significant when including resonators with complex geometrical layouts. For this reason, reduced order UC modelling, through e.g. Craig-Bampton reduction and generalized Bloch mode synthesis [5], is used to speed up calculations.

Besides numerical models based on periodic structure theory, analytical effective medium models are an appealing alternative to reduce computational costs [6]. In this case, the VM panel is studied through its effective dynamic mass density, i.e. as a homogeneous medium with equivalent mass properties that depend on the frequency of analysis. However, conventional effective medium models are developed only for infinite VM panels with translational single-degree-of-freedom resonators. In this work, effective medium models are developed to consider more complex distributed resonators, exhibiting multiple modes with both translational and rotational motion. Also, a methodology to compute the sound insulation of finite-sized VM panels, by including the effective medium model in the det-SEA framework, is described.

The effective dynamic mass density of VM panels with multi-modal resonators is first derived in Section 2. In Section 3, we describe the methodology to employ the effective medium model in the STL prediction of VM panels with both infinite dimensions, through analytical formulas, and finite dimensions, by including the effective medium model in the det-SEA framework. The proposed methodology is then validated against WFEM and det-SEA predictions that included full (periodic) FE models of the VM panel (Section 4). Finally, we give conclusions and remarks (Section 5).

2. EFFECTIVE MEDIUM MODEL FOR MULTI-MODAL VM PANELS

In this Section, the proposed effective medium model for multi-modal VM panels is described. The VM panel is studied through its effective dynamic mass density, i.e. as a homogeneous medium with equivalent mass properties that depend on the frequency of analysis. The expression of the effective dynamic mass density is derived by combining the inertial forces of the host panel with the inertial forces and bending moments transmitted to the panel by the local resonators [1].

The dynamic effective mass density of VMs with attached translational and rotational resonators can be found by considering single-degree-of-freedom (SDOF) resonators in each periodic unit cell (UC) (Figure 1). The translational and rotational inertia of the SDOF resonators are indicated as m_z and J_ry, and the related stiffnesses as k_z and k_ry. If the size of the UC is $L_\mathrm{x} \times L_\mathrm{y}$, and $n = 1/(L_\mathrm{x} L_\mathrm{y})$, the dynamic effective mass densities of VM panels with translational resonators and rotational resonators around the y- and the x-axis are, respectively [1]:

$$\begin{aligned}
\rho_\mathrm{eff,z} &= \rho + \frac{n k_\mathrm{z} m_\mathrm{z}}{h\left(k_\mathrm{z} - m_\mathrm{z}\omega^2\right)} \\
\rho_\mathrm{eff,ry} &= \rho + \frac{n k_\mathrm{x}^2 k_\mathrm{ry} J_\mathrm{ry}}{h\left(k_\mathrm{ry} - J_\mathrm{ry}\omega^2\right)} \\
\rho_\mathrm{eff,rx} &= \rho + \frac{n k_\mathrm{y}^2 k_\mathrm{rx} J_\mathrm{rx}}{h\left(k_\mathrm{rx} - J_\mathrm{rx}\omega^2\right)}
\end{aligned} \tag{1}$$

For VM panels with SDOF translational resonators, the effective dynamic mass density depends only on the frequency of analysis. For VM panels with SDOF rotational resonators, the effective dynamic mass density depends also on the wavenumber components k_x and k_y of the waves propagating in the panel.

Resonators with complex geometrical layouts cannot always be treated as SDOF systems. They often exhibit multiple modes, each one associated with both translational and rotational inertial contributions which need to be properly considered in the derivation of the effective dynamic mass density. Here we consider the translational and rotational contributions of each mode k in terms of its modal effective masses $m_{i,k}^\mathrm{eff}$ along the kinematic direction i = z, θ_x, θ_y. The modal effective masses can be found e.g. after discretizing the resonator layout by finite elements and computing its first n_m fixed-base modes [1]. The modal effective masses are then included in the effective dynamic mass density of the VM panel as:

$$\begin{aligned}
\rho_\mathrm{eff} =& \rho\alpha_\mathrm{static} + \sum_{k=1}^{n_\mathrm{m}} \frac{n}{h} \left(\frac{k_{\mathrm{z},k}^\mathrm{eff} m_{\mathrm{z},k}^\mathrm{eff}}{k_{\mathrm{z},k}^\mathrm{eff} - m_{\mathrm{z},k}^\mathrm{eff}\omega^2} \right. \\
&\left. + \frac{k_\mathrm{y}^2 k_{\theta_\mathrm{x},k}^\mathrm{eff} m_{\theta_\mathrm{x},k}^\mathrm{eff}}{k_{\theta_\mathrm{x},k}^\mathrm{eff} - m_{\theta_\mathrm{x},k}^\mathrm{eff}\omega^2} + \frac{k_\mathrm{x}^2 k_{\theta_\mathrm{y},k}^\mathrm{eff} m_{\theta_\mathrm{y},k}^\mathrm{eff}}{k_{\theta_\mathrm{y},k}^\mathrm{eff} - m_{\theta_\mathrm{y},k}^\mathrm{eff}\omega^2} \right)
\end{aligned} \tag{2}$$

where $\alpha_\mathrm{static} = 1 + m_\mathrm{ratio} - \sum_{k=1}^{n_\mathrm{m}} \frac{m_{\mathrm{z},k}^\mathrm{eff}}{m_\mathrm{UC}}$ includes the quasi-static contributions of higher order modes, and $k_{i,k}^\mathrm{eff} = m_{i,k}^\mathrm{eff}\omega_{\mathrm{r},k}^2$ are the modal effective stiffnesses. m_ratio is the ratio between the resonator mass and the mass of the host structure $m_\mathrm{UC} = \rho h L_\mathrm{x} L_\mathrm{y}$ within the UC.

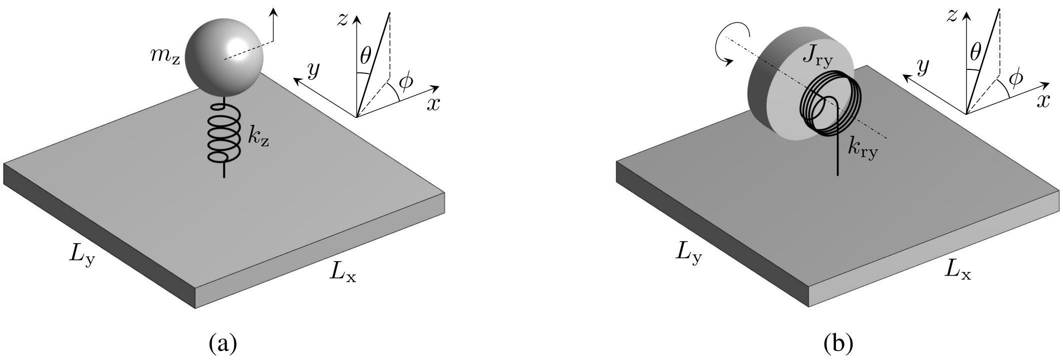


Figure 1: Scheme of the VM panel UC with attached (a) translational and (b) rotational single-degree-of-freedom resonators [1]

3. STL PREDICTIONS FOR VM PANELS WITH INFINITE AND FINITE SIZE

In this Section, methodologies are presented to employ the proposed effective medium model in the STL prediction of VM panels with both infinite and finite size.

3.1 VM panels with infinite size

The STL of VM panels with infinite size can be predicted through simple analytical formulas, by expressing the impedance of the panel in terms of its effective dynamic mass density.
For a plane incident sound wave with propagation direction defined by angles (ϕ, θ) (Figure 1), a bending wave in the panel is induced with trace wavenumber $k = \frac{\omega}{c} \sin \theta$, and its projections along the x and y directions $k_x = k \cos \phi$ and $k_y = k \sin \phi$. The STL R of the VM panel for a diffuse incident sound field can be analytically computed by integrating the directional transmission coefficient $\tau(\phi, \theta)$ [7]:

$$R = -10 \log_{10} \tau_d$$
$$\tau_d = \frac{\int_0^{2\pi} \int_0^{\pi/2} \tau(\phi, \theta) \sin \theta \cos \theta \mathrm{d}\theta \mathrm{d}\phi}{\int_0^{2\pi} \int_0^{\pi/2} \sin \theta \cos \theta \mathrm{d}\theta \mathrm{d}\phi}$$
$$\tau(\phi, \theta) = \left| \frac{2\rho_0 c/ \cos \theta}{2\rho_0 c/ \cos \theta - (\mathrm{j}/\omega)(D(\phi)k^4 - \rho_{\mathrm{eff}} h \omega^2)} \right|^2 \tag{3}$$

where h is the thickness of the host panel, c = 341 m/s and ρ_0 = 1.21 kg/m^3 are the speed of sound and the mass density in the surrounding air. When considering the general case of an orthotropic host panel whose principal material directions coincide with the coordinate axes, $D(\phi) = D_x \cos^4 \phi + 2H \cos^2 \phi \sin^2 \phi + D_y \sin^4 \phi$ is the bending stiffness of the panel along the direction ϕ, which depends on the bending stiffnesses D_x and D_y along the directions x and y, and on the torsional stiffness H.

3.2 VM panels with finite size

The STL of VM panels with finite size can be predicted by including the effective medium model of the panel in the det-SEA framework. In this framework, the finite-sized VM panel is embedded in a planar baffle, while the sound fields in the source room (1) and receiver room (2) are still taken to be diffuse, as for the finite panel [4].

The diffuse field sound transmission coefficient is evaluated by considering the interaction between the panel and the direct field response of the rooms, i.e. the sound fields that occur if the rooms behave as acoustic half-spaces. This interaction is expressed through the direct field dynamic stiffness matrices $\boldsymbol{D}_{d1}$ and $\boldsymbol{D}_{d2}$, for room 1 and room 2 respectively, which can be computed numerically via approximation of the Rayleigh integral, e.g. using a wavelet discretization of the baffled interface [8]. The direct field dynamic stiffness matrices are summed to the dynamic stiffness matrix of the panel $\boldsymbol{D}$ to obtain the total dynamic stiffness matrix $\boldsymbol{D}_t$:

$$\boldsymbol{D}_t := \boldsymbol{D} + \boldsymbol{D}_{d1} + \boldsymbol{D}_{d2} \tag{4}$$

The diffuse field sound transmission coefficient is finally evaluated as:

$$\tau_d = \frac{16\pi c^2}{L_{px} L_{py} \omega^2} \sum_{rs} \mathrm{Im}(\boldsymbol{D}_{d2})_{rs} (\boldsymbol{D}_t^{-1} \mathrm{Im}(\boldsymbol{D}_{d1}) \boldsymbol{D}_t^{-H})_{rs} \tag{5}$$

where L_{px}, L_{py} are the planar dimensions of the panel.

The dynamic stiffness matrix $\boldsymbol{D}$ of the VM panel is usually obtained through finite element schematization. However, when considering distributed resonators with complex geometrical layouts, the size of the finite element model and the computational cost of the associated calculations increase significantly. In this work, we instead propose to employ the analytical effective medium model to obtain the dynamic stiffness matrix of the VM panel. Assuming simply supported rectangular panels, the dynamic stiffness matrix is expressed with respect to the modal coordinates of the bare orthotropic panel without attached resonators, i.e. with respect to the following basis functions:

$$\begin{aligned} &\phi_j(x,y) = \frac{2}{\sqrt{\rho h}}\sin(k_{\mathrm{x},j}x)\sin(k_{\mathrm{y},j}y) \\ &k_{\mathrm{x},j} = \frac{n_{\mathrm{x},j}\pi}{L_{\mathrm{px}}}, \quad k_{\mathrm{y},j} = \frac{n_{\mathrm{y},j}\pi}{L_{\mathrm{py}}} \end{aligned} \tag{6}$$

The natural frequency associated with each mode shape $\phi_j(x,y)$ of the bare panel is:

$$\omega_j = \sqrt{\frac{1}{\rho h^2}\left(D_{\mathrm{x}}k_{\mathrm{x},j}^4 + 2Hk_{\mathrm{x},j}^2k_{\mathrm{y},j}^2 + D_{\mathrm{y}}k_{\mathrm{y},j}^4\right)} \tag{7}$$

The dynamic stiffness matrix of the VM panel is obtained from the equivalent diagonal modal mass matrix $\boldsymbol{M}$ and the diagonal modal stiffness matrix $\boldsymbol{K}$:

$$\begin{aligned} &\boldsymbol{D} = -\omega^2\boldsymbol{M} + \boldsymbol{K} \\ &\boldsymbol{M} = \mathrm{diag}\left(\frac{\rho_{\mathrm{eff},1}}{\rho}, \frac{\rho_{\mathrm{eff},2}}{\rho}, ..., \frac{\rho_{\mathrm{eff,n}}}{\rho}\right) \\ &\boldsymbol{K} = \mathrm{diag}\left(\omega_1^2, \omega_2^2, ..., \omega_n^2\right) \end{aligned} \tag{8}$$

The modal mass contributions include the modal dynamic effective mass densities, which depend on the frequency of analysis and on the modal wavenumbers $k_{\mathrm{x},j}$ and $k_{\mathrm{y},j}$:

$$\begin{aligned} \frac{\rho_{\mathrm{eff},j}}{\rho} =& \alpha_{\mathrm{static}} + \sum_{k=1}^{n_{\mathrm{m}}}\frac{n}{h\rho}\left(\frac{k_{\mathrm{z},k}m_{\mathrm{z},k}}{k_{\mathrm{z},k} - m_{\mathrm{z},k}\omega^2}\right. \\ &\left. + \frac{k_{\mathrm{y},j}^2 k_{\mathrm{rx},k}J_{\mathrm{rx},k}}{k_{\mathrm{rx},k} - J_{\mathrm{rx},k}\omega^2} + \frac{k_{\mathrm{x},j}^2 k_{\mathrm{ry},k}J_{\mathrm{ry},k}}{k_{\mathrm{ry},k} - J_{\mathrm{ry},k}\omega^2}\right) \end{aligned} \tag{9}$$

4. VERIFICATION OF THE PROPOSED EFFECTIVE MEDIUM MODEL

The proposed effective medium model is validated for the STL prediction of the VM panels in Figure 2, consisting of an orthotropic host panel (h = 15 mm, ρ = 1200 kg/m^3, D_{x} = 1500 Nm, D_{y} = 4500 Nm, H = 2598 Nm, η_0 = 0.05) with distributed multi-modal resonators in polymethyl methacrylate (PMMA, E = 4850 MPa, ν = 0.31, ρ = 1200 kg/m^3, η = 0.05). The first "double translational" resonator (Figure 2a) consists of two horizontal cantilever beams with attached end-point masses. The two main modes of interest are related to the vertical translational motion of the masses, while two secondary modes are related to the rotation of the masses. The second "double rotational" resonator (Figure 2b) consists of a mass suspended by a vertical beam, and the two main modes of interest are related to rotations of the mass around the x and y axes. While the orthotropic host panel exhibits a broadband STL dip due to coincidence effects in the range 1200-2100 Hz, the multiple resonator modes of the VM panels are tuned to suppress the STL dip of the infinite host panel by introducing corresponding STL peaks [1].

The validation is performed for VM panels with both infinite and finite dimensions. For infinite panel dimensions (Figure 3), the proposed effective medium model is validated against WFEM predictions. For finite panel dimensions (Figures 4 and 5), the use of the proposed effective medium model of the VM panel in det-SEA predictions is validated against the use of the full FE model. When reducing the dimensions of the panel, from infinite size to 150 cm $\times$ 125 cm and 60 cm $\times$ 45 cm, the effective medium model is able to accurately capture the increasing influence of the modal behaviour in the STL curves, along with the preserved improvements introduced by the VM within the broadband coincidence dip of the orthotropic host panel.

5. CONCLUSIONS

In this work, an analytical effective medium model has been developed to analyze the sound insulation of multi-modal vibroacoustic metamaterial panels, including the effect of multiple resonator modes with both translational and rotational contributions. The model has been formulated in terms of a dynamic effective mass density that depends on the frequency of analysis and on the wavenumbers of the waves propagating in the panel. This allows predicting the sound insulation of metamaterial panels with infinite dimensions by analytical formulas, while finite dimensions can be considered by including the effective medium model in

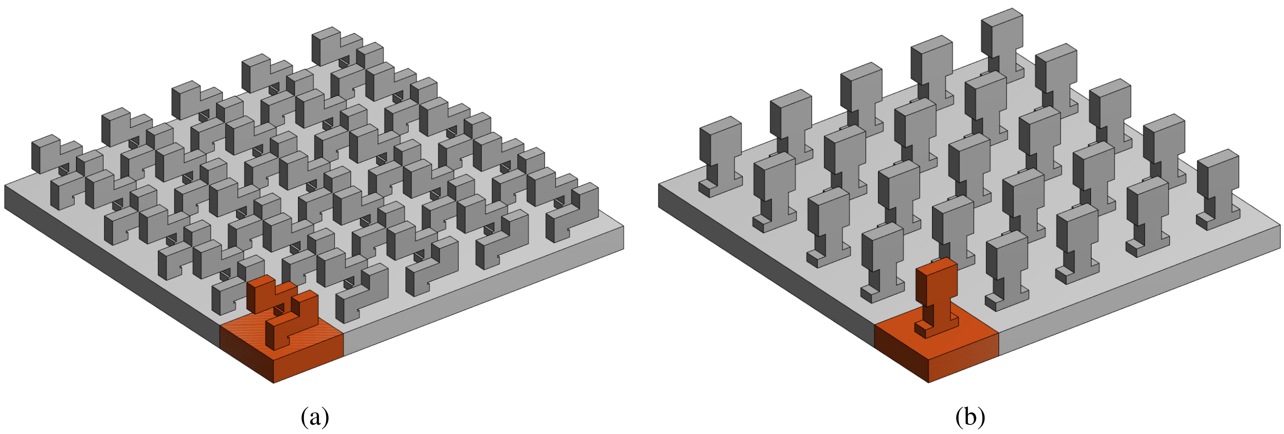


Figure 2: Considered multi-modal VM panels for the validation of the effective medium model: UC layouts with (a) double translational and (b) double rotational resonators.

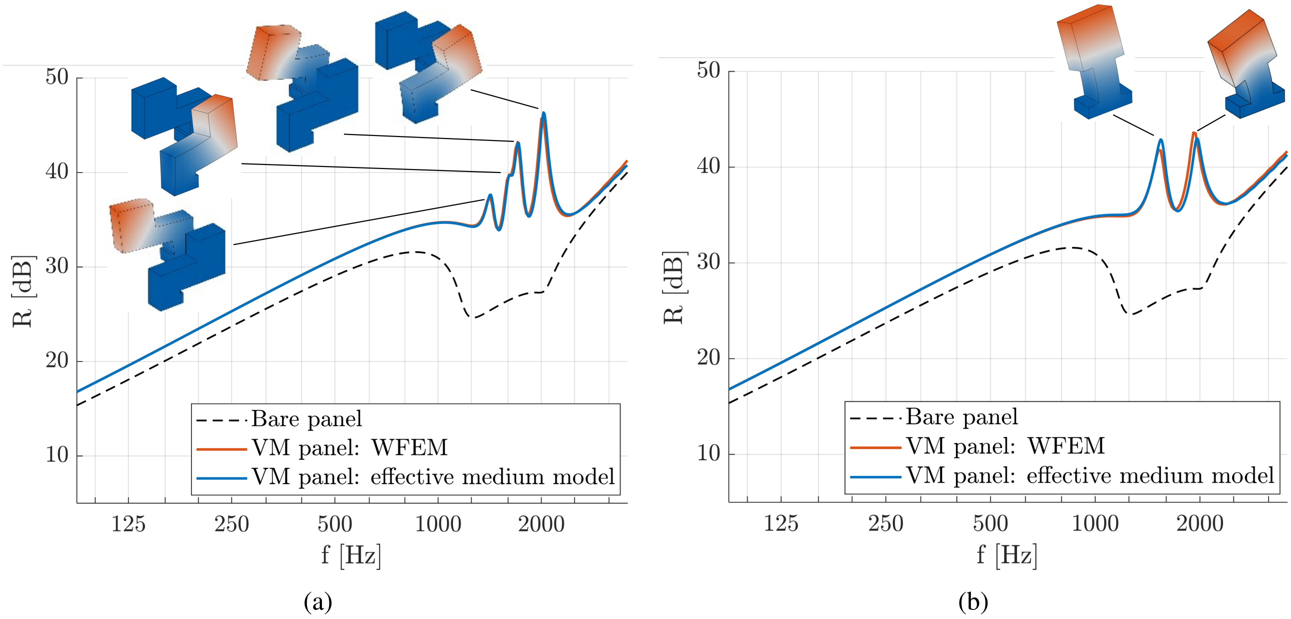
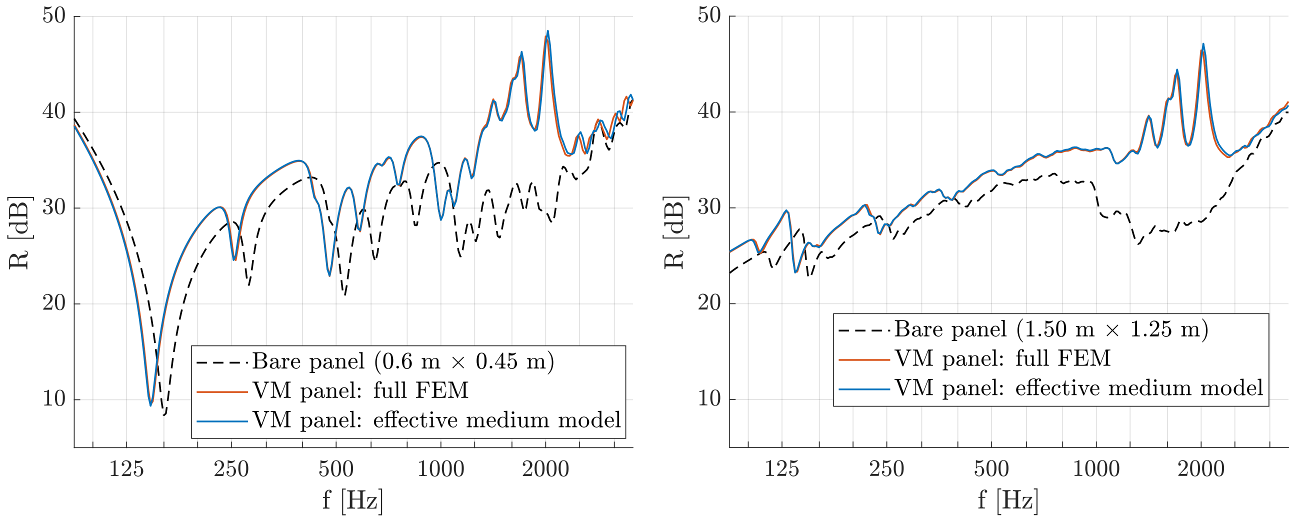
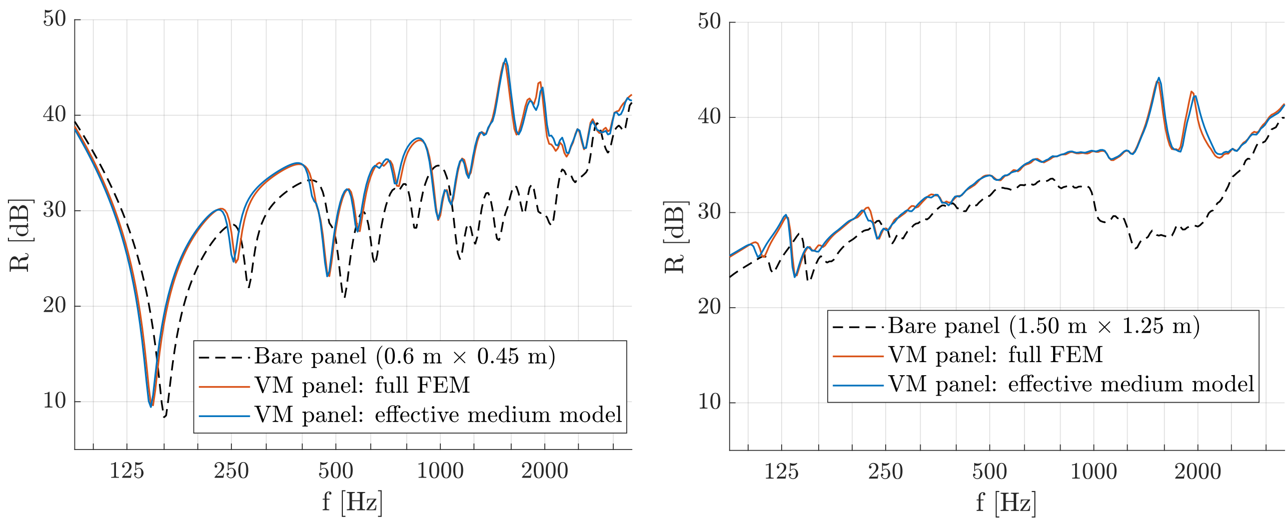


Figure 3: STL predictions for infinite VM panels dimensions: (a) VM panel with double translational resonators, (b) VM panel with double rotational resonators.



(a) (b)

Figure 4: Validation of the proposed effective medium model: det-SEA STL prediction for a rectangular VM panel with double translational resonators. The considered dimensions of the panel are (a) 60 cm × 45 cm and (b) 150 cm × 125 cm.



(a) (b)

Figure 5: Validation of the proposed effective medium model: det-SEA STL prediction for a rectangular VM panel with double rotational resonators. The considered dimensions of the panel are (a) 60 cm × 45 cm and (b) 150 cm × 125 cm.

the deterministic-statistical energy analysis framework. The accuracy of the presented prediction methodologies has been validated against numerical methods based on (periodic) finite element modelling. Two showcase metamaterial layouts have been considered, which target the suppression of the broadband sound insulation dip due to coincidence in orthotropic host panels. In both cases, the proposed methodologies are able to accurately predict the STL improvements within the coincidence dip due to the VM resonance peaks, along with the modal behaviour of the finite VM panel.

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