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PERIODIC BOUNDARY CONDITIONS IMPLEMENTATION FOR THE STUDY OF SONIC CRYSTALS

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ABSTRACT

In recent years, the study of metamaterials based on periodic structures such as sonic crystals has been gaining importance. However, their study depends on laboratory conditions that are impossible to achieve in practice. Therefore, the use of numerical simulations is required to study the sound propagation within these structures. In order to achieve an adequate depth of study, it is necessary to study the performance of this type of barriers at any incidence angle of an acoustic wave. In order to reduce the computational cost of the simulations, different approaches have been developed to implement periodic boundary conditions that simulate an infinite domain in which waves can propagate at a specific angle of incidence. In this work, different methods applied to the acoustic analysis of metamaterials will be studied using the finite difference method. Their suitability, the accuracy of their results and the parameters necessary for their correct performance will be compared in order to obtain the parameters that define the real behavior of a sonic crystal.

.**Keywords:** *Periodic boundary conditions, FDTD, Sonic Crystals.*

1. INTRODUCTION

The study of acoustic materials under perfectly diffuse field conditions is not possible to perform with the current means at our disposal [1]. That's the reason why computational simulations are used to obtain the acoustic properties under ideal conditions.

The finite differences method in time domain (FDTD) provides a time-dependent response from which multiple data can be obtained [2,3]. In this paper we are going to focus on obtaining the acoustic transmission coefficient provided by acoustic barriers, used for the calculation of the sound reduction index, R[4].

This simulation method has been widely used to obtain the characteristics of periodic materials, such as sonic crystals noise barriers (SCNB) [5], with good results in cases were the incident wave is normal to the barrier [6].

However, to obtain the behavior under perfectly diffuse field conditions of SCNB it is necessary to implement simulations using periodic boundary conditions. This allows us to obtain the properties of infinite barriers with complex inner sound propagation without having to waste enormous computational resources.

Several methods regarding periodic boundary conditions have been developed for the FDTD method [7].

In this paper we will compare the behavior and accuracy of two of these methods for the obtention of the sound reduction index under ideal diffuse field conditions provided by SCNB in two-dimensional simulations.

2. METHODS FOR THE IMPLEMENTATION OF PBCS

In this section, we introduce two of the most widely used methods for implementing Periodic Boundary Conditions using FDTD. These methods are based on the addition of different delay operations at the boundaries of the model to create a periodicity that simulates an infinitely periodic barrier.

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2.1 Multiple unit cells method

This method, developed for electrodynamics simulations [7, 8] and then applied to acoustic waves propagation, consists of stacking several rows of unit cells on top on each other, combined with absorbing boundaries on the sides and bottom of the model. The row on the top of the model shows a boundary condition that injects a delayed version of the particle velocity present at the lower part of the row. This defined as

$$u_y(x, y = B, t) = u_y(x, y = A, t - y_p sin\theta), \quad (1)$$

where u_y is the particle velocity in the y direction. It is measured at the position corresponding to the beginning of the last row of scatterers, A. Then, a time delay obtained from the width of the row y_p and the angle of incidence θ is applied at the top layer of particle velocities to update the model, creating a periodic boundary condition (see Figure 1).

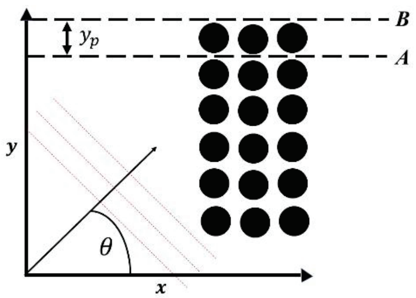

Figure 1. Schematic of the Multiple Unit Cells method for the implementation of PBC.

2.2 Sine-Cosine Method

Originally developed for simulating the propagation of electromagnetic waves [7, 9], this method splits the acoustic field into its real and imaginary parts. The acoustic pressure and velocity at the boundaries are defined applying a spatial delay for each the real and the complex components.
The excitation needed for this method consists of a complex angle- dependent single-frequency plane wave. The real part depends on cos (ωt) while the imaginary part depends on $\sin(\omega t)$.
The main limitation of this method is that only one frequency can be studied at a time, compromising one of the main advantages of the FDTD method. This occurs because the delay added at the boundaries of the model depends on the wavenumber of the injected wave and the angle of incidence.

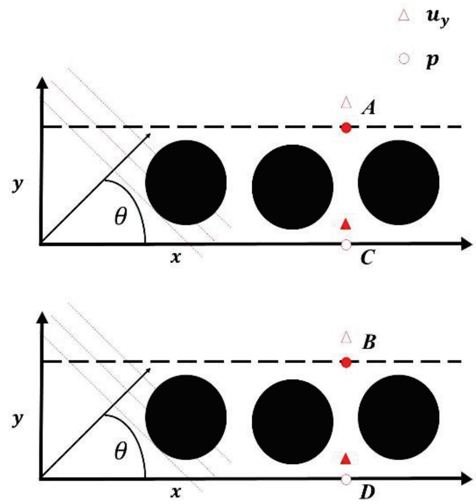

Figure 2. Schematic of the Sine-Cosine method for the implementation of PBC.

The expressions that define this method, using the nomenclature used in Fig.2, are

$$p(x, C) = Re\{[p(x, A) + jp(x, B)]e^{jk_y y_p}\} \quad (2)$$

$$p(x, D) = Im\{[p(x, A) + jp(x, B)]e^{jk_y y_p}\} \quad (3)$$

$$u_y(x, A) = Re\{[\, u_y(x, C) + j\, u_y(x, D)]e^{-jk_y y_p}\} \quad (4)$$

$$u_y(x, B) = Im\{[\, u_y(x, C) + j\, u_y(x, D)]e^{-jk_y y_p}\} \quad (5)$$

where A, B, C and D correspond to the boundary points for the real and the imaginary fields. The acoustic pressure, p, and the particle velocity in the y direction u_y are evaluated at the boundary points and are phase-shifted depending on the propagating wavenumber in the y direction, k_y, and the distance between the boundaries y_p.

3. THE HOMOGENEOUS BARRIER

To check the accuracy of each method in order to obtain the ideal sound reduction index under perfectly diffuse field conditions, we studied the insulation provided by infinitely

long sound barriers with a specific width and a density higher than the airs.
We decided to use this method of comparison since these barriers have an analytical way to obtain the sound transmission coefficient provided [10]. This analytical solution comes defined as

$$\tau(\theta, f) = \frac{1}{1+\frac{1}{4}\left(\frac{Z_2}{Z_1}-\frac{Z_1}{Z_2}\right)sin^2(kLcos(\theta))} \quad , \qquad (6)$$

where Z_1 and Z_2 denotes the air's impedance and the barrier's impedance respectively. This expression allows us to obtain the sound transmission coefficient, τ, for a given frequency, f, with an angle of incidence of a plane wave of θ. The width of the barrier comes defined as L and the propagating wavenumber inside the barrier is defined as k.
With this expression it is possible to obtain the sound transmission coefficient of a simple one-layered barrier analytically.
Since the sound transmission coefficient provided by Eq.(6) depends on the incident angle, we can obtain the sound transmission coefficient under diffuse field conditions by applying an angular average [11,12] defined for the 2-dimensional case as

$$\tau_{ideal}(f) = \frac{\int_0^{\theta max} \tau(\theta, f)\cos(\theta) d\theta}{\int_0^{\theta max} \cos(\theta) d\theta} \quad . \qquad (7)$$

The usual angle range for these results to provide reliable accuracy must be from $\theta = 0º$ to θ_{max} between 70º to 85º [12,13].

4. RESULTS

In this section we present the results of the simulations carried out to obtain the acoustic transmission coefficient under ideal diffuse field conditions of a barrier.
Firstly, we present the results for a homogeneous barrier with a thickness of 0.5m and a density 10 times the air's density. Once we assess the acoustic transmission coefficient with each simulation method and compare the accuracies of each one, we present the results under diffuse field conditions of a SCNB. The SCNB simulated has a Bragg's band-gap tuned at 1000 Hz, which is the most relevant frequency in the traffic noise spectrum, and its scatterer's have a diameter of a 75% of the lattice constant of the crystal.

4.1 Comparison procedure: the homogeneous barrier

For the homogeneous barrier case, the sine-cosine simulations were carried out with a frequency precision of four frequencies per 1/3 octave and an angular precision of 2º from 0º to 70º.
For the Multiple Unit Cells method, the angular precision remains the same, while the excitation is performed using a Ricker's wavelet.
As it can be observed in Fig.3, for the normal incidence case, the results for both the sine-cosine method and the multiple unit cells method are almost identical to those provided by the analytical solution for the transmission coefficient.

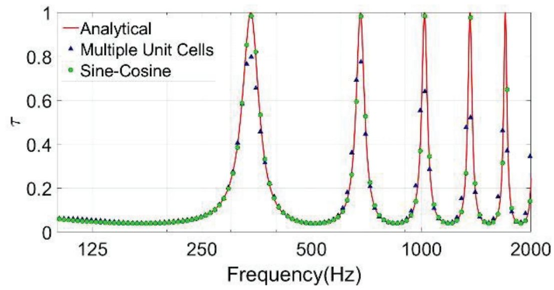

Figure 3. Acoustic transmission coefficient for the normal-incidence case of a 0.5m wide barrier with a density 10 times larger than the airs.

If we increase the angle of incidence, as seen in Fig.4 and Fig.5, we start to observe a mismatch between the results provided by the sine-cosine method and the multiple unit cells method. The sine-cosine method continues to provide highly accurate results even at large angles of incidence. On the other hand, the multiple unit cells method shows a loss in accuracy the larger the angle, showing poor results for angles close to the limit of 70º.

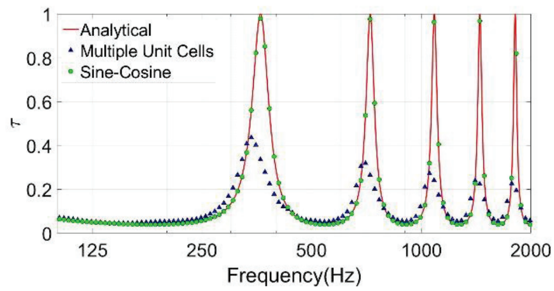

Figure 4. Acoustic transmission coefficient for a wave incident at 20º of a 0.5m wide barrier with a density 10 times larger than the airs.

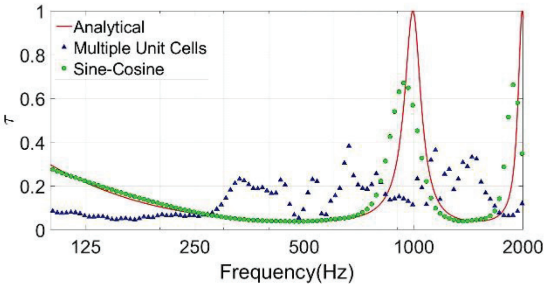

Figure 5. Acoustic transmission coefficient for a wave incident at 70º of a 0.5m wide barrier with a density 10 times larger than the airs.

From the acoustic transmission coefficients obtained for each angle of incidence, we can obtain the ideal sound transmission coefficient under perfectly-diffuse field conditions by applying the aforementioned angular-weighting average formula (see Figure 6).

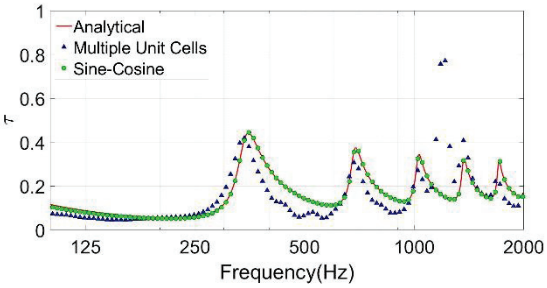

Figure 6. Transmission coefficient under ideal diffuse field conditions of a 0.5m wide barrier with a density 10 times larger than the airs.

We can observe a good accuracy of the sine-cosine method with the analytical results, while the multiple unit-cells method shows a worse performance at frequencies over 300 Hz for this case.

4.2 Accuracy of the sine-cosine method

Once we have checked that the sine-cosine method provides a better approximation to the ideal diffuse field case, we want to further study the accuracy of this method.
For doing so, we extended periodically along the y direction the acoustic pressure field obtained for the two-dimensional simulations. This way it is possible to obtain the complex acoustic pressure and, applying a two-dimensional FFT, we obtain the data corresponding to the frequency components in the acoustic field and its angle of propagation (see Figure 7).

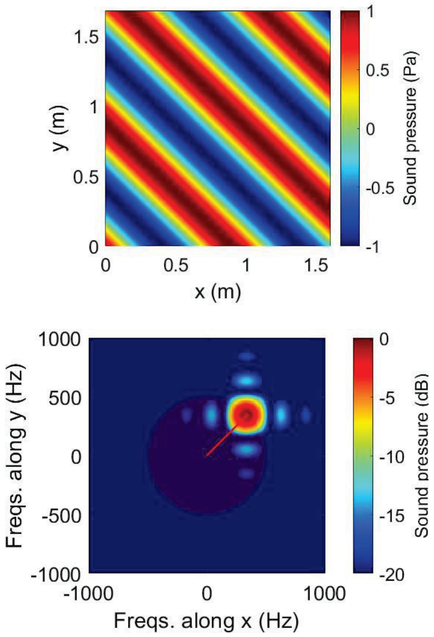

Figure 7. Absolute acoustic pressure (top) and 2-dimensional FFT (bottom) of a 500 Hz plane wave propagating at 45º obtained from the acoustic field. In the 2-D FFT it can be observed that the field has an approximated frequency of 500 Hz (distance to the center point) and an angle near to the 45º degrees specified (deviation from the horizontal).

From this two-dimensional Fourier transformation, we can obtain the perceived propagating frequency and angle in the simulations and compare it with the initial values. As it can be observed in Table 1, the error derived from this method underestimates the obtained frequency by a small factor, while showing a high accuracy angle-wise.

Table 1. Examples of perceived values of frequency and angle of incidence compared to the initial values.

Initial frequency	Initial angle	Perceived frequency	Perceived angle
500 Hz	0º	491 Hz	1º 14’
500 Hz	45º	491 Hz	43º 13’
500 Hz	70º	486 Hz	69º 13’
1000 Hz	0º	993 Hz	0º 37’
1000 Hz	45º	992 Hz	43º 11’
1000 Hz	70º	986 Hz	68º 1’

4.3 Sonic Crystals Noise Barrier

Once we have obtained the results for the homogeneous barrier and compared them with the analytical results, we now present the acoustic transmission coefficient obtained from a SCNB with a Bragg's band-gap centered at 1000 Hz. This band-gap dictates the frequencies at which acoustic waves do not propagate through the barrier due to inner reflections.

As it can be observed in Figure 8, for the case of normal incidence, the band-gap has a very notorious behavior, showing an acoustic transmission coefficient close to zero at the frequencies for which it was designed.

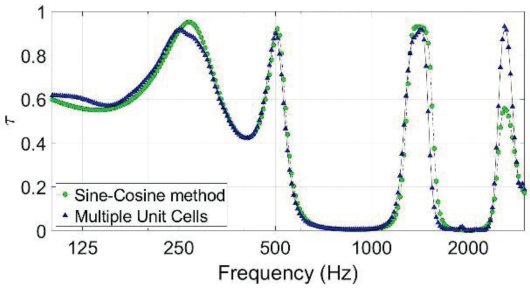

Figure 8. Acoustic transmission coefficient under normal incidence for a SCNB with a scatterer's diameter of 75% the lattice constant.

After running the simulations for each angle of incidence and averaging the results to obtain the ideal acoustic transmission coefficient, the results are shown in Figure 9.

It can be observed that the two methods give similar results for low frequencies, while the Multiple Unit Cells Method shows a lack of stability when evaluating higher frequencies.

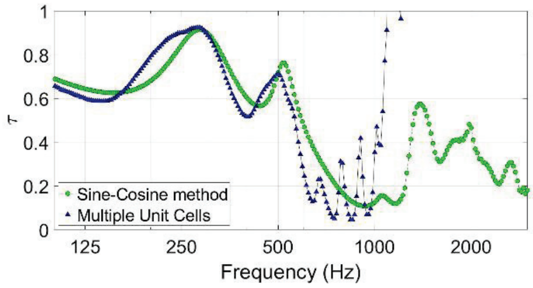

Figure 9. Acoustic transmission coefficient under diffuse field conditions for a SCNB with a scatterer's diameter of 75% the lattice constant.

5. CONCLUSIONS

The goal of this paper was to evaluate two different methods for the implementation of Periodic Boundary Conditions for the study of SCNB under random incidence.

Both methods work properly for small incidence angles. However, the Sine-Cosine method shows a higher accuracy for a larger angular and frequency span.

The main handicap of the sine-cosine method is that each frequency must be evaluated independently, so one of the most powerful advantages of the FDTD simulation technique is not applicable.

For high accuracy simulations, it is recommended to use the sine-cosine method to obtain the acoustic transmission coefficient under ideal diffuse field conditions over the Multiple Unit Cells method.

For smaller angles, the Multiple Unit Cells method can be of use since it allows us to evaluate all the frequency range simultaneously. The Multiple Unit Cells Method can also be of use when the maximum frequency considered is low enough. i.e., lower than 500 Hz (see Figs.6 and 9).

In future works, the acoustic transmission coefficient of more complex periodic metamaterials under diffuse field conditions will be obtained using the PBC methods described in this paper.

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