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DESIGN OF LOW DRAG REACTIVE SILENCERS BASED ON THE BOUND STATES IN THE CONTINUUM

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ABSTRACT

The phenomenon of bound states in the continuum and leaky resonances is employed to design a reactive silencer. It is targeted to prevent a significant spectral line from propagating in a flow-duct while maintaining low pressure drop and installation space (without any lateral extensions). The quasi bound states are achieved by tuning the geometrical parameters of thin plates embedded in a waveguide with dimensions and flow velocities typical for ventilation systems. In general, the procedure works for waveguides with plane wave propagation in the low Mach number regime. An optimization by evolution strategies used for this purpose is described and illustrated on two specific examples.

Keywords: *bound states in the continuum, flow ducts, numerical optimization, reactive silencer*

1. INTRODUCTION

Bound states in the continuum (BICs) are a type of eigensolutions found in wave systems, spanning quantum mechanics, optics, and mechanical waves (see e.g., the review [1]). BICs consists of discrete eigenfrequencies embedded into a continuous spectrum of states. In the context of acoustics, they were first studied by Parker [2, 3] and have since then been explored in-depth with various approaches and mathematical tools (see e.g., [4–13]). Recent experimental verifications [14–16] have confirmed their theoretical potential. In this article, quasi BICs are used to construct a silencer that suppresses a strong spectral line while maintaining favorable conditions for fluid transport and overall geometrical features.

This work aims to address the issue of high sound emission from fans in ventilation and air conditioning systems. The blade passing frequencies (BPF) are percieved as particularly disturbing [17] and can be several dB louder than the machine's broadband noise [18]. While silencers can reduce sound emissions, they often lead to additional pressure loss and high drag coefficients, which limits their use in ventilation systems. Beside that, space consumption and hygienic regulations might also play a role (e.g., foams are resctricted due to possible accumulation of bacteria [19] etc.). Researchers have explored solutions such as micro-perforated sound absorbers and mode filters [20, 21] to overcome these issues. An ideal silencer should be compact, have low drag, and selectively reduce tonality by lowering the BPF without requiring additional space.

2. THEORY

An acoustic waveguide infinite in at least one dimension supports a time-harmonic wave propagation for the whole frequency spectrum. The corresponding time independent wave solution for acoustic pressure $p'(\boldsymbol{x})$ can be found by solving the Helmholtz equation

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$$\nabla^2 p'(\boldsymbol{x}) + \frac{\omega^2}{c_0^2} p'(\boldsymbol{x}) = 0\,, \tag{1}$$

where ω stands for the angular frequency and c_0 is the adiabatic sound speed.

When there is an obstacle or a discontinuity in a waveguide, discrete eigenfrequencies ω_n can be found within the otherwise continuous frequency spectrum of Eq. (1) corresponding to bounded eigensolutions p_n known as the bound states in the continuum (BIC) [1]. We focus on the most common type of BIC, denoted as a quasi BIC or leaky mode, with a frequency of $\omega_n = \omega_{0n} + \mathrm{i}\gamma \in \mathbb{C}$, where ω_{0n} and γ are the real and imaginary parts, respectively, and γ is also known as a leakage rate. This represents a resonance mode with a quality factor of $\omega_{0n}/2\gamma$.

One of the possible procedures of finding the leaky resonances is to solve Eq. (1) at a finite domain with the Dirichlet or Neumann boundary conditions at the surface of the waveguide and additional outgoing wave condition at the boundaries of the chosen domain. As a single leaky resonance exhibits only a relatively thin spectral band, our design aims for placing multiple resonances close together, so their bands would overlap.

In this text a two-dimensional waveguide with three thin curved plates breaking its symmetry is studied.

To identify the leaky eigenstates localized around the plates, we use an optimization algorithm to vary the waveguide's geometry and solve the Helmholtz equation (1) with Neumann boundary conditions on the walls of the duct and the surface of the plates. Outgoing wave boundary conditions are imposed far from the silencer using the form $\frac{\partial p'}{\partial x} \pm \mathrm{i}\frac{\omega}{c_0} p' = 0$.

Making use of the symmetries in the studied geometries, the plates are described as one-dimensional curves. We choose to parametrize the individual plates as the Bézier curves of the 6th order:

$$\mathbf{B}(s) = \sum_{j=0}^{6} \binom{6}{j} (1-s)^{6-j} s^j \mathbf{P}_j\,, \tag{2}$$

where s is a parameter and $\mathbf{P}_i$ are vertices of the control polygon whose coordinates are used as parameters in the optimization.

The cost function we use has three key objectives. Firstly, it takes into account the distance between the detected resonant frequencies and the desired resonant frequencies, ensuring that there is sufficient leakage at the resonances. Secondly, the function seeks to minimize the curvature of the plates for aerodynamic reasons.

The cost function is minimized by employing the covariance matrix adaptation evolution strategy algorithm (CMA-ES, see e.g. [22]). All numerical simulations described below are conducted employing Comsol Multiphysics 5.5. The geometry of the model is controlled via LiveLink using the CMA-ES written in Matlab. The goal is to find groups of closely placed resonances while keeping the streamlined shape of the whole. Therefore the cost function takes into account the accuracy of the real part of the frequency, the leakage rate, and is penalized for the occurrence of high curvature.

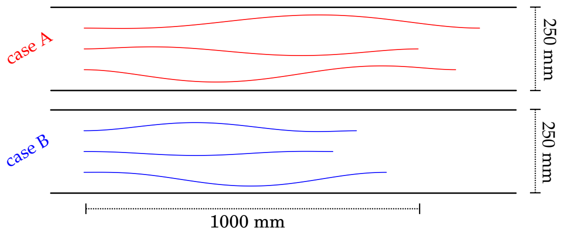

Figure 1. Geometry of the introduced cases.

3. RESULTS & DISCUSSION

To provide concrete examples of the introduced procedure, we present two cases labeled as A and B, with their geometries illustrated in Figure 1. The waveguide height, H, is set at 250 Hz, with a corresponding cut-off frequency of approximately 680 Hz. As such, the targeted frequencies (150 Hz and 200 Hz) are significantly below the cut-off frequencies, and therefore the plane waves far enough from the silencer are justified.

The optimization algorithm used to determine the geometry of each case is based solely on acoustical features. The Helmholtz equation is solved without flow and losses to determine the optimal geometry. To illustrate the functionality of the component, we depict the horizontal component of the acoustic intensity in Figure 2. The figure shows that the intensity (i.e., the energy flux) is directed by one or more channels forwards and by others backwards, forming a loop around the silencer with minor imperfections due to leakage. This phenomenon is related to the perfect BIC, which occurs because the phase delay on the path through the component is exactly 2π (see [8, 9]).

To demonstrate the properties of the silencers under real-world conditions, we will now consider the effects of

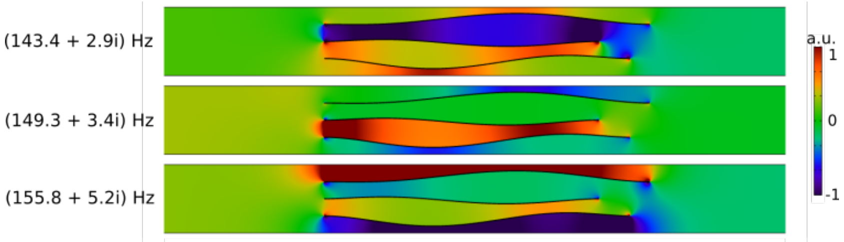

Figure 2. Maps of acoustic intensity component $i_1 = p'u'_1$ along the waveguide for three leaky eigenmodes of the Case A.

flow and thermoviscous losses. Firstly, we calculated the steady turbulent mean flow using the Reynolds-averaged Navier-Stokes equations, employing the k-ω SST model with resolved boundary layers. The plates were placed in a region of fully-developed flow. From the simulations, a hydrodynamic pressure drop (Δp_0) across the silencer was determined. We present the scaled values of $2\Delta p_0/(\rho_0 U_\infty^2)$, where ρ_0 and U_∞ denote the ambient air density and the flow speed at a significant distance from the silencer, respectively.

Figure 4 displays the pressure drops for various inflow velocities. The values indicate that the streamlining of the silencers is highly effective. The acoustic transmission properties were simulated by employing the time-harmonic Navier-Stokes equations, linearized around the turbulent mean flow velocity $\boldsymbol{u}_0$ and pressure p_0. From the amplitude transmission coefficient, $\mathcal{T}$, we calculated the transmission loss (TL) as $\mathrm{TL} = -20 \log 10\mathcal{T}$ dB. Figure 3 shows that case A exhibited a higher transmission loss and higher pressure drop compared to case B. The relative bandwidth of the proposed design is approximately 6-7%, indicating that it does not result in a filter that tends towards a very thin notch.

We confirmed that the transmission coefficients are independent of the specific location of the evaluation of acoustical variables behind the silencer. We also found that the transmission coefficient obtained by the perturbation velocity and pressure is the same as that obtained by the perturbation density alone. Furthermore, we verified that the low Mach number flow has minimal impact on the transmission properties (a direct comparison is not included here due to scope limitations). Consequently, the computationally cheaper optimization that employs only the Helmholtz equation is permissible.

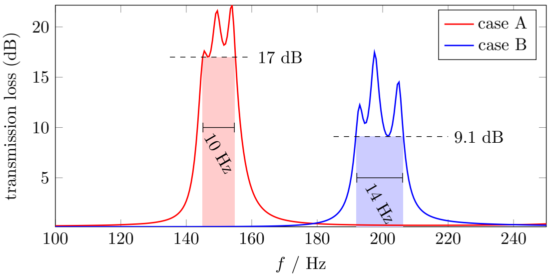

Figure 3. Transmission loss at $U_\infty = 8$ m/s for both cases of introduced BIC silencers. The targeted frequencies were 150 Hz and 200 Hz, respectively.

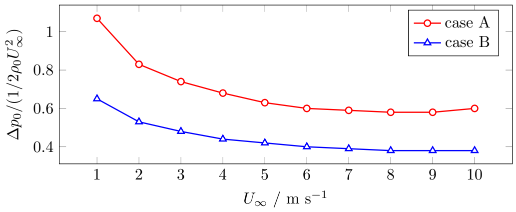

Figure 4. Scaled pressure drop Δp_0 in both introduced cases of BIC silencers.

4. CONCLUSIONS

Our research has shown that it is feasible to construct a low drag silencer that effectively eliminates a strong spectral line without resorting to methods such as laterally extending the duct or using foams, porous materials, and the like. Moreover, this silencer performs remarkably well in the low frequency region. The foundation of the underlying theory is the bound states in the continuum phenomenon, which serves as the basis of the optimization algorithm. This algorithm creates a stopband around the desired center frequency by placing several leaky resonances

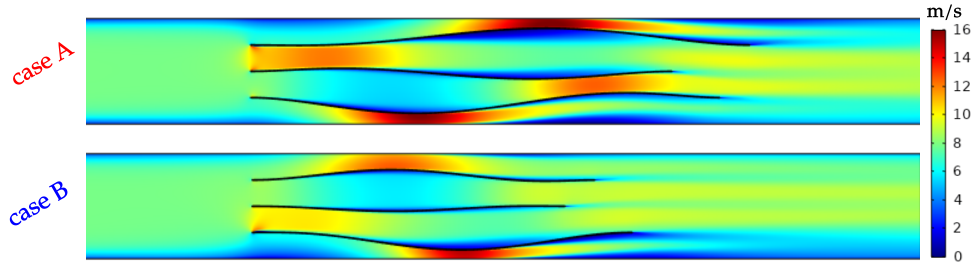

Figure 5. Mean flow velocity magnitude at $U_\infty = 8$ m/s for both cases of introduced silencers.

close together. The resulting silencer addresses the issue of unwanted noise in the form of spectral lines in a novel and effective way.

The proposed structure's effectiveness is demonstrated through two specific examples, which serve to illustrate how the silencer works in practice. In both cases, the results have been thoroughly validated using equations that account for the impact of low Mach number flow and thermoviscous losses. This validation process ensures that the silencer performs consistently and as intended under realistic operating conditions. The results obtained through this validation process further demonstrate the effectiveness of the proposed structure and provide evidence of its potential utility across a range of applications. Overall, the combination of practical demonstration and rigorous validation helps to establish the viability and value of this new approach to silencing unwanted noise.

The new approach to silencing unwanted noise is highly suitable for practical installations such as ventilation systems, where it can effectively reduce tone-like noise without requiring an increase in fan power due to its low pressure drop. This feature makes the proposed silencer not only effective but also energy-efficient, providing an additional advantage for practical applications. Moreover, the proposed procedure is straightforward to follow and its features are simple and straightforward for analysis. This combination of practicality and simplicity makes the proposed approach highly applicable and opens up a range of new possibilities for noise reduction across a variety of applications.

5. ACKNOWLEDGMENTS

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