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EXPLICIT MODELING AND OPTIMIZATION OF ACOUSTIC META-LENSES FOR BAFFLED SOURCES

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ABSTRACT

The recent emergence of personal sound zones applications instigates the development of acoustic solutions that generate spatial sound pressure level (SPL) contrast. While 'active' electro-acoustic solutions using multiple sources do exist, another way to do so is to manipulate the acoustic field radiated from a baffled source using an acoustic lens. In this work, we propose a metamaterial-based acoustic lens able to focus or steer an acoustic beam, using multiple slits loaded by periodic Helmholtz resonators. We control the acoustic radiation of a baffled duct by a meta-lens located at the output of the waveguide while excited at the other end by a plane-wave. A two-dimensional explicit model is developed, it is based on mode matching and accounts for thermo-viscous losses in the system. The pressure radiated by the meta-lens outside of the baffled duct is also explicitly calculated from integral formulations. The analytical results for near- and far-field radiation show excellent agreement with the solutions obtained by FEM. Finally, an optimal meta-lens is designed, for which the beam steering and focusing are reported experimentally on a 3D-printed prototype. This work paves the way of a complete analytical tool for optimizing the radiation of acoustic lenses using meta-materials.

Keywords: *meta-materials; acoustic lens; explicit modeling; finite elements; directivity control*

1. INTRODUCTION

Acoustic sound-zones are recently gaining interest and can be created and controlled using either active [1–4] or passive solutions. Passive solutions, such as acoustic lenses, are attractive as they are purely based on a structured geometry and require no additional sources of energy. Numerical simulations are typically used to model acoustic lenses [5, 6], but a new explicit model based on a mode-matching approach [7] is proposed in this work. The model uses an array of parallel slits loaded by periodic Helmholtz resonators as the meta-lens, which allows for flexible tuning. Furthermore, the proposed model accounts for thermo-viscous losses, inner-slit couplings and high-order modes. An optimization procedure using particle swarm optimization (PSO) [8] is carried-on in order to design a meta-lens able to steer the acoustic beam at a specific angle. The optimal meta-lens is then fabricated by 3D printing techniques [9] and tested experimentally in an anechoic chamber. This work provides a complete analytical tool for modeling and optimizing the radiation of

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acoustic lenses using meta-materials and can be extended to design 3D meta-materials.

2. METAMATERIAL-BASED ACOUSTIC LENSES

The acoustic propagation problem is set in the 2D Cartesian coordinate system. First, a semi-infinite waveguide of height $w^{(0)}$ is considered. Then, the meta-lens made of slits and Helmholtz resonators is positioned at one end of the main waveguide. The meta-lens itself is composed of $N = 10$ parallel slits and both the duct and meta-lens are baffled, radiating in a semi-infinite half-space. Finally, each slit is loaded by $Q = 5$ identical and evenly spaced Helmholtz resonators, the geometry of which may differ from one slit to another.

The wave propagation in the slits is governed by the geometric features of the resonant periodic structure. We analytically model the radiated pressure by the meta-lens, when a plane wave is imposed in the main duct. The mode-matching method [7] is used in order to account for high-order modes in the main duct, as well as coupling in-between the slits. The thermo-viscous losses in the slits and resonators are accounted for using equivalent fluid properties [10], and the length corrections are applied at the discontinuities of the slits and resonators parts [11]. The propagation through the meta-lens itself is based on the transfer matrix method [12].

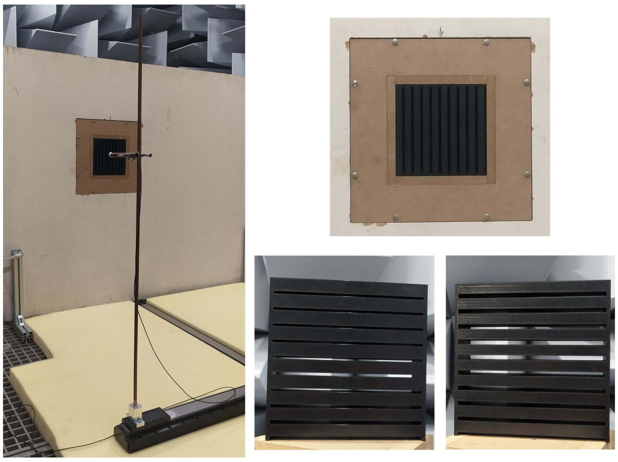

Figure 1. Photographs of: the experimental setup (left), the baffled meta-lens (top), 3D-printed meta-lens samples (bottom).

The analytical model is validated against the FEM results. The results of radiated pressure are in excellent agreement as long as the waves propagating in the slits are in the plane-wave regime. The numerical errors essentially come from the truncation of the high-order modes and radiation integrals, as well as additional couplings within the resonators which are not accounted for.

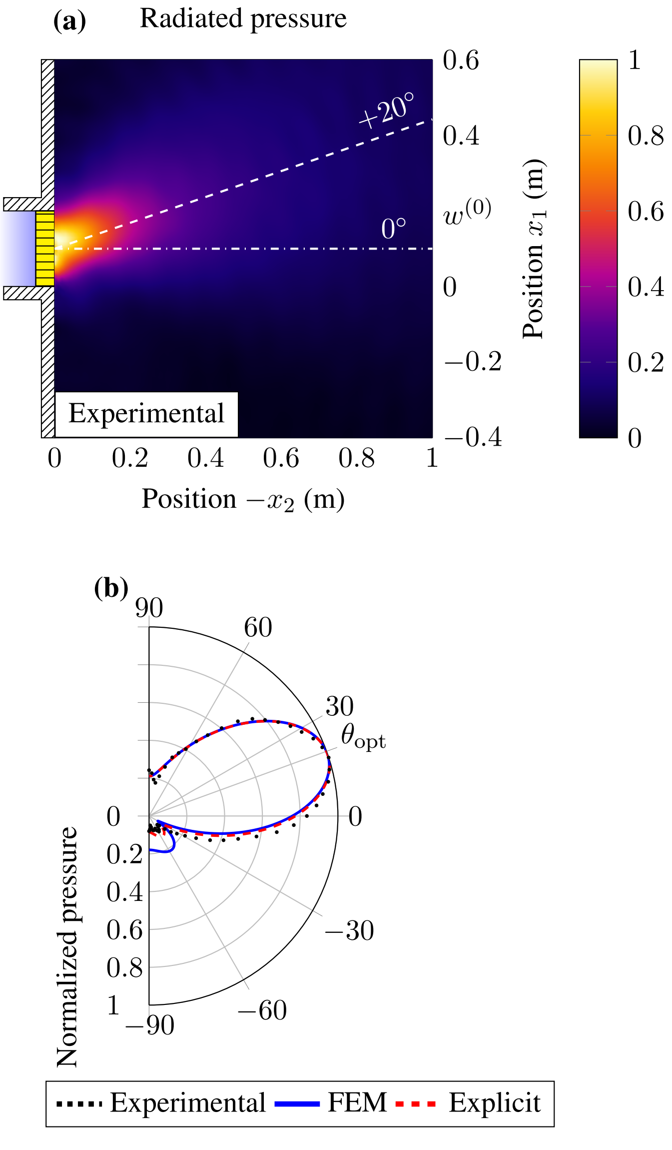

Figure 2. Radiated pressure field **(a)** normalised near-field angular response at $d = 50\,\mathrm{cm}$ **(b)**.

Finally, an optimisation procedure is carried-on, in order to design an acoustic meta-lens able to steer the beam at a specified angle $\theta_{\text{opt}} = +20°$. A sample having the optimal geometry is manufactured and tested in the controlled environment within of the anechoic chamber of the Acoustic Laboratory of Le Mans University (LAUM UMR CNRS 6613). The samples are manufactured by Stellantis from a fused-deposition modeling (FDM) process. The experimental setup as well as the meta-lenses prototypes are presented in Fig. 1. As observed in Fig. 2(**b**), the radiated pressure predicted by either the analytical and FEM models accurately matches the one measured experimentally.

3. CONCLUSION

We report on the behavior of meta-lenses based on slits loaded with Helmholtz resonators using mathematical, numerical, and experimental methods. Our explicit 2D model considers thermal and viscous losses, inter-slit couplings, and high-order modes in the baffled waveguide. We validate the model against FEM solutions and use it for an optimization procedure to design acoustic meta-lenses that focus or steer an acoustic beam. Experimental results show good agreement with the analytical and FEM models. The explicit model has some advantages over FEM solutions but has limitations related to truncation, length corrections, and inter-resonator couplings. We foresee extensions of the model to account for 3D geometries and improve optimization routines for faster minimization times. These improvements would enhance the modeling and thee design of acoustic meta-lenses and 3D meta-materials.

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