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SHAPE OPTIMIZATION OF 2D ACOUSTIC RADIATION PROBLEMS USING BEM AND ANALYTICAL SENSITIVITIES

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ABSTRACT

The Boundary Element Method (BEM) is a numerical method used to solve acoustic problems such as radiation, propagation and scattering by means of integral equations. Combining BEM with shape optimization procedures allows engineers to design components or ambients with some desired acoustic characteristics. This work presents a node-based shape optimization procedure in which the derivatives of the BEM matrices with respect to the nodal coordinates of a predefined mesh are obtained analytically, being calculated in the same way as for each element contribution. Examples with a well-known base geometry are presented to illustrate the potential of this procedure for shape optimization of 2D time-harmonic acoustic radiation problems.

Keywords: BEM, Radiation, Shape optimization

1. INTRODUCTION

Few works are found in the literature concerning discrete approaches for optimization with BEM in acoustic problems. Andersen et al. [1] pointed that node-based procedures can be expensive due to the large number of variables and the necessity of mesh regularization to avoid distorted elements. Considering scattering problems, they propose the usage of a moving averaged Gaussian filter for each design variable (position of each node in a BEM mesh) at the normal direction. Moreover, the authors applied the semi-analytical adjoint sensitivity analysis to obtain the sensitivities of acoustic measures of interest, in which the derivatives of the BEM matrices were given by the finite difference method with a fixed step size. Since BEM global matrices are dense, the cost of performing finite difference on the matrix level is expensive, and the authors proposed a procedure based on a cleaver assembly of the perturbed entries [1].

Since solving acoustic problems by BEM relies on using the Green's function as the fundamental solution of the time-harmonic Helmholtz differential equation [2], and the numerical method behind the discretization procedure involves solving the corresponding Helmholtz integral considering the contribution of all elements on the acoustic behavior of each node [3], the resultant system's matrices are complex-valued and dense. Thus, the evaluation of derivatives of BEM matrices by using finite differences can be expensive, and the choice of the step size can be strongly dependent on the mesh size and on the analyzed frequency.

There is a lack in the literature regarding the use of discrete approaches with BEM in acoustic problems with efficient analytical evaluation of sensitivities. This is the aim of the present work, where the adjoint method is used. Although the procedure is applied to the minimization of radiation efficiency in 2D problems, it is carried out in such a manner that it can be easily adapted to other acoustic problems in 2D or 3D.

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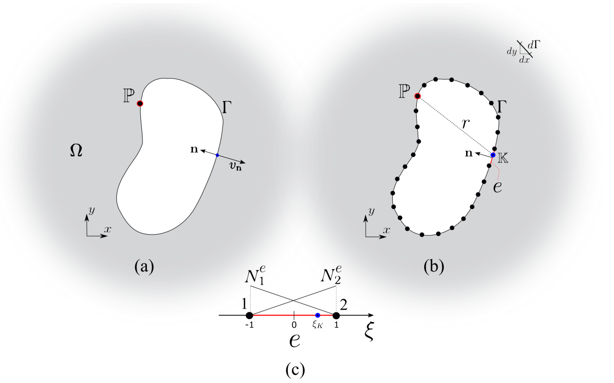

Figure 1. a) outer domain Ω, interior boundary Γ, normal direction $\mathbf{n}$, normal component of velocity $v_{\rm n}$ and collocation point $\mathbb{P}$. b) nodes, elements e, integration point $\mathbb{K}$ and relative distance r. c) Two-node linear boundary element e.

2. 2D TIME-HARMONIC ACOUSTIC RADIATION PROBLEMS SOLVED BY BEM

Some practical acoustic problems can be described by the linearized wave equation [4,5],

$$\nabla^2 \mathrm{p}(\mathbb{X},t) - \frac{1}{c^2}\frac{\partial^2 \mathrm{p}(\mathbb{X},t)}{\partial t^2} = 0, \qquad (1)$$

where t is time, p is the pressure field evaluated at the point $\mathbb{X} = (x,y,z)$ in the acoustic domain of interest and c is the speed of sound in the acoustic medium. For time-harmonic waves with angular frequency ω, Eq. (1) can be rewritten in terms of $\mathrm{p}(\mathbb{X},t) = p(\mathbb{X},\omega)\mathrm{e}^{\mathrm{j}\omega t}$, where $\mathrm{j} = \sqrt{-1}$, as the *Helmholtz equation*,

$$\nabla^2 p(\mathbb{X},\omega) - \kappa^2 p(\mathbb{X},\omega) = 0, \qquad (2)$$

where p is the complex amplitude of the pressure at $\mathbb{X}$, and $\kappa = \omega/c$ is known as the wave number.
Consider an exterior problem defined in a 2-D unbounded acoustic domain Ω representing the radiation from a vibrating structure placed at the internal boundary contour Γ. The boundary integral equation is obtained applying Green's second identity to functions p and ψ. Thus, the boundary problem (Helmholtz Integral Equation) is defined by [3,6,7]

$$C(\mathbb{P})p(\mathbb{P}) = -\int_\Gamma \left(\mathrm{j}\rho\omega v_{\rm n}\psi + p\frac{\partial\psi}{\partial\mathbf{n}}\right)\mathrm{d}\Gamma, \qquad (3)$$

where $v_{\rm n}$ is the amplitude of the normal component of the particle velocity at a point in Γ, which is pointing away from the interior domain, and the normal $\mathbf{n}$ is pointing away from the exterior domain. The leading coefficient $C(\mathbb{P})$ is defined as [3,6,7]

$$C(\mathbb{P}) = 1 - \int_\Gamma \frac{\partial\psi_{\rm L}}{\partial\mathbf{n}}\,\mathrm{d}\Gamma, \qquad (4)$$

where $\psi_{\rm L}$ is the fundamental bi-dimensional solution of the Laplace equation ($\psi_{\rm L} = -\ln r/2\pi$).
Considering an infinite 2D domain Ω with an internal boundary Γ (see Fig. 1 (a)), the boundary Γ can be split into a finite number $n_{\rm el}$ of elements Γ_e as shown in Fig. 1 (b). Coordinates x^e and y^e in each finite segment e can be mapped to local coordinates $\xi \in [-1,1]$ as

$$x^e(\xi) = \sum_{i=1}^{n_n} x_i N_i^e(\xi) \quad \text{and} \quad y^e(\xi) = \sum_{i=1}^{n_n} y_i N_i^e(\xi), \qquad (5)$$

where x_i and y_i are Cartesian 2D coordinates at points i at e, n_n is the number of points at e and $N_i^e(\xi)$ are interpolation functions on e. This work considers linear segments, Fig. 1(c), such that $n_n = 2$ and

$$N_1^e = \frac{1}{2}(1-\xi) \quad \text{and} \quad N_2^e = \frac{1}{2}(1+\xi). \qquad (6)$$

Pressure and normal velocity fields, p and $v_{\rm n}$, can be described in a given element e as

$$p(\xi) = \sum_{i=1}^{2} p_i N_i^e(\xi) \quad \text{and} \quad v_{\rm n}(\xi) = \sum_{i=1}^{2} v_{{\rm n}i} N_i^e(\xi), \qquad (7)$$

where p_i and $v_{{\rm n}i}$ are the nodal values at point i. A 2-node element is considered sufficient to model the 2D radiation problem discussed in this work due to the fact that, in general, a detailed shape optimization employs a mesh with more elements than that required to correctly describe the wave phenomena.
The Helmholtz integral (3) is solved using the expressions in Eq. (7) and performing numerical integration along Γ considering $k_{\rm el}$ Gauss-Legendre quadrature points per element. The system of equations that represents the contribution of all points on the acoustic behavior of Γ can be represented in a matrix form for a given ω as

$$(\mathbf{C} + \mathbf{D})\,\mathbf{p} = \mathbf{G}\mathbf{v}_{\rm n}, \qquad (8)$$

where $\mathbf{p}$ is the vector pressure at each node, and $\mathbf{v}_{\rm n}$ is the vector of normal velocities. The dense matrices $\mathbf{C}$, $\mathbf{D}$ and

$\mathbf{G}$ are resultant of the numerical integration of (4) over the boundary Γ [3, 8].

2.1 Obtaining the radiation efficiency in terms of BEM element parameters

The radiated power W_{rad} and the radiation efficiency σ_{rad} can be obtained by BEM. Considering time-harmonic acoustics, the sound intensity $\boldsymbol{I}$ (which is a vector quantity) at a given point with coordinates (x, y) and for a given angular frequency ω in a 2D-domain is given by [5]

$$\boldsymbol{I}(x, y) = \frac{1}{2}\Re\left\{p(x, y)\boldsymbol{v}^*(x, y)\right\}, \tag{9}$$

where $\boldsymbol{v}^*$ is the complex conjugate velocity and $\Re$ is the real part. Thus, the radiated power W_{rad} through the boundary Γ in Fig. 1 can by obtained by [4]

$$W_{\text{rad}} = \int_\Gamma \boldsymbol{I}\cdot\mathbf{n}\,\mathrm{d}\Gamma = \frac{1}{2}\int_\Gamma \Re\left\{p(-v_\text{n}^*)\right\}\mathrm{d}\Gamma, \tag{10}$$

where the normal direction points inward.
The total radiated power can be found by summing up the contribution W_{rad}^e of all elements in the mesh [8]

$$W_{\text{rad}}^e = -\frac{1}{2}\Re\left\{\mathbf{p}_e^T\mathbf{B}_e\mathbf{v}_{\text{n}e}^*\right\}, \tag{11}$$

where, for a straight 2-node element e, $\mathbf{p}_e = [p_1\ p_2]^T$, $\mathbf{v}_{\text{n}e} = [v_{\text{n}1}\ v_{\text{n}2}]^T$, and [8]

$$\mathbf{B}_e = \int_{\Gamma_e}\mathbf{B}^0\mathrm{d}\Gamma_e \approx \sum_{K=1}^{k_{\text{el}}}\mathbf{B}_K^0 w_K J_e, \tag{12}$$

considering numeric integration with weights w_K, where J_e is the determinant of the Jacobian of element e and [8]

$$\mathbf{B}^0 = \begin{bmatrix} N_1N_1 & N_1N_2 \\ N_2N_1 & N_2N_2 \end{bmatrix}. \tag{13}$$

Global matrix $\mathbf{B}$ is obtained by assembling all $\mathbf{B}_e$. Consequently, Eq. (10) can also be written as [8]

$$W_{\text{rad}} = -\frac{1}{2}\Re\left\{\mathbf{p}^T\mathbf{B}\mathbf{v}_\text{n}^*\right\}. \tag{14}$$

It is important to remark that only the pressure field at Γ is needed to evaluate W_{rad} (a field point mesh is not needed). Considering an infinite-length cylinder modeled in two-dimension, the radiation efficiency can be obtained by [5], for a given wave number κ,

$$\sigma_{\text{rad}}(\kappa, a) = \frac{W_{\text{rad}}(\kappa, a)}{\rho c S\overline{|v_\text{n}|^2}/2}, \tag{15}$$

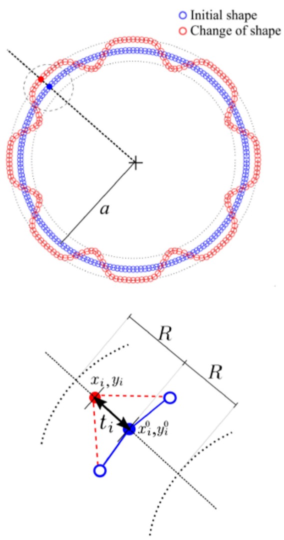

Figure 2. Parameterization of nodes position. Node i moves at radial direction $\mathbf{n}_i^\text{N}$ within bounds R. Radial direction $\mathbf{n}_i^\text{N}$ is obtained from normal directions of elements in the vicinity of node i. t_i is the radial change in position for node i, according to the design variable μ_i.

where $\overline{|v_\text{n}|^2}$ is the spatially averaged, mean-square velocity, and S is the surface area of the structure. For an infinite-length cylinder (2D), S is replaced by the perimeter $2\pi a$.

3. PARAMETERIZATION AND OPTIMIZATION

The optimization is performed by changing the shape of Γ, by means of its nodal positions, Figs. 2. In this work, we assume that nodes can move only in the radial direction, within the dashed lines shown in Fig. 2. Normal direction for each node is defined is obtained from the normals of the elements in the vicinity of node i, elements e_1 and e_2. This direction $\mathbf{n}_i^\text{N} = (n_i^{\text{N}^x}, n_i^{\text{N}^y})$ is kept fixed along the optimization process such that

$$x_i = x_i^0 + \mu_i R n_i^{\text{N}^x}, \text{ and } y_i = y_i^0 + \mu_i R n_i^{\text{N}^y}, \tag{16}$$

where $\mu_i \in [-1, 1]$ is the parameter that controls the position of each node i, and R is the maximum distance from the initial position.
The optimization problem is thus written as

$$\begin{aligned} \min_{\boldsymbol{\mu}} \quad & \sigma_{\text{rad}}(\omega_f) \\ \text{subject to} \quad & (\mathbf{C}+\mathbf{D})\,\mathbf{p}=\mathbf{G}\mathbf{v}_{\text{n}} \\ & S \leq S_{\max} \\ & -\mathbf{1} \leq \boldsymbol{\mu} \leq \mathbf{1} \end{aligned} \tag{17}$$

where $\boldsymbol{\mu}$ is the vector containing all design variables, ω_f is the frequency of analysis, S is the length of Γ with limit value of $S_{\max}$. In this research, the problem is solved by using the Method of Moving Asymptotes (MMA) [9], with the parameters recommended by [10–12]).

3.1 Sensitivity analysis

In this work, the vibration of the 2D structure is imposed and it is assumed that $\overline{|v_{\text{n}}|^2}$ does not depend on the shape. By this way, the derivatives of W_{rad} with respect to the 2D nodal coordinates are given by [8]

$$\frac{\partial W_{\text{rad}}}{\partial x_i} = -\frac{1}{2}\Re\left\{\mathbf{p}^T\frac{\partial \mathbf{B}}{\partial x_i}\mathbf{v}_{\text{n}}^*\right\}+\Re\left\{\boldsymbol{\lambda}^T\left(\frac{\partial \mathbf{D}_c}{\partial x_i}\mathbf{p}-\frac{\partial \mathbf{G}}{\partial x_i}\mathbf{v}_{\text{n}}\right)\right\}, \tag{18}$$

and

$$\frac{\partial W_{\text{rad}}}{\partial y_i} = -\frac{1}{2}\Re\left\{\mathbf{p}^T\frac{\partial \mathbf{B}}{\partial y_i}\mathbf{v}_{\text{n}}^*\right\}+\Re\left\{\boldsymbol{\lambda}^T\left(\frac{\partial \mathbf{D}_c}{\partial y_i}\mathbf{p}-\frac{\partial \mathbf{G}}{\partial y_i}\mathbf{v}_{\text{n}}\right)\right\}, \tag{19}$$

where $\boldsymbol{\lambda}$ is obtained from the adjoint problem [8]

$$\mathbf{D}_c^T\boldsymbol{\lambda} = \left(\frac{1}{2}\mathbf{B}\mathbf{v}_{\text{nR}} - \mathrm{j}\frac{1}{2}\mathbf{B}\mathbf{v}_{\text{nI}}\right) = \frac{1}{2}\mathbf{B}\mathbf{v}_{\text{n}}^*\,. \tag{20}$$

Thus, the sensitivities of matrices $\mathbf{C}$, $\mathbf{G}$, $\mathbf{D}$ and $\mathbf{B}$ with respect to x_i and y_i are needed. This is the most laborious part of the methodology, but strictly analytical, being obtained using general and element equations listed along this paper. The complete calculation of the sensitivities is presented in a paper recently submitted by the author, under review [8].

3.2 Smoothing

A linear spatial filter is applied to avoid possible local solutions with a very irregular distribution of nodes in a zigzag pattern. Here, a weighted sum of the variables $\boldsymbol{\mu}$ is performed at a control region

$$\tilde{\mu_i} = \frac{1}{\sum_{j\in \mathrm{N}_i}\Lambda_{ij}}\sum_{j\in\mathrm{N}_i}\Lambda_{ij}\mu_j, \tag{21}$$

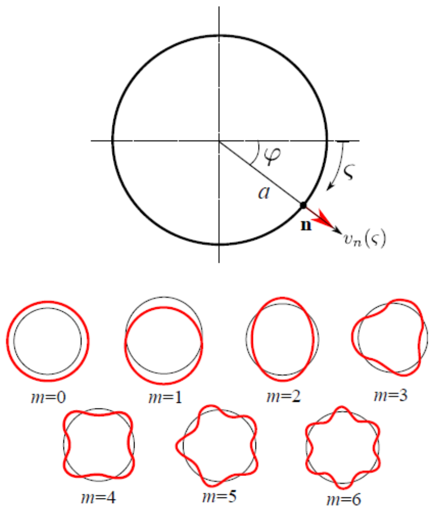

Figure 3. Vibration of an infinite cylinder. 2D modeling.

where $\tilde{\boldsymbol{\mu}}$ is the new set of design variables and N_i is the set with the $2n_{\text{N}}+1$ nodes adjacent to node i (equally distributed on both sides). The parameter $\Lambda_{ij} = n_{\text{N}} - \Delta_{ij}$ is a weight depending on the relative position of nodes i and j in N_i where Δ_{ij} is the numbered distance between nodes i and j.

4. RESULTS

Consider an infinite-length cylinder with radius a=0.5 m vibrating with angular frequency ω, as shown in Fig. 3. It is assumed in-phase vibration along the length, with circumferential normal velocity distribution defined by

$$v_{\text{n}}(\varsigma) = \hat{v}_{\text{n}}\cos\left(2\pi m\varsigma\right), \tag{22}$$

where $\varsigma = \varphi/2\pi \in [0,1]$ represents the normalized path along the perimeter of the circumference and $\hat{v}_{\text{n}}$ is the peak of the normal velocity distribution. The integer parameter m defines the shape of the distribution (the number of complete spatial periods along the perimeter), as shown in Fig. 3. For a given target angular frequency ω, m can be changed to simulate different structural configurations that lead to different vibration shapes at the same frequency.

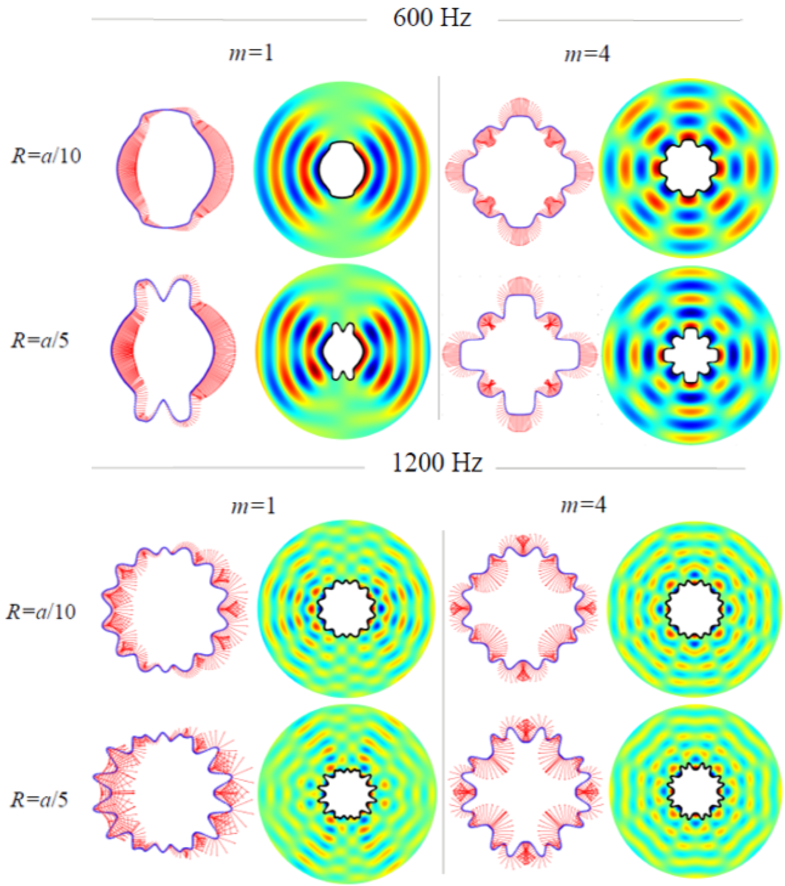

Figure 4. Vibration of an infinite cylinder. 2D modeling.

The optimization procedure proposed is studied with a mesh of 360 nodes to ensure detailed final shapes, and initially considering $n_{\mathrm{N}} = 8$ aiming to avoid irregular distribution of nodes. Two different values of R are used: $R = a/10$ and $R = a/5$. Two excitation frequencies are considered: 600 and 1200 Hz. Two configuration of vibration are assumed: m=1 and m=4. The final shapes and the resultant pressure surrounding the structure is presented in Fig. 4.

The increase in the perimeter is a consequence of the appearance of winding curves in the final designs, which cause significant changes in the direction of the element normals and, consequently, in the direction of the element normal velocities. Depending on the frequency (and related wavelength) and on m, this effect causes a more diffuse distribution of the normals, promoting energy canceling and/or hydrodynamic short cuts. The frequency plot of the radiation efficiency for these two excitation frequencies, considering $R = a/5$, are presented for several values of m in Fig. 5. We can see the effect of the optimization at the frequencies of interest.

4.1 Conclusion

The minimization of radiation efficiency in 2D acoustic problems is discussed in this work, taking as example an infinite-length cylinder. The equilibrium problem is solved by using the Boundary Element Method (BEM) and the complete analytical expressions to evaluate the sensitivities are presented

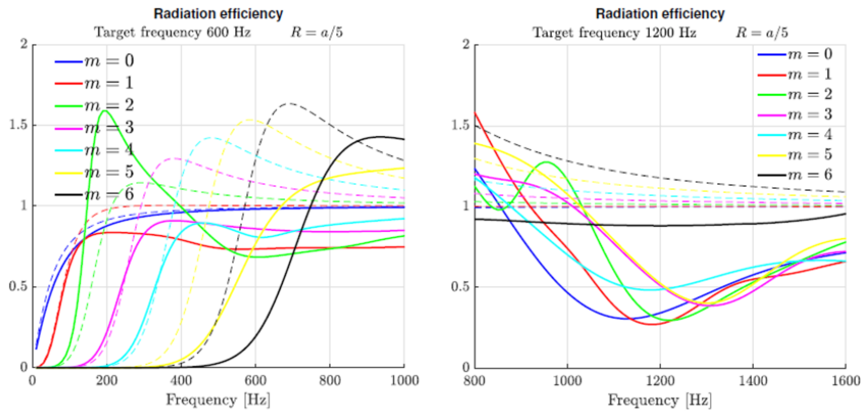

Figure 5. Radiation efficiency [no unit].

and developed in details. Mesh dependency and distortions are addressed by using a regularization technique (filtering). The use of analytical sensitivities makes the problem efficient, hindering the limitations of directly using nodal positions as design variables as well as avoiding numerical problems and the need to investigate the proper level of perturbation in finite differences and semi-analytical approaches.

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