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NEW MEASUREMENT METHOD FOR BENDING LOSS FACTOR AND THE BENDING STIFFNESS: TEST METHOD

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ABSTRACT

In order to reduce the sound radiation or the transmission of sound, plate-shaped components are often coated with damping materials. In the case of bending waves, the descriptive dynamic-mechanical properties are the bending loss factor and the bending stiffness. The most frequently used method is the resonance curve method (also called Oberst method), in which the dynamic-mechanical properties are determined in the one-dimensional bending eigenmodes of a rectangular bar. However, this very robust method has two serious disadvantages for many applications. Firstly, the loss factor can only be reliably determined up to a value of $\eta \leq 0.1$, as the neighbouring modes overlap too much for higher values of the loss factor. Secondly, the properties can usually only be determined at two to three resonance frequencies.

Based on the measurement setup of the resonance curve method, a new test method was developed that overcomes the above-mentioned disadvantages of the resonance curve method. The bending loss factor and the bending stiffness can be measured as a continuous spectrum and the upper measurement limit for the bending loss factor is not limited due to the system. This new bending wave method is based on the theory of the so-called rigid Bernoulli beam.

Keywords: *loss factor, bending stiffness, bending wave method, Oberst method*

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1. INTRODUCTION

Coatings on sheet metal structures are also often applied for acoustic reasons. Various principles of action are pursued here:

- Reduction of airborne sound radiation by reducing the local velocity: A local coating reduces the velocity in this area. In the case of predominant radiation by free bending waves, the mass per unit area and the loss factor have a radiation-reducing effect. In the case of predominant radiation by forced bending waves, the influence of the loss factor is small and the reduction in airborne sound radiation results from the applied mass per unit area.
- Structure-borne noise attenuation during propagation over a coated area: For attenuation of structure-borne noise propagation, the loss factor is the most important parameter.
- Generation of an impedance jump at the coating boundary: For the reflections of sound waves at the impedance jump, the change in the mass per unit area and the bending stiffness due to the coating is decisive. The loss factor, on the other hand, plays a subordinate role.

Various methods are used in practice for the metrological determination of the dynamic-mechanical properties of coatings. The bending wave method has been further developed by the authors, as it has several advantages compared to other methods. For a better understanding of the advantages and disadvantages, three standard methods are explained first, followed by the bending wave method.

2. OVERVIEW OF TEST METHOD

2.1 Introduction

For the determination of the loss factor various methods are known. The methods differ both in the dimensions of the specimens and thus also in the excited types of waves, as well as in the measurement methods used. The three most important methods are explained in the following.

2.2 Resonance curve method

The most commonly used method is the resonance curve method according to Heinrich H. Oberst, in which the dynamic mechanical properties are determined in the one-dimensional bending eigenmodes of a bar. The determination of the bending loss factor $\eta = \tan(\delta_f)$ and the flexural storage modulus E is described in ISO 6721-3 [3].
The typical sample dimensions are length x width x thickness = (200 mm … 300 mm) x (10 mm … 20 mm) x 1.0 mm.
The loss factor η is determined according to the following equation:

$$\eta = tan(\delta_f) = \frac{\Delta f_n}{f_{rn}} \quad (1)$$

with:

Δf_n width of a peak of the resonance curve of the nth mode in Hz at amplitude $v_{max}/\sqrt{2}$, where v_{max} is the maximum amplitude

f_{rn} natural frequency of the nth mode in Hz.

The bending stiffness B in Nm^2 is determined according to the following equation:

$$B = \frac{64 m'' b f_{rn}^2 l^4}{\pi^2 \beta_n^4} \quad (2)$$

with:

m'' mass per unit area in kg/m2

n ordinal number of the vibration n = 1, 2, …

f_{rn} natural frequency of the nth mode in Hz

b width of the bar in m

l length of the bar in m

β_n $\beta_1 = 1.1944$, $\beta_2 = 2.9860$, $\beta_n = 2n\text{-}1$.

The flexural storage modulus E in N/m^2 can be determined according to the following equation:

$$E = \frac{12B}{bh^3} \quad (3)$$

with:

h height of the bar in m.

The resonance curve method has the disadvantage that loss factors of $\eta > 0.1$ are systematically overestimated due to modal overlap. In addition, the narrow bars of only 10 mm to 20 mm width commonly used in this method lead to artifacts in liquid-applied coatings due to edge influence. The drying behavior of the bar edge and the bending of the coating over the bar width are particularly critical in the case of water-based coatings.
The evaluation of the bending stiffness is associated with an increased uncertainty, since the freely vibrating length of the bar is included in the result with the fourth power. In addition, to clamp coated bars, the coating on the clamping surface must be removed, so that an undefined termination is produced at the clamping point.

2.3 Power injection method

In the power injection method, the loss factor is calculated from the power balance between the power injected and the power remaining in the system [4]. The test is typically performed on approximately 0.04 m^2 to a maximum of 1 m^2, roughly square panels.
The loss factor η is determined according to the following equation:

$$\eta = \frac{P}{2\pi f E} \quad (4)$$

with:

P structure-borne noise power introduced into the system in W

f frequency in Hz

E spatially averaged energy in the system in Ws.

The structure-borne noise power P introduced into the system is determined using a load cell and a velocity sensor at the force transmission point:

$$P = |F||v| \cos(\phi_F - \phi_v) \quad (5)$$

with:

F force (r.m.s.) at the force transmission point in N

v velocity (r.m.s.) at force transmission point in m/s

$\phi_F - \phi_v$ phase shift between velocity and force.

The energy existing in the system is determined according to the following equation:

$$E = m\overline{v_{eff}^2} \quad (6)$$

with:

m mass of the plate in kg

$\overline{v_{eff}^2}$ spatially averaged velocity square over the entire plate in (m/s)^2.

The frequency-dependent direct sound field around the force transmission point must be excluded from the evaluation and the higher energy density at the plate edge must be taken into account [5].
The test plates allow the analysis of coatings applied over large areas. In contrast to the narrow bars used in the resonance curve method, the edge influence is negligible due to the small proportion compared to the area. If the airborne sound velocity in the near field is determined in addition to the structure-borne noise velocity, the radiation-relevant sound power can also be determined.

However, the comparatively large test plates make investigations in the climatic chamber more difficult.

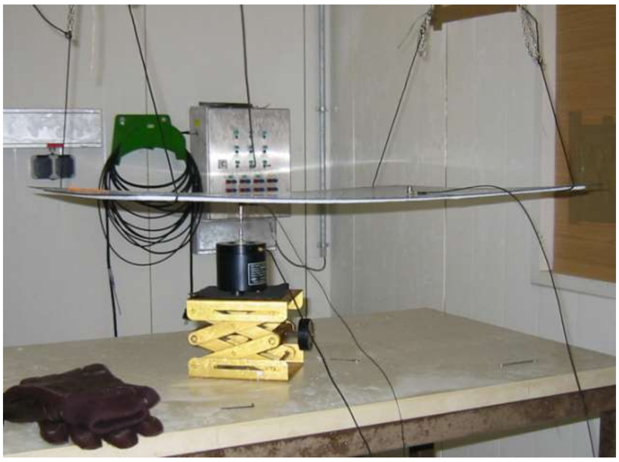

Figure 1. Test set-up for the power injection method.

The bending stiffness cannot be derived from the power injection method. In principle, however, the measured velocities on the plate can be used for a modal analysis to determine the effective bending stiffness of the first modes.

2.4 Reverberation method

The reverberation method determines the loss factor from the exponential decay of the structure-borne noise velocity after a pulse-like excitation. Strictly speaking, a diffuse structure-borne noise field, i.e. a sufficiently high modal density, is a prerequisite for the application of the method. Ultimately, however, the decay behaviour of a single mode can also be analysed, although in this case the attenuation must not be too high, since otherwise an exponential amplitude response will not be obtained. The measurements are usually carried out on square plates of approx. 1.5 m^2, which allow a lower frequency limit of approx. 100 Hz with a 1.0 mm thick steel plate.
The loss factor η is determined according to the following equation:

$$\eta = \frac{2.2}{T f_m} \tag{7}$$

with:
T reverberation time of velocity in s
f_m centre frequency of the analysed frequency range in Hz.
The reverberation method requires that the decay time of the bandpass filter used is shorter than the decay time of the test object to be analysed. Reliable results can be obtained using the time-inverse Schröder backward integration if the condition $BT > 4$ is met, where B is the filter bandwidth in Hz. For low attenuations, the reverberation method is very well suited. However, the large dimensions of the test objects compared with the resonance curve method make measurements in a climatic chamber difficult. The bending stiffness cannot be determined using this method.

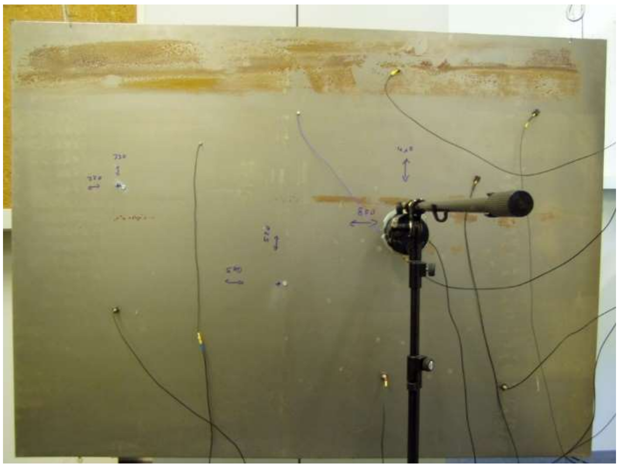

Figure 2. Test set-up for the reverberation method.

3. BENDING WAVE METHOD

3.1 Motivation

With the further developed bending wave method, a method has been standardized which overcomes some disadvantages of the methods described above. The bending wave method determines the dynamic-mechanical properties of a unilaterally clamped bar which is excited to one-dimensional bending vibrations without contact.

3.2 Test objects

The bar dimensions are preferably length x width = 500 mm x 50 mm when using 1.0 mm thick spring steel sheet as the support bar for the coating to be tested.
The larger bar dimensions compared to the resonance curve method were chosen for the following reasons:

- The bar length of 500 mm results in a higher stability of the measurement results compared to local material variations along the bar, as the coating length is longer and the analysis is carried out at many points over the entire length of the bar.

- The bar width of 50 mm significantly reduces the edge influence compared to the narrow bars of 10 mm to 20 mm commonly used in the resonance curve method. The bar width was not chosen even higher since this would reduce the upper frequency limit of the method due to cross modes.

3.3 Description of the test arrangement

The complex velocity is determined at least 20 measuring points along a bar and a best fit to the bending wave equation is performed by iteration.
The developed test facility allows the measurement of up to 18 bars with different dimensions in one fully automated test cycle (see Fig. 3).

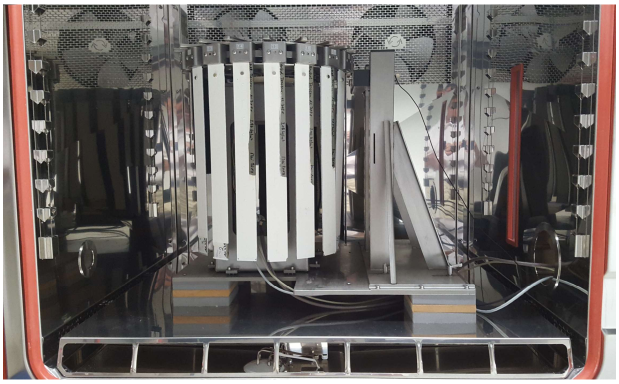

Figure 3. Test set-up for the bending wave method.

3.4 Evaluation

The bending wave method is based on the theory of the so-called shear-rigid Bernoulli beam. Considering various associated assumptions, the linear homogeneous differential equation for the bar including the terms for the near fields at the bar ends is obtained [1]:

$$v(x) = v_+ e^{-jk\ } + v_{+j} e^{-kx} + v_- e^{jkx} + v_{-j} e^{kx} \quad (8)$$

with:

$v(x)$ amplitude of velocity at point x in m/s
v_+ amplitude of the velocity of the outgoing wave in m/s
v_- amplitude of the velocity of the returning wave in m/s
v_{+j} amplitude of the velocity of the near field of the outgoing wave in m/s
v_{-j} amplitude of the velocity of the near field of the returning wave in m/s
k complex wave number $k = k' - jk''$ in 1/m.

No assumptions are made about the boundary conditions during the evaluation, since it has been shown experimentally that the clamping conditions are not ideal despite the solid steel clamps used to hold the bar. Ideally, the boundary conditions are free of velocity and bending at the clamping point and free of moment and force at the freely vibrating end of the bar.
For evaluation, the bending wave equation is used to determine the complex wave number from the experimentally determined velocities along the bar. For each frequency to be analysed, the following system of equations is set up:

$$\begin{bmatrix} e^{-jk\ _1} & e^{-kx_1} & e^{jkx_1} & e^{kx_1} \\ e^{-jkx_2} & e^{-kx_2} & e^{jkx_2} & e^{kx_2} \\ \dots & \dots & \dots & \dots \\ e^{-jkx_n} & e^{-kx_n} & e^{jk\ _n} & e^{kx_n} \end{bmatrix} \begin{bmatrix} v_+ \\ v_{+j} \\ v_- \\ v_{-j} \end{bmatrix} = \begin{bmatrix} v_{x1} \\ v_{x2} \\ \dots \\ v_{xn} \end{bmatrix} \quad (9)$$

$$[A][v] = [v_x]$$

with:

$x_{1\dots n}$ measuring positions along the bar in m
$v_{x1\dots xn}$ measured complex velocity in m/s.

The overdetermined linear system of equations is solved for each frequency using singular value decomposition (SVD), i.e. the amplitudes v_+, v_{+j}, v_-, v_{-j} are calculated.
In an iterative process, the complex wavenumber with the smallest squared error of the standard is searched for. This minimizes the sum of the difference squares between measured velocity $[v_x]$ and calculated velocity $[A][v]$.

$$||[A][v] - [v_x]||_F \rightarrow min \quad (10)$$

From the complex wave number $k = k' - jk''$ with the minimum squared error, the required quantities loss factor and bending stiffness can be calculated:

$$\eta = 4\frac{k''}{k'} \quad (11)$$

$$B = \frac{m'' b (2\pi f)^2}{(k')^4} \quad (12)$$

with:

m'' mass per unit area of the bar in kg/m^2
b width of the bar in m.

To limit the computational effort, the complex wavenumber is determined for the centre frequencies of the 1/96 octave bands. The results are output in 1/24 octaves, with four values arithmetically averaged in each case.
An example of the amplitudes of equation (10) calculated from the measured velocity using the bending wave method is shown in Fig. 4.

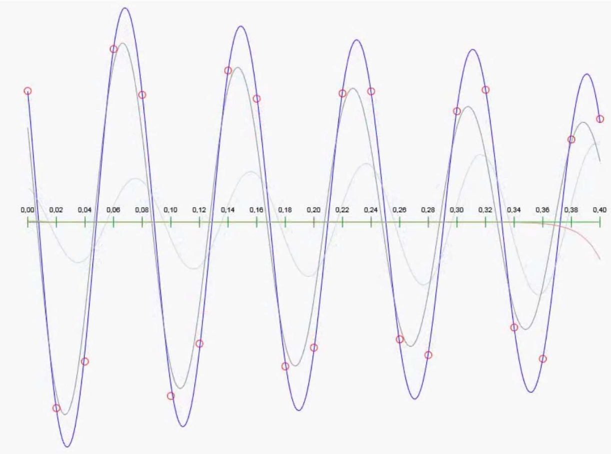

Figure 4. Measured velocity (red dots), calculated resulting velocity (blue curve), outgoing bending wave from left to right (dark grey curve), returning bending wave from right to left (light grey curve) and near field at the free bar end (purple curve).

3.5 Frequency range of the bending wave method

The upper working frequency of the bending wave method is determined by the position of the first cross mode of the bar. Below the first cross mode, there is a one-dimensional wave field, which is the basis for the model of the bending wave method. Below the first cross mode, the method is therefore unrestrictedly applicable.
The frequency of the first cross mode f_0 is determined by the following equations:

$$\lambda_B > 2\,b \tag{13}$$

$$f_o < \frac{2\pi}{(2\,b)^2}\sqrt{\frac{B/b}{m''}} \tag{14}$$

Under the idealized assumption of homogeneous bar properties over the entire bar length and bar width, as well as an ideal measurement point grid at the same phase points of the cross modes, the cross modes have no influence on the analysis of the longitudinal modes.
For a 50 mm wide and 1.0 mm thick spring steel bar, the first cross mode is about 1000 Hz. With coatings made of sheet material that allow homogeneous properties over the entire bar length and width, an upper frequency limit of up to 2000 Hz can be achieved if the loss factor was above approx. $\eta > 0.05$. In the case of liquid-applied coatings, there is usually no homogeneous coating, which typically results in an upper frequency limit of 1600 Hz.
The lower working frequency of the bending wave method results from the maximum permissible phase error between the individual measuring points. With long bending waves, only small spatial phase gradients occur. Experimental investigations have shown that the lower working frequency f_u of the test method corresponds to the frequency at which the free vibrating length of the bar is approximately 1.7 times the free bending wavelength.
The lower operating frequency f_u of the test method can be estimated as follows:

$$\lambda_B < \frac{l}{1.7} \tag{15}$$

$$f_u > \frac{2\pi}{\left(\frac{l}{1.7}\right)^2}\sqrt{\frac{B/b}{m''}} \tag{16}$$

For the 500 mm long spring steel bars described above, reliable measurement results can be determined from 125 Hz.
The method is not limited to specimens with spring steel bars as supports. The dynamic mechanical properties of bars made of silicone and CSM can also be determined using the bending wave method described. The dimensions of the specimens must be adapted according to the frequency range to be investigated.

4. COMPARISON OF BENDING WAVE METHOD AND RESONANCE CURVE METHOD

The velocities on the bar determined for evaluation according to the bending wave method also allow evaluation according to the resonance curve method. On the same test object and on the basis of the same measurement data, the evaluation can be carried out according to both methods. Since the resonance curve method only provides results in the natural frequencies, the comparison with the results of the bending wave method is only made in the resonance frequencies of the resonance curve method.
For comparison of methods, six spring steel bars each were investigated for six different nominal coating thicknesses in the temperature range from -20 °C to +70 °C. The coating material used was a liquid-applied coating with masses per unit area between 2 kg/m^2 and 4 kg/m^2. The maximum loss factors were between 0.1 and 0.3 depending on the coating thickness and temperature.
The evaluation was carried out for the modes with ordinal numbers 4 and 5, whereby the associated frequencies for the investigated bars were between 100 Hz and 400 Hz depending on the coating thickness and the temperature. In the resonance curve method, the evaluation was carried out on the basis of the resonance width for all values, i.e. also for very small values of the loss factor. Fig. 3 shows the mean value and standard deviation of the ratio between the loss factor according to the resonance curve method η_R and the loss factor according to the bending wave method η_B

depending on the absolute value of the loss factor according to the bending wave method η_B. Mean value and standard deviation were calculated in value ranges of the loss factor with a width of $\Delta\eta_B = 0.01$ or $\Delta\eta_B = 0.05$.

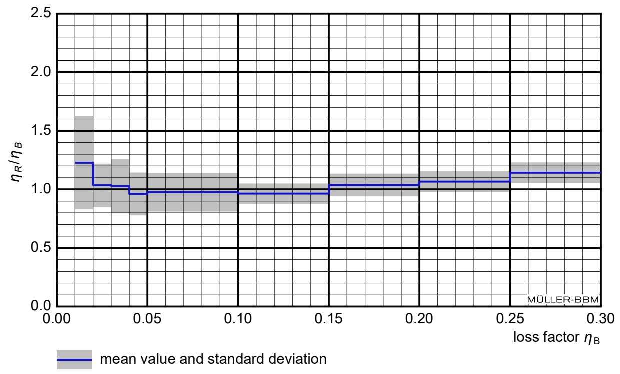

Figure 5. Ratio of the loss factor according to the resonance curve method and the loss factor according to the bending wave method depending on the absolute value of the loss factor according to the bending wave method, evaluation at the frequencies of the modes with ordinal numbers 4 and 5 of the resonance curve method: mean value and standard deviation.

The results show that for loss factors between 0.02 and 0.15, both methods give good agreement. For loss factors of more than 0.15, the values obtained by the resonance curve method are systematically higher on average than those obtained by the bending wave method. According to ISO 6721-3 [3], loss factors can only be determined with the resonance curve method at values of $\eta_R \leq 0.1$, which is confirmed by the present measurement results. Due to the superposition of the resonance curves at a higher damping, the loss factors are systematically overestimated with the resonance curve method. For loss factors of less than 0.02, the methods provide deviating results with massive scattering. It should be noted that for these very low loss factors, both the resonance curve method, due to the narrow resonance curves, and the bending wave method, due to the small spatial amplitude decrease, are less suitable. For very low damping, the reverberation method, which can also be applied to bars, is more suitable.

5. EXAMPLE

Fig. 6 shows the loss factor of a spring steel bar (dimensions 500 mm x 50 mm x 1.0 mm) with a 4.0 mm thick sound deadening coating determined using the bending wave method. Fig. 7 shows the results of the same bar determined using the resonance curve method.

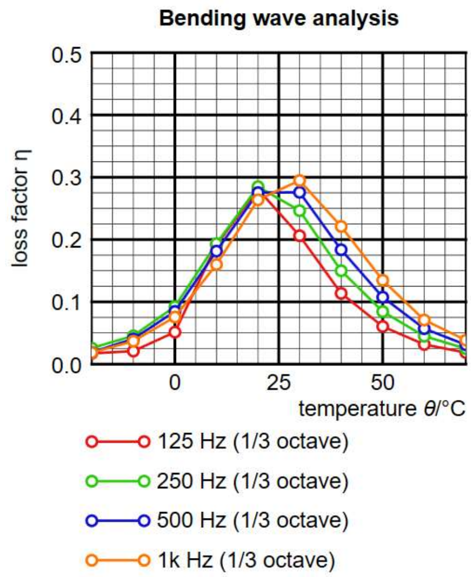

Figure 6. Loss factor of a 1.0 mm tick spring steel bar with 4 kg/m^2 sound deadening coating according to bending wave method.

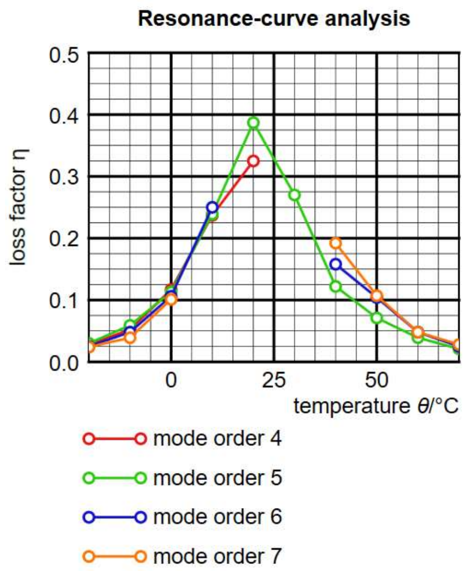

Figure 7. Same test sample as in Fig. 6 but analysed according to resonance curve method.

This example shows the overestimated loss factor when using the resonance curve method. As already explained, this behaviour occurs on statistical average with loss factors above $\eta > 0.15$ but not systematically with every bar.
For easier interpretation of the frequency- and temperature-dependent data, the representation with the help of a colour scale has proven to be useful (see Fig. 8).

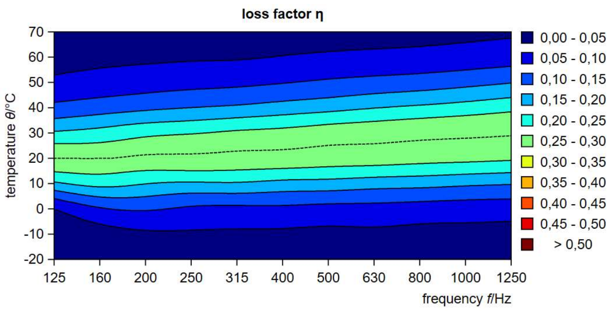

Figure 8. Same results as in Fig. 6 but representation of loss factor with colour scale and with bandwidth 1/24 octaves.

6. SUMMARY

The advantages and disadvantages of three common methods (resonance curve method, power injection method and reverberation method) were described for the determination of the loss factor. Based on the test device for the resonance curve method and the bending wave equation according to the theory of the so-called shear-rigid Bernoulli beam, another method, the bending wave method, was developed and explained in detail here. The main advantages of the bending wave method compared to the resonance curve method are the extended range of validity to very high loss factors ($\eta > 1$), the high spectral resolution and the measurement range extended to high frequencies.
A metrological comparison of the resonance curve method and the bending wave method on the same test objects has shown that the loss factors of both methods agree very well on average in their validity range of $\eta = 0.02…0.15$. With the resonance curve method, the loss factors above the validity range of the method, i.e. at loss factors of $\eta_R > 0.15$, are overestimated on statistical average due to the overlap of the resonance curves.

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