DOI: 10.61782/fa.2023.0888

Subwavelength Broadband Perfect Absorption For Unidimensional Open-duct Problems

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ABSTRACT

Passive metamaterials provide efficient solutions for sound absorption in the low-frequency regime with deep subwavelength dimensions. They have been extensively applied in unidimensional reciprocal problems considering that an incident wave is either reflected or transmitted at the outlet boundary. However, the generalized problem with impedance boundary condition is not well understood yet. This work presents a general design methodology of metamaterial absorbers for open-duct problems, which is a special case of impedance outlet boundary encountered in broad practical applications, for example, noise-attenuation problems in heat ventilation and air conditioning (HVAC) systems. By properly using monopolar point scatterers made of arrays of Helmholtz resonators, the design process is significantly simplified: the transfer matrix modelling is sufficiently accurate to describe the acoustic response of the system. A single monopolar point scatterer is insufficient to attenuate both the reflected and radiated waves; a frequency-dependent maximum absorption exists and is derived analytically. To go beyond this absorption bound and achieve perfect absorption, at least two point scatterers are necessary. Specific designs are provided and validated experimentally for maximum or perfect absorption, either at single frequencies or over specific frequency bands.

Keywords: *absorption bound, open-duct problems, perfect absorption, radiation problems*.

1. INTRODUCTION

Among the design strategies of passive meta-absorbers, the use of coupled resonators, of either monopolar or dipolar types, has been extensively studied (see, for example, Chapter 3 of [1], Chapter 5 of [2], etc.). In the unidimensional (1D) reflection problem (either with a rigid boundary [3] or a soft boundary [4]), perfect absorption can be realized at a given frequency by using a single resonator. In the opposite, the maximum absorption coefficient that can be achieved with either a single monopolar or a dipolar type resonator is $\alpha max=1/2$ in the 1D transmission problem [5]; to yield perfect absorption, at least two coupled resonators are necessary, because both types of resonances at the same frequency are required to suppress the reflection and the transmission simultaneously [5]. Notice that the aforementioned strategies are based on the mirror symmetries arising from the boundary of the considered system where the acoustic wave is either totally reflected or totally transmitted. Thus, they can be well applied merely in the reflection and transmission problems. In contrast, the outlet boundary does not preserve mirror symmetries in the 1D open-duct problem, because the corresponding acoustic boundary condition has both resistive and reactive contributions that are frequency dependent. Due to this added complexity, the design strategies of passive absorbers in open-duct systems are not yet well developed, and thus, related applications are still limited.

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The general aim of this work is to reveal the absorption behaviors of passive metamaterials in the 1D open-duct problem and to develop corresponding design strategies. To ensure the ventilation in the duct, as well as to simplify the design process, only monopolar resonators loaded to the waveguide are considered. In this scenario, either a single resonator or multiple resonators at the same axial position along the wave direction can be modelled as a point scatterer in the subwavelength regime. It is proved that the absorption coefficient of a single point scatterer cannot exceed a frequency-dependent upper bound αmax. General design methodologies are provided to realize αmax either at a single frequency or over a specific frequency band. In a further step, we show how to break this absorption bound toward perfect absorption by employing coupled point scatterers.

2. OUTLINE OF THE MODELING APPROACH

The absorption of both reflected and radiated acoustic waves by using monopolar point scatterers in the unidimensional (1D) open-duct problem is studied in this work. The absorption coefficient is defined as

$$\alpha = 1 - E_{\mathrm{Ref}} - E_{\mathrm{Rad}}, \tag{1}$$

where E_{Ref} and E_{Rad} are the reflection and radiation energies normalized by the incidence energy, respectively.

In the case that a single point scatterer is used, which is illustrated in Figure 1(a), the maximum absorption coefficient is derived as

$$\alpha_{\max} = \frac{1}{2}\left[1 + \sqrt{\frac{(\theta_{\mathrm{Rad}} - 1)^2 + \chi_{\mathrm{Rad}}^2}{(\theta_{\mathrm{Rad}} + 1)^2 + \chi_{\mathrm{Rad}}^2}}\right], \tag{2}$$

which is in general frequency dependent and less than unity. Note that in the 1D problem, the acoustic response of the open end can be modelled by the specific radiation impedance [6] $\zeta_{\mathrm{Rad}} = \theta_{\mathrm{Rad}} + \mathrm{i}\chi_{\mathrm{Rad}}$ under the $e^{\mathrm{i}\omega t}$ convention. To achieve $\alpha_{\max}$ at a single frequency, two conditions are necessary: (1) the scatterer provides the optimal impedance ζ_{opt} and (2) the distance between the scatterer and the open end is the optimal value L_{opt}. In contrast, to achieve perfect absorption ($\alpha = 1$), at least two point scatterers are needed as illustrated in Figure 1(b). Moreover, it is required that the scatterers should achieve the optimal impedance values:

$$\begin{cases} \zeta_{\mathrm{HR1}} = 0 \\ \zeta_{\mathrm{HR2}} = -\mathrm{i}\sin(k_0 L_2)\mathrm{e}^{\mathrm{i}k_0 L_2} \end{cases}, \tag{3}$$

The above definition (Eq. (1)) and results (Eqs. (2) and (3)) in the open-duct problem are direct generalizations of the 1D reflection [3,4] and transmission [5,7] problems. The reflection or transmission problem corresponds to the special case when the boundary impedance ζ_{Rad} approaches to infinity or 1+0i, respectively.

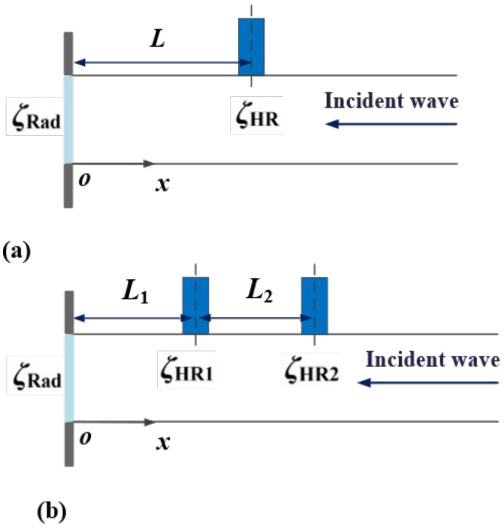

Figure 1. Schematic illustrations of metamaterials made of point scatterers in the 1D open-duct problem: (a) Single point scatterer. (b) Two coupled point scatterers.

3. SPECIFIC DESIGNS FOR BROADBAND MAXIMUM AND PERFECT ABSORPTION

To realize αmax and perfect absorption as well, we propose a general design strategy, in which monopolar point scatterers are employed. A circular waveguide (whose radius is 5 cm) with an open end in the baffled wall is considered. Arrays of Helmholtz resonators in parallel with the waveguide, i.e., in both the circumferential and the wave directions, are used to play the roles of the point scatterers. As predicted by Eq. (2), the maximum absorption is close to perfect in the low frequency range. Specifically, for this circular waveguide, αmax ≥ 0.9 when f ≤ 500 Hz. Thus, a compact and efficient absorber is realized by utilizing this property with a single point scatterer made of detuned Helmholtz resonators, which possesses deep subwavelength size and achieves broadband (250 Hz to 500 Hz) maximum absorption as shown in Figures 2(a) and 2(b). In contrast, coupled point scatterers are necessary to break the absorption bound and thus to achieve perfect absorption. A specific design is provided with numerical and experimental validations over the frequency band 450 Hz to 1000 Hz as shown in Figures 2(c) and 2(d).

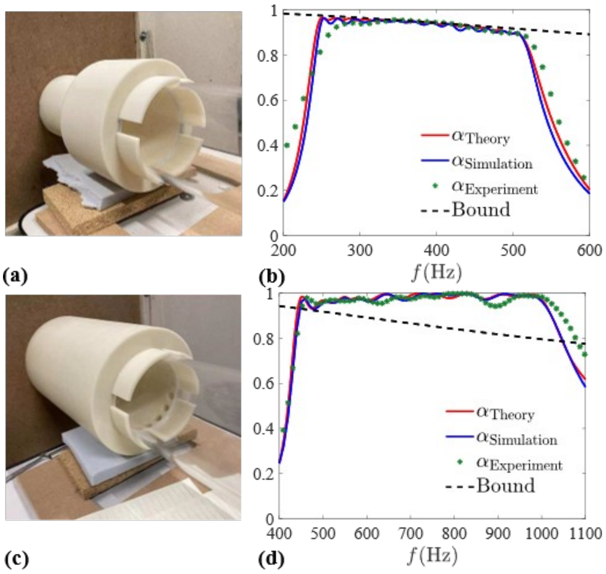

Figure 2. Single-point-scatterer metamaterial for broadband maximum absorption: (a) Sample working from 250 Hz to 500 Hz. (b) Absorption coefficient of the sample: comparison of the theory, simulation, and measurement. Coupled-point-scatterer metamaterial for broadband perfect absorption: (c) Sample working from 450 Hz to 1000 Hz. (d) Absorption coefficient of the sample: comparison of the theory, simulation, and measurement.

4. CONCLUSION

The absorption of both reflected and radiated acoustic waves by using monopolar point scatterers in open-duct problems is studied in this work. With a single point scatterer, the maximum absorption coefficient ($\alpha_{\max}$) is frequency dependent and less than unity in general. A general design method is proposed to achieve broadband $\alpha_{\max}$ and perfect absorption, by using arrays of Helmholtz resonators. The method is validated both numerically and experimentally.

For more details, the reader can refer to [8].

5. ACKNOWLEDGMENTS

All the authors gratefully acknowledge the support of Valeo. J.-P. G. and V. R.-G. would like to acknowledge the support of the ANR-RGC METARoom project (ANR-18-CE08-0021).

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