DOI: 10.61782/fa.2023.0898

forum acusticum 2023

EVALUATION OF A GENERAL POWER-FLOW METHOD FOR VIBROACOUSTICS MID-FREQUENCY PROBLEMS

Thiago Morhy Cavalcante[1*] **Júlio Apolinário Cordioli**[1]
[1] Laboratory of Vibration and Acoustics (LVA), Federal University of Santa Catarina (UFSC), Brazil

ABSTRACT

Most engineering structures are composed of two distinct sets of components: rigid elements, e.g., a pillars/column, and more flexible elements, such as plates, shells or cavities. At a specific frequency, these components may vibrate in distinct wavelengths simultaneously, resulting in the so-called mid-frequency problem. Different methods have been proposed to deal with the problem, with the Hybrid FE-SEA Method standing out. However, the Hybrid FE-SEA method displays a limitation when modeling flexible elements using SEA: their vibrational behavior needs to be approximated from elementary components (beam, plates, etc), resulting in large approximations for complex components. To address this limitation, a novel method denoted the Generalized Hybrid FE-SEA has been proposed, allowing for irregular elements to be fully described in a power-flow framework. So far, the method has been evaluated only for high-frequency problems and has yet to be analyzed in the case of mid-frequency problems, which is the aim of this work. The work presents an overview of the method formulation and compares the results obtained by the novel method with the established Hybrid FE- SEA and Monte Carlo Simulations for two numerical cases. Results show that the Generalized Hybrid FE-SEA displays a superior performance when compared to the Hybrid FE-SEA.

Keywords: *Vibro-acoustics, Mid-frequency problems, Hybrid FE-SEA, Statistical Energy Analysis, Finite Element Method*

**Corresponding author*: *morhyt@gmail.com.*

1. INTRODUCTION

Vibro-acoustic systems in engineering industries, such as automotive, aerospace, and marine, are analyzed using various numerical methods. The choice of method is guided by the application being considered, the system's configuration, and the degree of complexity. Additionally, computational cost is a crucial consideration, and sometimes a method that is capable of optimal results is not suitable for the task due to its high computational cost.

These aspects are typically related to the frequency spectrum adopted for the application. For low frequency analysis, fewer modes need to be represented, resulting in lower processing costs and more coherent deformation. Methods like the Finite Element Method (FEM) [1] for structural vibration and the Boundary Element Method (BEM) for acoustical applications exhibit optimal performance, as coarser meshes are required at these frequency regions. These methods can also describe the highest level of detail for the system's configuration.

For high frequency analysis, the highly incoherent deformation and concentration of modes demand the model to account for a high level of detail in the system, while the uncertainties in the model require a statistical description of the results. Approaches like the Statistical Energy Analysis (SEA) [2] became an important alternative for this type of scenario, as the space and ensemble-averaged descriptions made in the framework serve as a superior approximation for the uncertain diffuse field produced in the system. However, these descriptions are generally obtained from analytical formulations for simple structural elements, resulting in reduced processing costs but with some strong simplifications of the system.

In the case of mid-frequency problems, where small and large wavelength deformations are present at the same time in different components of the system, the established method is the Hybrid FE-SEA [3]. This method connects the system's dynamic equilibrium with a power-flow model, established through the use of the reciprocity relationship between the direct-field radiation and the diffuse reverberant loading [4]. This allows the diffuse reverberant field of the components to be directly derived from the direct-field's impedance from their boundary. Most implementations of this method in commercial software assume that deformations in a diffuse field can be considered to be spatially incoherent, resulting in enormous computational cost reductions as the power-flow model can then be computed by already robust and established analytical formulations of SEA.

Although these analytical formulations are excellent approximations to diffuse field descriptions, their range of possible configurations is limited to elementary structural elements (beams, plates, etc.). As a result, irregular components that assume small wavelength deformations in Hybrid FE-SEA (and SEA as well) are sectioned into multiple elementary sub-components/subsystems. Commercial software typically offers a set of possible subsystem configurations, such as symmetric acoustic cavities, flat and curved plates [5].

In complex systems, irregular components are not only sectioned but also considerably simplified to ensure a minimum amount of modes in most subsystems. This simplification may imply the loss of important information about the subsystem's deformation, and therefore, a more generic description of the subsystem becomes appealing. This could be obtained with the support of an FE model of these components. In the case of a periodic FE model, the direct field impedance is captured by the dispersion curves [6]. If a standard FE model is used, ensemble averaging techniques can be applied to derive the direct field impedance [7]. These direct-field impedances are used to model the exchange of energy between the components of the system, but the power dissipated by these components is still analytically computed in both SEA and Hybrid FE-SEA.

These discussed generic descriptions were also evaluated in a power-flow model [8, 9], where high-frequency problems were considered in the analysis. In this context, this more generic approach is denoted as Numerical SEA, presenting a direct comparison to the established SEA. The results obtained exhibited the versatility of the method for different configurations, including elementary and orthotropic materials, as well as complex junctions. It also showed the superiority of Numerical SEA over SEA for irregular geometries. When there are components vibrating in large wavelengths in the system, a hybrid formulation is employed by the novel method. To reduce confusion, in this context, the method is denoted as Generalized Hybrid FE-SEA.

The goal of the present work is to evaluate the performance of the novel method in a mid-frequency problem, which has not been done yet, and compare it with Hybrid FE-SEA. A brief description of the two hybrid methods is presented in the following sections. The implementation required for the novel method to be applied in the hybrid context and the obtained results are also presented.

2. HYBRID FE-SEA METHOD

The established method for mid-frequency problems involves dividing the system into two types of subsystems: deterministic subsystems that vibrate in large wavelengths and contain a small number of modes in the analyzed frequency spectrum, and statistical subsystems that represent the rest of the subsystems and exhibit small wavelength deformations and a high concentration of modes. Due to the highly coherent behavior of deterministic subsystems, they can be represented using a coarse mesh for their deformation, and are modeled using a dynamic stiffness matrix $\mathbf{D_d}$. In the case of statistical subsystems, their deformations are modeled by diffuse reverberant fields, which can be fully described by direct-field impedances $\mathbf{D_{dir}}$ from the boundaries/connections and an intensity parameter C [4].

The cross-spectral response of the coupled system is given by [3]

$$\mathbf{S_{qq}} = \mathbf{D_{tot}}^{-1}\left[\mathbf{f_{ext}}\mathbf{f_{ext}}^{H} + \sum_{i} C_i \,\mathrm{Im}\left\{\mathbf{D_{dir,i}}\right\}\right]\mathbf{D_{tot}}^{-H}, \tag{1}$$

where the total dynamic stiffness matrix is the sum of the deterministic dynamic stiffness matrix and the direct-field dynamic stiffness matrices of the statistical subsystems

$$\mathbf{D_{tot}} = \mathbf{D_d} + \sum_{i} \mathbf{D_{dir,i}}, \tag{2}$$

where the i subscript corresponds to the ith statistical subsystem. The vector $\mathbf{f_{ext}}$ represents the external loading. The parameter C_i represents the intensity of the ith diffuse-field and is derived by idealizing a power-flow model between the statistical subsystem's diffuse wavefields. This parameter is usually denoted as diffuse or diffuse-field amplitude. Similar to SEA, the steady-state power-flow balance equation for the ith subsystem is defined as

$$\langle \Pi_i^{\rm in,dir} \rangle = \langle \Pi_i^{\rm out,rev} \rangle + \langle \Pi_i^{\rm diss} \rangle, \tag{3}$$

where $\langle \Pi_i^{\rm in,dir} \rangle$, $\langle \Pi_i^{\rm out,rev} \rangle$ and $\langle \Pi_i^{\rm diss} \rangle$ are, respectively, the ith statistical subsystem's ensemble average power being injected to its diffuse-field (by the radiation of the direct-field), the ensemble average power being ejected by its diffuse-field to other subsystems and the ensemble average power dissipated by the subsystem. The first two power-flow contributions ($\langle \Pi_i^{\rm in,dir} \rangle$ and $\langle \Pi_i^{\rm out,rev} \rangle$) are derived in accordance with the dynamic equilibrium of the system, taking into account the subsystems' impedances $\mathbf{D_{dir,i}}$ and $\mathbf{D_d}$ [3]. Similar to SEA, the dissipated power $\langle \Pi_i^{\rm diss} \rangle$ is defined as

$$\langle \Pi_i^{\rm diss} \rangle = \pi \eta_i \langle E_i \rangle = \pi \omega \mathcal{M}_i C_i, \tag{4}$$

where η_i and $\mathcal{M}_i$ are, respectively, the damping loss factor and the dissipation coefficient of the ith statistical subsystem. The relationship between the average vibrational energy $\langle E_i \rangle$ and the diffuse amplitude C_i has already been established [4] and is defined as

$$\langle E_i \rangle = \pi \omega n_i C_i, \tag{5}$$

where n_i is the modal density of the ith statistical subsystem. Therefore, the dissipation coefficient becomes equivalent to the modal overlap factor

$$\mathcal{M}_i = \pi \eta_i n_i. \tag{6}$$

Finally, the power-flow balance equation can be expressed as

$$\begin{bmatrix} h_1 & \dots & -h_{1,\rm N} \\ \vdots & \ddots & \vdots \\ -h_{\rm N,1} & \dots & h_{\rm N} \end{bmatrix} \begin{bmatrix} C_1 \\ \vdots \\ C_{\rm N} \end{bmatrix} = \frac{1}{\pi\omega} \begin{bmatrix} \Pi_{\rm in,1}^{\rm ext} \\ \vdots \\ \Pi_{\rm in,N}^{\rm ext} \end{bmatrix}, \tag{7}$$

where $h_{\rm i}$ is defined as

$$h_{\rm i} = \mathcal{M}_{\rm i} + \mathcal{M}_{\rm d,i} + \sum_{j \neq i} h_{\rm j,i}, \tag{8}$$

and where $\mathcal{M}_{\rm d,i}$, $h_{\rm i,j}$ and $\Pi_{\rm in,i}^{\rm ext}$ represent, respectively, the dissipation coefficient from the deterministic subsystems, the transfer coefficient from the ith to the jth statistical subsystem, and the input power from external loads to the ith statistical subsystem. The derivations related to all these coefficients were omitted in this paper but can be found in the references [3, 9, 10]. Once the diffuse-field amplitudes C_i are determined, they can be related to their subsystem's energies and, therefore, to the statistical subsystem's engineering units. The response of the deterministic subsystems can be directly obtained by solving Eqn. (1), as the diffuse amplitudes have already been determined.

3. GENERALIZATION OF THE METHOD

The power-flow balance of the ith statistical subsystem is fully described by considering their direct field dynamic stiffness and dissipation coefficient. In this work, we generalize this method by deriving these parameters for subsystems with generic configurations beyond elementary scope. To achieve this, we employ standard finite element (FE) models of the statistical subsystems and average their FE matrices in an ensemble to derive proper diffuse contributions for the power-flow derivation. In order to handle the computational costs associated with large-scale problems, we utilize an efficient averaging technique [7], which allows for significant reduction in processing time. The derivation of $\mathbf{D_{dir,i}}$ is achieved by employing a reduced model of the ith statistical subsystem. Similarly, for $\mathcal{M}_i$, we utilize the same averaged matrices in combination with the FE formulation for intrinsic mechanical damping [11]. The detailed derivations of these parameters in this generalized context have been previously presented [9, 10].

Implementing these generic derivations involves performing matrix multiplications, which can be computationally expensive due to the use of FE models. To improve computational efficiency when modeling complex systems in mid and high-frequency problems, model reduction techniques are employed. Specifically, projections into the modal basis are utilized for the interior of the statistical subsystems, where most of the degrees of freedom are concentrated, as well as for the deterministic subsystems [10]. After reducing the model, the only remaining nodal degrees of freedom are those located at junctions/excitation points connected solely to statistical subsystems, which are retained to enforce compatibility. Another important aspect in implementing these generalized descriptions is related to the energetics of the diffuse wavefields. If no post-processing is applied to the FE matrices, a single lumped generic wavefield is defined for each statistical subsystem. However, this condition may lead to an overestimation of energy dissipation as it assumes equipartition of energy between all possible wavefields inside the subsystem. For example, in the case of a flat plate, this assumption is not reliable since in-plane modes dissipate and exchange far less energy than out-of-plane modes. However, for irregular geometries, the wavefields tend to become coupled, and this overestimation tends to diminish. A detailed process for partitioning the out-of-plane and in-plane wavefields from a lumped wavefield of a flat plate has been previously presented [10].

4. NUMERICAL EVALUATION

The novel method was evaluated through two numerical cases and compared with the results of the established Hybrid FE-SEA method as well as a reference curve. The reference curve was obtained from a Monte Carlo analysis using finite element (FE) models, where an ensemble of randomized systems was generated, and the ensemble average served as the reference curve. The degree of convergence between the hybrid methods and the reference curve indicates the effectiveness of each method in modeling the system. In this study, the randomization of the FE Monte Carlo samples was achieved by applying constraints (either clamped or pinned) to random portions of the statistical subsystems' domain. A convergence analysis was conducted to determine the required number of samples for each numerical case in the FE Monte Carlo ensemble.

The FE matrices and 3D information used in both the novel method and the FE Monte Carlo approach were extracted from the VAOne software [5] and post-processed in MATLAB [12]. For the established Hybrid FE-SEA method, the results were directly obtained from VAOne. Additionally, a damping loss factor of 1% (0.01) was assumed for all analyzed subsystems, FE models were discretized with six elements per wavelength, and modes with natural frequencies up to twice the maximum analyzed frequency were extracted. The material properties of the analyzed subsystems, which consist of aluminum or steel, are listed in Table 1.

In each case, a single subsystem was excited by a transverse point force of 1N, represented by a purple arrow. In the FE Monte Carlo model, the point force was randomly positioned within the excited subsystem for each sample, with the condition of being far from discontinuities to avoid wavefield coupling.

Table 1. Aluminum and Steel material properties.

Properties	Aluminum	Steel
Density ρ [kg/m³]	2700	7800
Young's Modulus E [GPa]	71	210
Shear Modulus G	26.7	80
Poisson's Ratio ν	0.329	0.3125

4.1 Co-planar flat plates coupled by a beam

The first numerical case involves two co-planar aluminum plates connected to a rigid steel beam at four specific points (two on each plate), as shown in Figure 1. The connections are represented by yellow circles in the figure, with one circle hidden behind the beam. The plates have an area of 0.723 m^2, and the excited plate (green) has a thickness of 1 mm, while the receiver plate (orange) has a thickness of 2 mm. The steel beam, with a length of 1.1 m, has a rectangular cross-section measuring 0.1 m by 0.08 m and a thickness of 10 mm. The plates are modeled as statistical subsystems with partitioned wavefields (in-plane and out-of-plane), while the beam is considered deterministic.

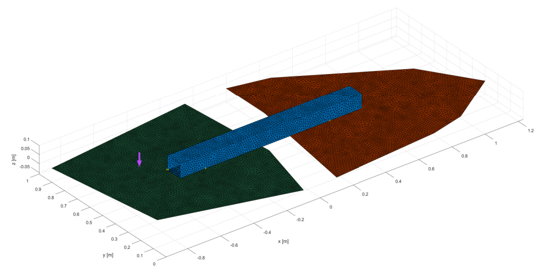

Figure 1. Numerical case 1 - 3D visualization.

The results for the excited and receiver plates are shown in Figure 2 and Figure 3, respectively. In these graphs, the gray curves represent the results from each sample of the FE Monte Carlo analysis, while the black curve represents the mean value, which serves as the reference result. For the excited plate, the response is mainly influenced by the internal mechanics of the subsystem and the external loading, resulting in a simple frequency-dependent decay for all methods. This is due to the high modal overlapping observed in the flat plate. On the other hand, the receiver plate exhibits specific predominant modes in the results, which is a consequence of

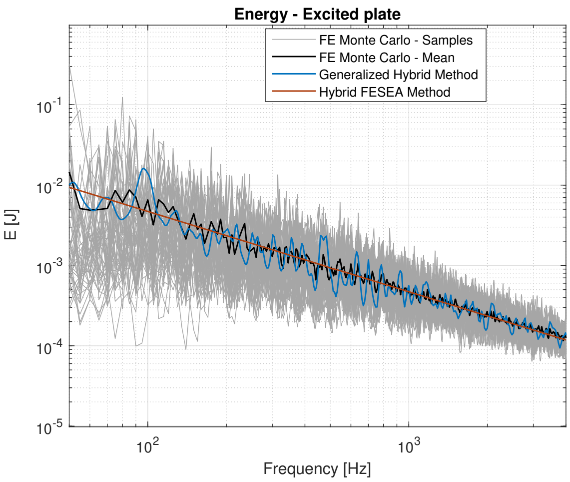

Figure 2. Numerical case 1 - Excited plate's vibrational energy results.

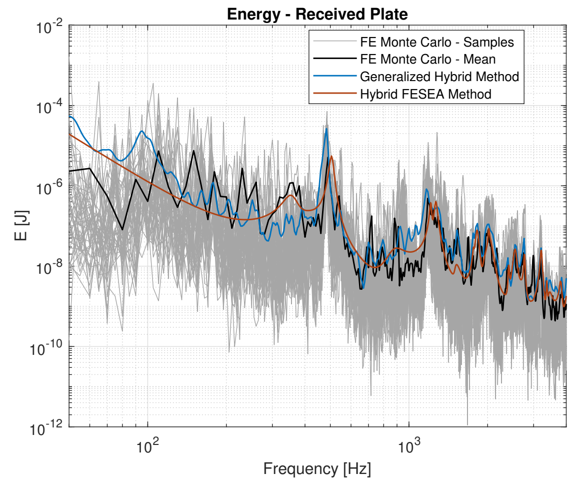

Figure 3. Numerical case 1 - Receiver plate's vibrational energy results.

the modal behavior of the steel beam that connects both plates. The stiff beam exhibits strong and spatially coherent deformation due to its low modal density, thus significantly influencing the response of the receiver plate. In both cases, the hybrid methods demonstrate similar results to the reference, indicating that the novel method can accurately describe the wavefields of the statistical subsystems. Assuming a lumped wavefield for the statistical subsystems leads to an overestimation of energy dissipation in the in-plane wavefield, as shown in Figure 4 (FE Monte Carlo sample results are omitted). In terms of computational cost, the established Hybrid FE-SEA method outperforms the other methods (Figure 5), thanks to the analytical formulations used to model the wavefields of the statistical subsystems.

4.2 Cube beam framework

The second case, illustrated in Figure 6, consists of four flat plates (green) connected by their edges to form a cube-shaped system with open top and bottom sides. The plates are connected to a single beam framework (orange) that has a hollow structure with a square cross-section measuring 2.52 cm on each side and a thickness of 3 mm. The beam framework is made of steel. Each plate is identical, with a square area of 0.4724 m², a thickness of 2 mm, and is made of aluminum. One of the plates is excited, while all the edges of the plates are connected to the beam framework. In addition, a clamped boundary condi-

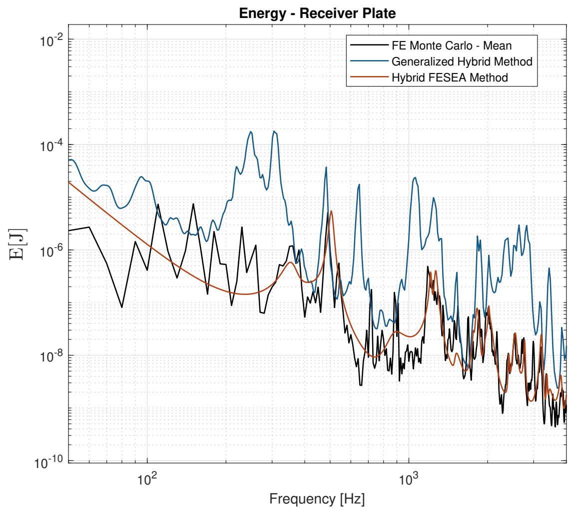

Figure 4. Numerical case 1 - Vibrational energy results for the receiver plate with a lumped wavefield.

tion is applied to the two bottom outer edges of the beam framework (Figure 7). The plates are modeled as statistical subsystems, while the beam framework is considered deterministic. The wavefields of the plates are divided into out-of-plane and in-plane components.

The vibrational energy results for the front excited plate and a side plate are presented in Figure 8 and Fig-

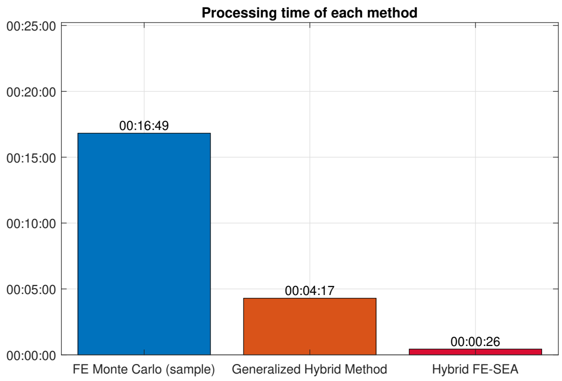

Figure 5. Numerical case 1 - Computational processing time demanded.

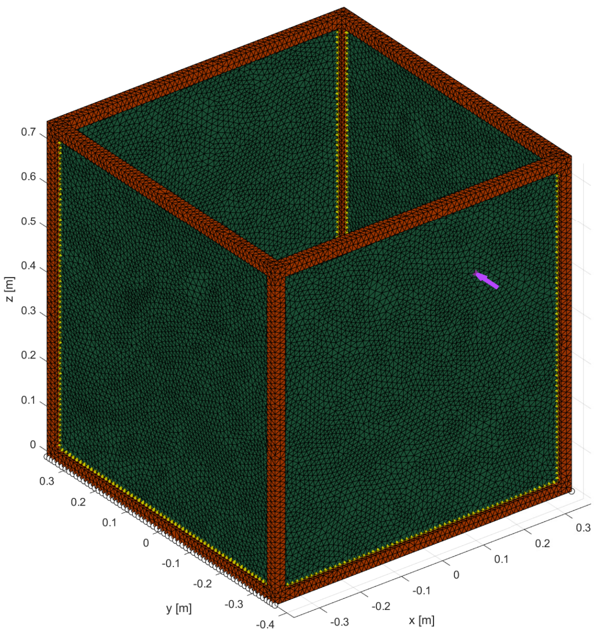

Figure 6. Numerical case 2 (3D visualization).

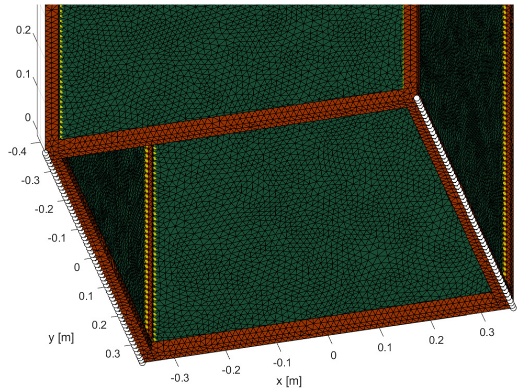

Figure 7. Numerical case 2 - Constrained nodes (white balls).

ure 9, respectively. Both methods yield comparable results to the reference for the excited front plate. However, a discrepancy is observed between the established hybrid method and the reference for the side plate's response at higher frequencies. This discrepancy arises from the simplifications made by the analytical formulations used in the established hybrid method, which assume straight-line junctions between the plates and the beam framework. In this second case, where square junctions are present, the simplifications lead to deviations. In contrast, the novel method demonstrates excellent convergence with the reference, indicating that the generalized descriptions significantly improve the analysis of complex vibro-acoustic problems. Moreover, in terms of computational cost, the novel method is much more efficient compared to a single sample of the FE Monte Carlo ensemble and the established Hybrid FE-SEA method (Figure 10). The discrepancy in computational cost between the novel method and the established method is mainly attributed to the software's simplifications, which assume four separate straight-line junctions for each plate instead of considering a generic junction.

5. CONCLUSIONS

The Generalized Hybrid FE-SEA method shows promising performance as an alternative for modeling complex vibro-acoustic systems, particularly in mid-frequency problems, when compared to established methods. In cases where the system primarily consists of elementary configurations (as observed in the first numerical case), the novel method produces equivalent results to the established hybrid approach, albeit with higher computational costs. However, for cases involving complex configurations (as demonstrated in the second numerical case), the analytical formulations used in the established Hybrid FE-SEA method encounter difficulties in accurately

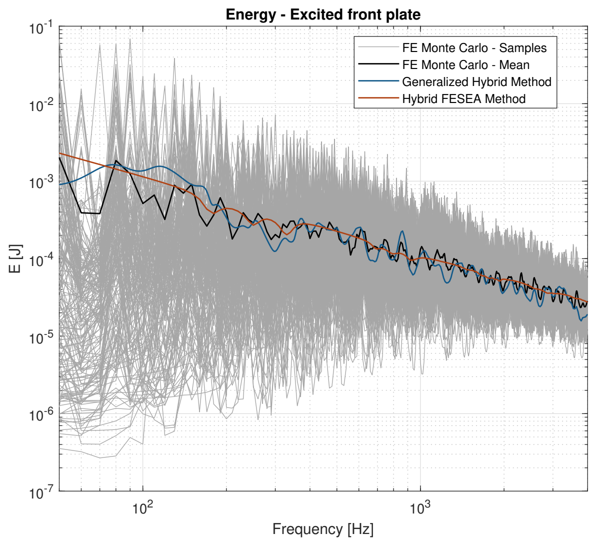

Figure 8. Numerical case 2 - Excited front plate's vibrational energy results.

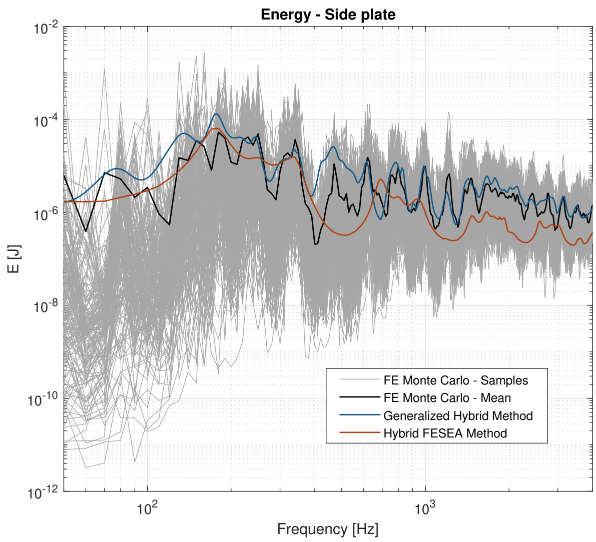

Figure 9. Numerical case 2 - Receiver side plate's vibrational energy results.

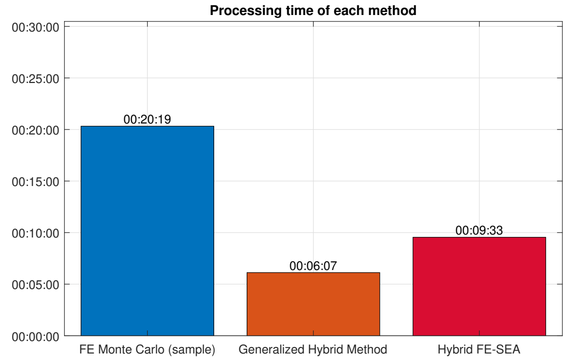

Figure 10. Numerical case 2 - Computational processing time demanded.

defining wavefields for the statistical subsystems. On the other hand, the generic descriptions employed in the novel method successfully model the power flow between the diffuse wavefields of the statistical subsystems with reduced computational processing, making it a powerful tool for vibro-acoustic modeling.

Further evaluations and developments of the generic descriptions are still required. The inclusion of acoustic cavities is essential in vibro-acoustic scenarios, necessitating the exploration of area connections between the structural and acoustic domains. A more in-depth analysis of the partitioned wavefield is also necessary to develop a generic process for identifying and partitioning wavefields in irregular configurations. Numerous opportunities exist for future research in the development of complex vibro-acoustic modeling.

6. ACKNOWLEDGMENTS

This work is the result of a partnership between the Laboratory of Vibration and Acoustics (LVA) from the Federal University of Santa Catarina (UFSC) and the research and development group from the software developer company ESI-Group.

7. REFERENCES

[1] L. Meirovitch, *Fundamentals of Vibrations*. United States: Waveland Press, 2010.

[2] R. H. Lyon and R. G. Dejong, *Theory and application of statistical energy analysis*. United States: Butterworth-Heinemann, 1995.

[3] P. J. Shorter and R. S. Langley, "Vibro-acoustic analysis of complex systems," *Journal of Sound and Vibration*, 2005.

[4] P. J. Shorter and R. S. Langley, "On the reciprocity relationship between direct field radiation and diffuse reverberant loading," *Journal of Acoustical Society of America*, 2005.

[5] ESI-Group, "Vaone 2019.1." Software, 2019. https://www.esi-group.com/products/va-one.

[6] V. Cotoni, R. S. Langley, and P. Shorter, "A statistical energy analysis subsystem formulation using finite element and periodic structure theory," *Journal of Sound and Vibration*, 2008.

[7] H. Devriendt *et al.*, "Coupling loss factor calculation through lorentzian-weighted frequency averaging of the direct field dynamic stiffness," in *NOVEM*, 2015.

[8] L. Alimonti *et al.*, "The diffuse field reciprocity relationship as a framework for a general power flow model," in *NOISE-CON*, 2019.

[9] M. Hinz, "Statistical energy analysis modeling of deterministic subsystems," Master's thesis, Federal University of Santa Catarina (UFSC), Florianópolis, 2021.

[10] T. C. Morhy, "Evaluation of novel and established methods used in complex vibro-acoustic analysis," Master's thesis, Federal University of Santa Catarina (UFSC), Florianópolis, 2022.

[11] B. R. Mace and P. J. Shorter, "Energy flow models from finite element analysis," *Journal of Sound and Vibration*, 2000.

[12] Mathworks, "Matlab 2020b." Software, 2020. https://www.mathworks.com/products/matlab.