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A COMPARATIVE ANALYSIS OF DIFFUSION MODEL AND RAY TRACING IN ROOM ACOUSTICS THROUGH GENERATIVE GEOMETRIC FORMS

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ABSTRACT

Room acoustics design of various spaces of different forms and functions necessitates reliable tools of analysis. Computer-aided simulations have replaced physical models, since 1960s, in both research and mostly in practice, due to both time and cost efficiency. There are various room acoustics modeling approaches in sound field analysis. Different methods have different strengths and weaknesses. This study aims to compare diffusion model (DEM) to ray-tracing, to investigate potentials or shortages of both techniques, and coherence to one another in quantification of basic acoustical parameters but as well in visualization of sound propagation in rooms. This research uses primitive geometric forms and their different combinations. Produced ten different geometric entities, which may well represent a real architectural space, are then utilized to produce room impulse responses in a ray-tracing simulation and as well in a diffusion equation model computation. Identical source-receiver configurations and material input are used for controlled experiments. T30 values and relative sound energy level differences are compared. Energy flow vectors are traced in time-dependent solutions of DEM, to check the characteristics of energy distribution within different geometric domains. Materials are varied over specific surfaces to check the consequence of inhomogeneous absorption for investigated methods.

Keywords: *diffusion equation model, ray-tracing simulations, room acoustics, primitive forms*

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1. INTRODUCTION

Room acoustics design in architecture is critical for many types of functional spaces. The typology and the scale of an acoustically critical indoor space can vary from an auditorium to a classroom, from an airport terminal or sports arena to a worship space [1-4]. Acoustical comfort parameters are essential to consider during the early design phases of projects. Computer aided acoustic modeling and simulations are highly utilized to evaluate a space under generation, as physical scale models are both time consuming and economically less efficient in comparison to today's computational technologies.

There are various room acoustic modeling techniques, which can basically be grouped under wave based and geometric acoustics simulations [5]. Although being the most accurate, wave-based techniques are computationally not efficient for mid to high frequencies and especially challenging for complex spaces as in real-case architectural scenarios. Geometrical acoustics (GA) simulations try to approximate the sound waves as rays, neglecting mostly wave-based phenomena, so less accurate but computationally much plausible to be used in both research and practice. Among the GA approaches ray tracing and image-source method have been applied over 50 years by now. Image source method has high accuracy in estimating early reflections and is employed in real-time auralizations [6], but cumbersome to apply for higher order reflections [7-8]. Commercially available room acoustics simulation software is mostly using hybrid models; typically, a combination of ray and image tracing [9]. The methods in relation to real-time auralizations are out of the scope of this study, so not listed here.

A comprehensive study on GA can be found in Savioja and Svensson's review [7], which excludes diffusion model. Diffusion equation-based modeling (DEM) of sound energy decay is much recent in comparison to ray-tracing as a GA method. DEM has found application area in room acoustics analysis of monumental spaces as well as coupled volumes [4, 10-13]. One advantage of DEM is that instead of tracing rays from source to receiver to collect RIRs, DEM utilizes a volumetric grid (meshed domain), where the sound energy propagation can be traced in a time dependent solution providing information on the energy flux. More discussion on the progress of DEM applications in room acoustics is provided under methodology section.

Different models have different limitations as well as strengths. As, there are still many research questions in the field in relation to efficiency or purpose-specific benefits, this research compares DEM, one comparatively recent technique in room acoustics computations, to a more prevalent and established method that is ray-tracing. There are a couple of studies utilizing DEM and ray-tracing in analysis of different venues, by the aid of input from field tests [4, 13] for majorly complex structures. In order to test the two methods in a much-controlled environment, this study utilizes simple structures generated out of primitive forms. Accordingly, ten different geometric domains varying from a simple cube to coupled ellipsoids are produced, which may well represent a real architectural space. Obtained RIRs for specific source-receiver configurations are analyzed initially to compare basic acoustic parameter values, specifically T30s. Relative sound energy distributions are also checked for stationary solution of ray-tracing and time-dependent solutions of DEM. Different materials over specific surfaces are tested to check the effect of inhomogeneous absorption for the investigated methods. The results aim to compare DEM and ray-tracing methods, by a controlled set of simulations on acoustical models with the same room and material input.

2. METHODOLOGY

2.1 Computational techniques

Since the very early publications [14], ray-tracing technique in room acoustic computations has been advanced much, as briefed in Savioja and Svensson's review [7]. This study utilizes ODEON v17.12 as a hybrid room acoustics simulation software to assess ray-tracing. Diffusion equation model (DEM), on the other side, assumes that particles travel along straight lines at the speed of sound in the interior space and multiple diffuse reflections occur on the room boundaries which can be conceived as scattering objects. These diffuse reflections of the sound particles establish a reverberation process building up a so-called diffuse sound field in the enclosure. Picaut et al. [15] by using the physical analogy with the diffusion of particles in a medium containing spherical scattering objects, have proposed a model, to describe the local acoustic energy density in rooms with perfectly diffuse reflecting walls. The numerical implementation of DEM in room acoustics predictions was very first studied by Valeau et al. [15]. Their results point out the possibility of this new model to be a solution of the acoustic analysis of various room shapes. Billon et al. [17] applied the diffusion model to the coupled volume configuration. Later, Jing and Xiang [18] introduced mixed-boundary condition, making the diffusion equation applicable in broader room-acoustic conditions as well as different room shapes.

One of the attractive features of the diffusion equation in either finite-element or finite-difference implementation, is that the mean-free path length of the space under consideration primarily dictates the meshing condition in enclosures, independent of the wavelengths. Rooted in Fick's law, the diffusion equation predicts sound energy fluxes more efficiently than existing numerical tools [12]. The DEM equations applied in the scope of this study are listed in the following. The time-dependent sound energy density *w*, in a unit time (t) and position (r), in the presence of an omni-directional sound source, *q (r,t)* can be estimated by [16]:

$$\frac{\partial w(r,t)}{\partial t} - D\nabla^2 w(r,t) = q(r,t), \quad \in V \tag{1}$$

where *D* is the diffusion coefficient, which takes into account the room morphology via its mean free path (D=λ*c/3). The boundary equation is as follows [18]:

$$-D\frac{\partial w(r,t)}{\partial n} = \frac{c\alpha}{4(1-\alpha/2)} w(r,t), \text{ on } S \tag{2}$$

where α is the absorption coefficient of the specific surface or boundary. The energy flow level is then defined as:

$$J_L(r,t) = 10\log_{10}\left\{\left[\frac{\partial w(r,t)}{\partial x}\right]^2 + \left[\frac{\partial w(r,t)}{\partial y}\right]^2 + \left[\frac{\partial w(r,t)}{\partial z}\right]^2\right\}^{1/2} \tag{3}$$

The solid models of computed geometric entities in this study are meshed for finite element analysis, by making sure that the maximum mesh size of each entity is much smaller than the mean-free-paths of individual domains. Due to its reliance on mean-free-path for its meshing limits, the speed of computation is much higher for DEM than for other numerical tools, especially for complex geometries. DEM has the potential to trace the sound propagation in a time-dependent solution by the help of energy flow vectors which are some of its benefits over ray-tracing. In order to further discuss the application areas of these two techniques for different cases, initially a comparison study is held to check their likeliness in assessing basic objective room acoustics metrics. The used geometric entities within the scope of this study are detailed in the following.

2.2 Generative geometric forms

The physical properties of geometric domains that are tested comparatively by ray tracing and DEM in this study are listed in Table 1. The forms are generated from simple cubes, rectangular prisms, domes and ellipsoids later some forms are coupled to each other to increase the complexity of the system. However, none of these form combinations should be considered as coupled volume systems as coupling interfaces are much larger than a typical coupled volume aperture. Coupled systems are currently out of the scope of this paper but will be investigated in the broader version of this research. The systems although currently small in scale, for the speed of assessments, can typologically represent many kinds of architectural spaces with different styles and functions. For instance, a cube can be viewed as an abstracted studio environment, a cube-dome combination can be a mosque or a synagogue, an elongated prism can be a corridor and so on.

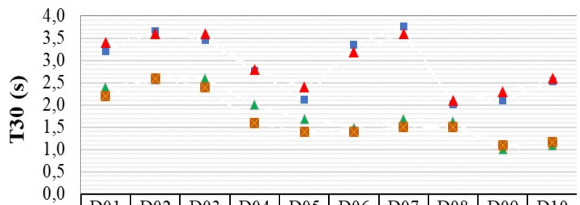
Some domains have identical volumes as of Domains D01, D04, D05 & D07, and Domains D02 & D03 to compare the effect of geometry on sound energy distribution and decays. Within each domain, considering the comparatively small sizes of the domains, one source and maximum two receivers are located exactly at the same position in ray-tracing model and within the solid model for the DEM computation. Two different states of absorption area are tested. Initially an alpha of 0,06 is attained to all surfaces representing a typical plastered solid surface for 500 Hz. Later, only floor surfaces are replaced with alpha 0,25 representing a typical carpet surface. The comparisons are only held for 500 Hz in the scope of this paper, as a characteristic speech frequency, which is not affected much by the wave properties of sound for given sizes of geometric entities.

Table 1. Physical parameters of assessed geometric entities, and mean-free-paths (λ)

Domain Code#	Name	Surface Area (m^2)	Volume (m^3)	Mean-free-path (λ, m)
D01	Cube	384	512	5,33
D02	Domed-cube	433	647	5,98
D03	Big-cube	449	647	5,76
D04	Rectangular Prism	448	512	4,53
D05	Tube	544	512	3,76
D06	Triple-caves	555	673	4,85
D07	Semi-dome	364	512	5,46
D08	Inverse-dome	431	391	3,62
D09	Ellipsoid	202	180	3,56
D10	Double-ellipsoid	370	348	3,76

3. RESULTS

In this section two different results, out of DEM and ray-tracing simulations are presented. Initially room impulse responses are collected for fixed source-receiver configurations to estimate reverberation times (T30) for ten different geometric domains. Fig. 1 presents the comparative graph of T30 values for ray-tracing and DEM.



	D01	D02	D03	D04	D05	D06	D07	D08	D09	D10
Ray	3,2	3,7	3,5	2,8	2,1	3,4	3,8	2,0	2,1	2,5
DEM	3,4	3,6	3,6	2,8	2,4	3,2	3,6	2,1	2,3	2,6
Ray-c	2,4	2,6	2,6	2,0	1,7	1,5	1,7	1,6	1,0	1,1
DEM-c	2,2	2,6	2,4	1,6	1,4	1,4	1,5	1,5	1,1	1,2

Figure 1. T30 values, for 500 Hz, comparison for domains D01 to D10, ray tracing (ray) versus DEM, without and with carpet (-c)

When Table 1 is compared to estimated T30 values (Fig.1) of various room forms, it can be stated that the lower the mean-free-path, the lower the estimated T30s for the domains with similar volumes are. As inherent in the formula, the smaller the surface area volume ratio is, the higher the mean-free-path. Dome and semi-dome type of enclosures in that sense have the greatest potential of long reverberation in comparison to any other geometric entity with a similar volume. On the opposite side, elongated forms, disproportionate long enclosures, display the lowest reverberation as they have highest surface area to volume ratio. For disproportionate rooms, when even one surface becomes absorptive, the deviation between T30 values of DEM versus ray-tracing increases.

According to Fig. 1 the comparison of ray tracing to DEM when all surfaces are reflective (painted block or concrete) indicates that the deviation of T30 values are less than 5%, which is acceptable considering JND of reverberation times. The highest difference is 0,3 s for D05-Tube, which is a disproportionate long enclosure. Still the deviation is not very high when all the surfaces are reflective. Ray-c and DEM-c indicates the state of the spaces, when floor surface is absorptive. In this case for ordinary rooms the difference of T30 is at maximum 0,2 s. Greater deviations are observed for D04-rectangular prism, around 0,4 s, and again for D05-Tube, around 0,3 s. D04 is a flat room, with a height to length ratio of 1:4. This may pose a problem with the estimated difference in T30 values, especially for the case when one surface of the room is comparatively much more absorptive (inhomogeneous absorption). D05-Tube, on the other hand is a long enclosure, again another type of disproportionate room.

In order to improve the accuracy of computations in long enclosures, many studies have been conducted. Whether DEM or ray-tracing simulated RIRs are much closer to the real measurements is a research question that was previously discussed for metro stations [19]. The applicability of diffusion coefficient for such long spaces has been an argument of some other research. Visentin et al. [20] observed that the theoretical value of the diffusion coefficient is only valid close to the source. They defined a spatially varying diffusion coefficient for non-homogeneous diffusion, for long enclosures. The concern at this point is that the diffusion coefficient is not known beforehand, when needed in design phase, but obtained after the intensities are already known within the enclosure. Foy et al. [21], applied the particle path statistics to propose a semi-analytical expression to the diffusion coefficient. A purely analytical expression of the diffusion coefficient for long enclosures is still necessary that can be the subject of a future research.

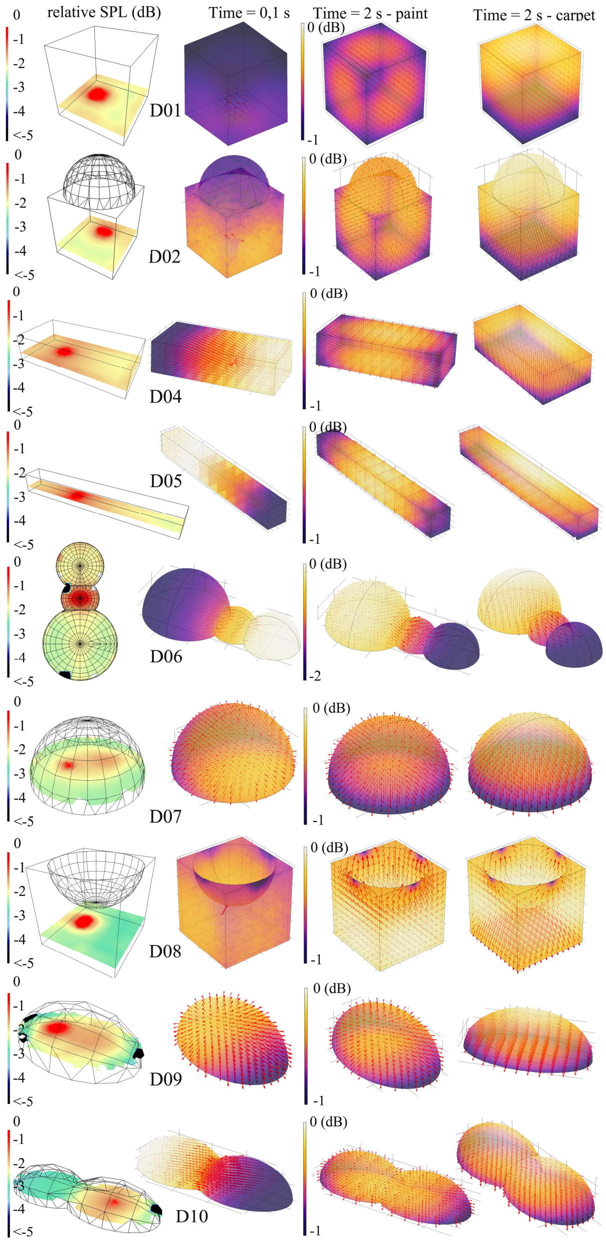


Figure 2. Relative SPL (dB) maps, for 500 Hz obtained from ray-tracing (first column), energy flow vectors obtained from DEM solution for the time instants 0,1 s, and 2 s for all surfaces reflective (second and third column) and only floor surface absorptive (fourth column); for geometric entities D01 to D10 (excluding D03).

Fig. 2 illustrates relative SPL (dB) maps, for 500 Hz obtained from ray-tracing. As the domains are comparatively small in scale the difference between grid surfaces is mostly at around 2 dB. The largest difference is found for the D10-double ellipsoid with around 5 dB, where the source is under one ellipsoid, and the source-receiver distance is greater in this case, while the flow is restricted with the intermediate narrow zone at the crossing of two ellipsoids. This is not to be pronounced as a coupling aperture because the intersection area is much larger than those apertures in a typical coupled volume scenario, but still an obstructive factor.

In Fig.2. ray-tracing results are used to observe the stationary distribution of energy levels with a point-source. On the other hand, DEM is utilized to visualize the energy flux, and concentration of energy within the domains at different time instants. Fig. 2 only excludes D03, which is the larger version of D01-cube. Time 0.01 s indicates the start of the flow as the impulsive signal is just stopped. And time 2 s is a representative time instant for all cases when the sound has filled up all the domains and arrived to a steady-state. At that time instant the reflective floor versus carpet floor is compared for all cases. The difference for the energy distribution is 1-2 dB for a specific time instant so, not repeated in Fig. 3 and Fig. 4, but indicated only in the scale legends of Fig. 2. The difference in dB scale is only meaningful to compare different time instants as shown in Fig. 5. Colors are kept in the figures to emphasize the flow direction.

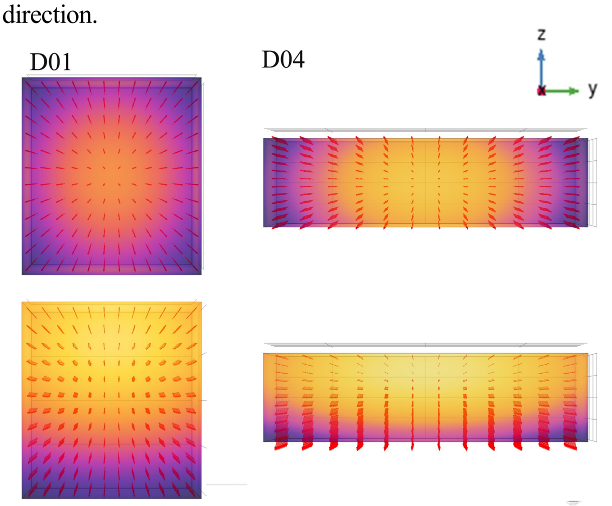


Figure 3. Energy flow vectors obtained from DEM solutions at time instant 2 s, for domains D01 and D04, all surfaces reflective (above) versus absorptive floor (below), cross-sectional views

The concentration of energy is worth examining for absorptive versus reflective floor solutions. As can be better seen from a zoom-in section for D01 and D04, in Fig. 3, in a very cubic domain when stabilized, the energy is distributed from the center to the room boundaries. For the same case, when the floor is absorptive the energy flux on the two axis of cross-section is from ceiling towards the floor. The same applies for most symmetric cases but more visible or dominant in pure single volumes as for D07. When it comes to the rectangular or tubular forms for all surfaces reflective state energy flux is towards to the boundary of the small cross section, like the end of the corridor for a tunnel or tube case. When floor surface is absorptive the energy flux for the tube and the rectangular prism is towards the floor. Fig. 3 and Fig 4. include the elevation views gathered from the sideview of axonometric arrow plot. This is why there is a crowd of arrows, that include the 3d arrow plot layers (towards different directions) stacked on top of each other in given side views.

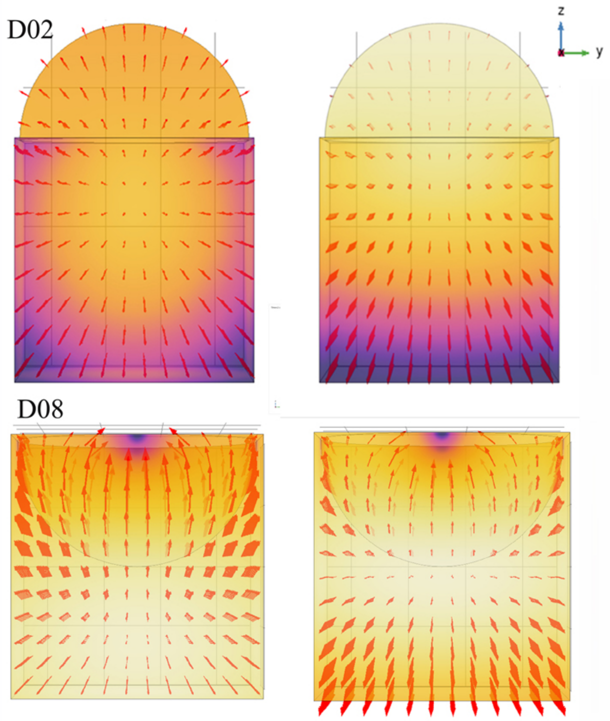


Figure 4. Energy flow vectors obtained from DEM solutions at time instant 2 s, for domains D02 and D08, all surfaces reflective (left column) versus absorptive floor solutions (right column), cross-sectional views

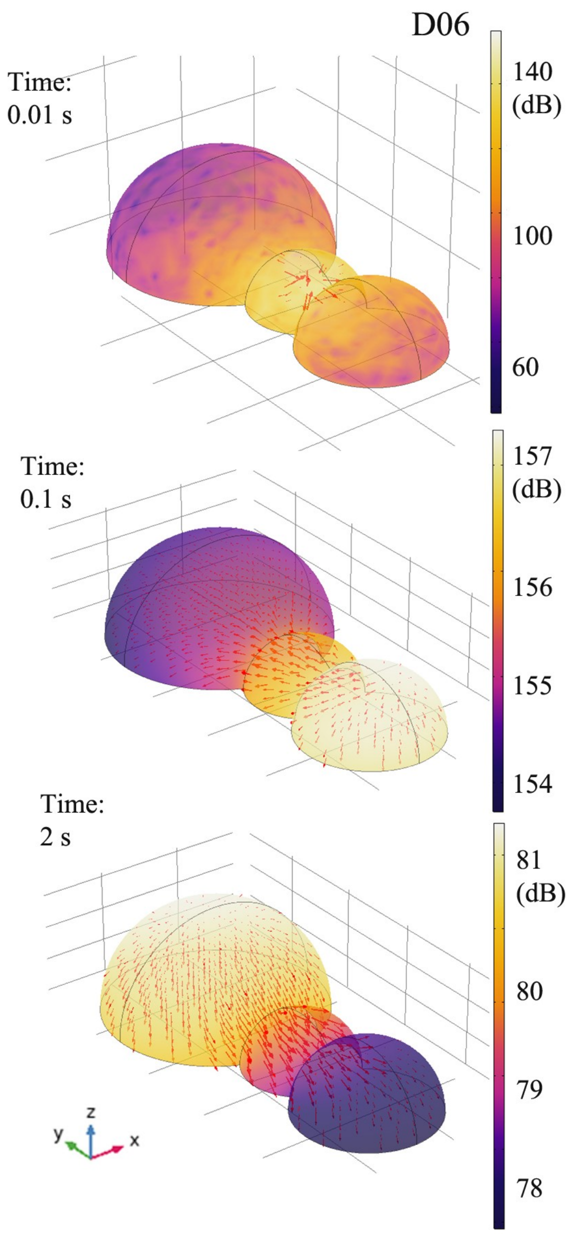


Figure 5. DEM solution for D06 for the time instants 0,01 s, 0,1 s and 2 s for floor absorptive and dome surfaces reflective state, axonometric views

In Fig. 4, D02-domed cube and D08-inverse dome are further analyzed over the cross section of energy flow vectors within volumes for both states of the floor (reflective and absorptive) for the time instant 2 s. D02 is a combination of dome with a prismatic base as in the case of many religious [4,12] building typologies but just simplified to two basic primitive forms. The energy flux, as in D01 and D07, is from the center of the cross-sectional axis towards the prismatic base borders, when all surfaces are reflective, but the situation immediately changes from dome to floor as the floor becomes absorptive. This creates an energy division within the space. For these small-scale volumes, the uneven distribution of absorption does not cause non-exponential sound energy decay, at least for the analyzed cases from D01 to D10. However, as the scale becomes too large, it has the potential to create multiple sound energy decays as previously observed in monumental structures [22].

D08 is a half-dome subtracted cube, which is generated to investigate the diffusive properties of a convex boundary in comparison to a concave dome. In this case the energy flow vectors are towards the overall boundary surfaces instead of high concentration from ceiling to floor in the case of carpeted floor. In carpeted state there is a slight shift of flow vectors towards the floor when it is compared to the reflective state of D08. The distribution is much even within the domain when compared to other carpeted examples of different domains.

Lastly, D06-triple caves is highlighted in Fig. 5 to illustrate the time dependent shift of the energy right after the impulsive source is started (time: 0,01s), then stopped (time: 0,1 s), finally reaching the steady state (time: 2 s). In this case the source is under the middle half dome, which is at the same time the smallest volume. For the first time steps, the energy flow is from central semi-dome towards the medium-sized half dome at one side, meaning that the source at the center initially fills up the medium volume and later at the steady state the higher energy concentration is shifted to the largest semi-dome, and the energy flux after that time is from the largest semi-dome to the smaller domes. The floor reflective or absorptive states do not distinctively change the energy flow patterns for this triple semi-dome coupled configuration.

4. DISCUSSION

Sound field analysis is an ongoing research area, where applicable methods and tools of investigation have been continuously updated. On the other hand, accurate characterization of the acoustics of a room in its design phase, is critical for practitioners. Research necessitates in depth analysis, whereas practice applies the product of research, and in that sense requires speed in simulations. Ray-tracing simulations are heavily applied in acoustical design as a practical tool. On the other hand, DEM, recently have been used in research to understand the behavior of sound energy propagation within rooms for case specific problems.

This research has compared ray-tracing to DEM, in a controlled set-up of primitive forms, as previous studies focus on complex real-size structures. As the complexity of architecture increases so does the materials, it is harder to discuss the differences of different methods and geometric outcomes. As presented in the results section the RIRs obtained in compared geometric entities produce quite identical energy decays, when T30 values are compared especially for the cases when overall surfaces are highly reflective. The deviation starts, for disproportionate rooms and specifically for the cases when one surface has been absorptive. It is previously discussed that in long enclosures [20], a single diffusion coefficient in DEM may not represent the sound behavior especially as the source-receiver distance increases. This is also observed in this study that the deviation of ray-tracing and DEM results are some of the highest for long enclosures. However, it is also still debatable that the ray-tracing simulations are consistent with the field measurements for long enclosures when one surface is highly absorptive [19]. The second case where T30 values deviate more than 5%, is for the flat room (D04) - or comparatively flat room in comparison to other domains. When a room should be considered flat or long, in other words what are the proportions for room to be considered as a disproportionate room may necessitate more alternative proportions to be tested and compared, through the same methodology applied in this study, which will be done in the future steps of this research. Still, the results of both ray-tracing and DEM for disproportionate rooms should be validated with real measurements. This can only be tested through scale models in a systematic set-up, meaning that the physical models are still necessary in research to further tune the existing theoretical models.

Lastly, both ray-tracing and DEM are computationally fast tools to be applied in room acoustics design with current technologies. But DEM additionally provides the time dependent sound energy flow in a room for the whole volume. Such visualization is especially useful for better explaining the sound propagation due to different geometric relations as shown in this study over some primitive forms. As the forms get more complex the need to explain some acoustical phenomena may even become more critical, as in the case of coupled-room systems where multi-slope sound energy decay may be observed [4, 11-13,17]. The comparison of ray-tracing to DEM for coupled volume systems in a controlled-experiment is currently out of the scope of this paper, but will be systematically investigated in the next step.

5. REFERENCES

[1] Z. Sü Gül, Z., M. Eşmebaşı, and Z. Bora Özyurt: "The effects of stage house coupling on multipurpose auditorium acoustics," Appl. Acoust. vol.198, 2022, https://doi.org/10.1016/j.apacoust.2022.108996

[2] Z. Sü Gül, Z., M. Çalışkan, and Z. Bora Özyurt, "Acoustical design challenges of public interiors: Cultural centers and terminal buildings," in *Proceedings of Meetings on Acoustics (POMA)*, 42, pp. 015002:1-13, 2020.

[3] M. Eşmebaşı, M., Z. Bora Özyurt, and Z. Sü Gül, "Contemporary sports arena room acoustics design," in ICSV28-International Congress on Sound and Vibration, (Singapore), 24-28 July, 2022.

[4] Z. Sü Gül: "Exploration of room acoustics coupling in Hagia Sophia of İstanbul for its different states," *J. Acoust. Soc. Am.*, vol. 149.1, pp. 320-339, 2021.

[5] L. Savioja, and N. Xiang: "Simulation-based auralization of room acoustics," *Acoustics Today*, vol. 16. 4, pp. 48-56, 2020.

[6] M. Vorländer: "Are virtual sounds real?," *Acoustics Today*, vol. 16.1, pp. 46-54, 2020.

[7] L. Savioja, and U. P. Svensson: "Overview of geometrical room acoustics modeling techniques," *J. Acoust. Soc. Am.,* Vol. 138, pp. 708-730, 2015.

[8] L. Savioja, and N. Xiang: "Introduction to the special issue on room acoustic modeling and auralization," *J. Acoust. Soc. Am.,* vol.145.4, pp. 2597-2600, 2019.

[9] G. Naylor: "Odeon-another hybrid room acoustical model," *Appl. Acoust.,* vol.38, pp. 131-143, 1993.

[10] P. Luizard, J.-D. Polack, and B. F. G. Katz: "Sound energy decay in coupled spaces using a parametric analytical solution of a diffusion equation," *J. Acoust. Soc. Am.,* vol. 135, pp. 2765-2776, 2014.

[11] Z. Sü Gül, N. Xiang N., and M. Çalışkan: "Investigations on sound energy decays and flows

in a monumental mosque," *J. Acoust. Soc. Am.*, vol. 140.1, pp. 344-355, 2016.

[12] Z. Sü Gül, N. Xiang N. and M. Çalışkan: "Diffusion equation based finite element modeling of a monumental worship space," *J. Comput. Acoust.,* vol. 25.4, pp. 1750029 1-16, 2017.

[13] Z. Sü Gül, E. Odabaş, N. Xiang, and M. Çalışkan: "Diffusion equation modeling for sound energy flow analysis in multi domain structures," *J. Acoust. Soc. Am.*, vol. 145.4, pp. 2703-2717, 2019.

[14] A. Krokstad, S. Strøm, and S. Sørsdal: "Calculating the acoustical room response by the use of a ray tracing technique," *J. Sound Vib.,* vol. 8.1, pp. 118-125, 1968.

[15] J. Picaut, L. Simon, and J.-D. Polack: "A mathematical model of diffuse sound field based on a diffusion equation," *Acta Acust. Acust.*, vol. 83.4, pp. 614-621, 1997.

[16] V. Valeau, J. Picaut and M. Hodgson: "On the use of a diffusion equation for room-acoustic prediction," *J. Acoust. Soc. Am.*, vol.119.3, pp. 1504-1513, 2006.

[17] A. Billon, V. Valeau, A. Sakout, and J. Picaut: "On the use of a diffusion model for acoustically coupled rooms," *J. Acoust. Soc. Am,* vol. 120.4, pp. 2043-2054, 2006.

[18] Y. Jing, and N. Xiang: "On boundary conditions for the diffusion equation in room acoustic prediction: Theory, simulations, and experiments," *J. Acoust. Soc. Am.*, vol. 123.1, pp. 145-153, 2008.

[19] Z. Sü Gül, E. Odabaş, and M. Çalışkan: "Comparative evaluation of ray tracing and diffusion equation modeling in room acoustics design of subway stations," *Acoust. Aust*., vol. 48.1, pp. 93-105, 2020.

[20] C. Visentin, N. Prodi, V. Valeau, and J. Picaut, "A numerical investigation of the Fick's law of diffusion in room acoustics," *J. Acoust. Soc. Am.*, vol.132.5, pp. 3180-3189, 2012.

[21] C. Foy, V. Valeau, C. Prax, J. Picaut, and A. Sakout, "Inhomogeneous diffusion process in elongated rooms: An interpretation based on the dynamics of sound particles," *JASA Express Lett.,* vol. 1.5, pp. 051601-1-6, 2021.

[22] Z. Sü Gül, M. Çalışkan, N. Xiang, and A. Tavukçuoğlu: "Assessment of acoustical indicators in multi-domed historic structures by non-exponential energy decay analysis," *Acoust. Aust*., vol. 46.2, pp. 181-192, 2018.