DOI: 10.61782/fa.2023.0973

forum acusticum 2023

IMPEDANCE TUBE MEASUREMENTS OF THE EFFECTIVE ACOUSTIC PROPERTIES OF ANISOTROPIC POROUS MEDIA

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ABSTRACT

Porous media such as foams and wools can be described as effective anisotropic fluid materials that are characterized by a bulk modulus and a full symmetric density tensor. This paper presents a method for retrieving the bulk modulus and all six components of the density tensor from reflection and transmission coefficients measured in an impedance tube. The reflection and transmission properties are estimated for six different orientations of the measuring sample (one per coefficient of the density tensor) and an inverse problem is formulated to infer the seven effective fluid parameters. After diagonalization of the density tensor and determination of the principal directions, the effective parameters, namely the Johnson-Champoux-Allard-Lafarge (JCAL) parameters, are recovered via a deterministic minimization procedure. The validity of the method is examined experimentally on a commercial glasswool material, with principal axes tilted in a priori unknown directions.

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Keywords: *Anisotropic porous media, impedance tube, reflection and transmission coefficients*

1. INTRODUCTION

Porous materials are usually characterized and modeled as homogeneous isotropic fluids although they are in fact anisotropic media. This article aims at partially filling this gap by proposing a reliable and robust method for the characterization of homogeneous anisotropic porous materials from impedance tube measurements. We use the Jonhson-Champoux-Allard-Lafarge (JCAL) model for porous media with a motionless frame [1,2] to describe the anisotropic properties of the equivalent fluid. Inspired by Terroir et. al in 2019 [3], the six elements of the symmetric density tensor as well as the bulk modulus are first reconstructed using at least as many measurements of the reflection and transmission coefficients on a number of samples. To maximise the amount of information, the samples are taken at different and judiciously chosen angles with respect to the bulk material coordinate system. The JCAL parameters as well as the principal directions are then reconstructed from these measurements. The key difference with Ref. [3] is that while the original work presented a numerical validation in free-field, the present study is experimental and uses an impedance tube to acquire data.

This paper is organized as follows: the first part introduces the theory of homogenized anisotropic media and the inverse problem, then we present the experimental results and we conclude with future perspectives.

2. THEORY

Let us consider a layer of homogeneous anisotropic material Ω of thickness L considered as an equivalent fluid of bulk modulus B and density tensor $\boldsymbol{\rho}$. In the reference coordinate system $(\mathrm{O}, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ with position coordinates (x_1, x_2, x_3), the layer's boundaries are positioned at $x_3 = 0$ and $x_3 = L$. This layer is surrounded on both sides $x_3 \leq 0$ and $x_3 \geq L$ by a homogeneous isotropic fluid Ω_0 of bulk modulus B_0 and scalar density ρ_0. The density tensor $\boldsymbol{\rho}$ is symmetric; that is $\boldsymbol{\rho} = \boldsymbol{\rho}^\top$, where $^\top$ denotes transposition. Effectively, the density tensor can be written as $\boldsymbol{\rho}^\star = \mathbf{diag}(\rho_{\mathrm{I}}, \rho_{\mathrm{II}}, \rho_{\mathrm{III}})$, where ρ_{I}, ρ_{II} and ρ_{III}, in the material principal directions $(\mathbf{e}_{\mathrm{I}}, \mathbf{e}_{\mathrm{II}}, \mathbf{e}_{\mathrm{III}})$. In the reference coordinate system, the density tensor then reads $\boldsymbol{\rho} = \mathbf{R}\boldsymbol{\rho}^\star\mathbf{R}^\top$, where $\mathbf{R} = \mathbf{R}_3(\theta_{\mathrm{III}})\mathbf{R}_2(\theta_{\mathrm{II}})\mathbf{R}_1(\theta_{\mathrm{I}})$ is the rotation matrix between the two coordinate systems, with $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$ being elementary matrices of rotations and θ_{I}, θ_{II} and θ_{III} the roll, pitch and yaw angles, respectively. For the sake of simplicity, we choose to work with the inverse density tensor $\mathbf{H} = \boldsymbol{\rho}^{-1} = \mathbf{R}\mathbf{H}^\star\mathbf{R}^\top$, with $\mathbf{H}^\star$ the diagonal inverse density tensor in the material principal directions, i.e. $\mathbf{H}^\star = \mathbf{diag}(H_{\mathrm{I}}, H_{\mathrm{II}}, H_{\mathrm{III}}) = \mathbf{diag}(1/\rho_{\mathrm{I}}, 1/\rho_{\mathrm{II}}, 1/\rho_{\mathrm{III}})$.

2.1 Direct and inverse problem

The acoustic pressure and velocity fields, namely p and $\mathbf{v}$ in the layer are governed by the equations of mass and momentum conservation:

$$\mathrm{j}\omega p = B\boldsymbol{\nabla}\cdot\mathbf{v}\,, \qquad \mathrm{j}\omega\cdot\mathbf{v} = \mathbf{H}\boldsymbol{\nabla}p\,, \tag{1}$$

where $\omega = 2\pi f$ is the angular frequency and the time dependency $\mathrm{e}^{-\mathrm{j}\omega t}$ is omitted.

The problem now consists in retrieving the six components of the symmetric (inverse) density tensor $\mathbf{H}$ and the value of the bulk modulus B from the knowledge of the material thickness and the scattering coefficients R and T.

This method has been described by Terroir et al. [3]. As we measure reflections and transmission coefficients in an impedance tube, the method uses acoustic waves with normal incidence for different rotations of our material instead of considering oblique incidences which cannot be achieved with the present set-up. To relate the measured quantities (R, T) to the equivalent acoustic properties ($\mathbf{B}$, $\mathbf{H}$), we start from the linear system representing the direct problem (the developments are not reminded here, see [3] or [4] for details). These equations are valid for any slab of the material of thickness L and excited by waves with normal incidence:

$$\mathbf{W}_{\mathrm{L}} = \frac{1}{2}\begin{bmatrix}\tilde{Z} & \tilde{Z}\\ 1 & 1\end{bmatrix}\begin{bmatrix}\mathrm{e}^{-\mathrm{j}\tilde{k}L} & 0\\ 0 & \mathrm{e}^{\mathrm{j}\tilde{k}L}\end{bmatrix}\begin{bmatrix}\tilde{Z}^{-1} & -1\\ \tilde{Z}^{-1} & 1\end{bmatrix}\mathbf{W}_0\,, \tag{2}$$

where we have introduced $\mathbf{W}_L = (1+R\,,\,(1-R)/Z_0)^\top$ and $\mathbf{W}_0 = (T\,,\,-T/Z_0)^\top$, the two state vectors at both ends of the system.

After inversion of the system, we obtain the following relations:

$$\tilde{Z} = Z_0\frac{(1+R)^2 - T^2}{(1-R)^2 - T^2}\,, \tag{3}$$

$$\mathrm{e}^{-\mathrm{j}\tilde{k}L} = \left(1 + \frac{(Z_0 - \tilde{Z})}{(Z_0 + \tilde{Z})}R\right)\frac{1}{T}\,. \tag{4}$$

The equivalent fluid impedance $\tilde{Z}$ is directly retrieved from the knowledge of R and T, while for the equivalent fluid wavenumber $\tilde{k}$, we need to invert the relation Eq. (4), which leads $\forall n \in \mathbb{Z}$:

$$\tilde{k}L = -\mathrm{ang}(\mathrm{e}^{-\mathrm{j}\tilde{k}L}) + \mathrm{j}\log|\mathrm{e}^{-\mathrm{j}\tilde{k}L}| + 2n\pi\,, \tag{5}$$

where ang is the phase angle and log is the natural logarithm. From these quantities, we directly obtain the equivalent fluid inverse mass density and bulk modulus via:

$$H_{\mathrm{eq}} = \frac{\omega}{\tilde{Z}\tilde{k}}\,, \tag{6}$$

$$B_{\mathrm{eq}} = \frac{\omega\tilde{Z}}{\tilde{k}}\,. \tag{7}$$

2.2 Estimation of the bulk modulus and the inverse density tensor via impedance tube measurements

In order to estimate the full (inverse) density tensor, we need to perform at least six measurements (for the six unknown quantities of the inverse symmetric density tensor). Terroir et al. [3] considered impinging sound waves at six oblique incidences. As we performed impedance tube measurements, we do not vary the incidence angle for the incident waves, but rather apply six rotations in the 3D space to the material sample and consider incident plane wave.

From these six measurements, we estimate the value of the inverse density and the bulk modulus for each direction.The bulk modulus being a scalar, it should not vary depending on the direction and although it is a by-product of the methods, determining it six times allows us to evaluate the method accuracy and measurement qualities by looking at the standard-deviation of B for all measurements.

For the three diagonal values of the density, we take samples of the material in the three orthogonal directions of the reference frame $(\mathrm{O}, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$. We thus obtain the three diagonal values of the density tensor H_{11}, H_{22} and H_{33}.

To recover the three extra-diagonal terms, we take three samples with an arbitrary angle in the three planes defined by the frame directions, namely $(\mathbf{e}_1, \mathbf{e}_2)$, $(\mathbf{e}_1, \mathbf{e}_3)$ and $(\mathbf{e}_2, \mathbf{e}_3)$.

This allows us to retrieve the three extra-diagonal terms as a function of the diagonal ones and the in-plane angles θ_{ij} and the measured equivalent inverse density H_{ij}^+, computed from the measurements.

2.3 Estimation of the principal directions and the JCAL parameters via minimization

After estimating the effective acoustic properties of the material, we need to fit the closest JCAL model (defined by 12 parameters for a full anisotropic material, plus the 3 Euler angles defining the principal axes directions, leading to 15 unknown parameters) in order to find values for the JCAL parameters of the studied material.

This is achieved via a minimization procedure based on the following cost function $J(\mathbf{q}, \boldsymbol{\theta})$:

$$\begin{aligned} J(\mathbf{q},\boldsymbol{\theta}) &= \mathcal{S}\left(\rho_0\left[H_{ij}^+ - H_{ij}(\mathbf{q},\boldsymbol{\theta})\right]\right) + \left\|\frac{B^+ - B(\mathbf{q})}{\gamma P_0}\right\|^2, \\ H_{ij}(\mathbf{q},\boldsymbol{\theta}) &= \mathbf{R}(\boldsymbol{\theta})\mathbf{diag}\Big[H_I(\mathbf{q}), H_{II}(\mathbf{q}), H_{III}(\mathbf{q})\Big]\mathbf{R}^\top(\boldsymbol{\theta}), \\ \mathcal{S}(f(\mathbf{q},\boldsymbol{\theta})) &= \sum_{i\geq j}^{3}\left[\|f(\mathbf{q},\boldsymbol{\theta})\|^2\right], \end{aligned} \tag{8}$$

where the quantities denoted with the superscript $^+$ are the measured quantities, $\mathbf{q}$ represents the vector of the twelve JCAL parameters to be found, $\boldsymbol{\theta}$ is a vector containing the three Euler angles of the material principal directions, ρ_0, P_0, and γ are respectively the mass density, the atmospheric pressure, and the adiabatic constant of the saturating and surrounding air medium. The minimisation is performed with the Covariant Matrix Adaptation Evolutionary Strategy (CMA-ES) [5] algorithm.

3. EXPERIMENTAL VALIDATION

The chosen material to be characterized is a glasswool material used for sound insulation and provided by Saint-Gobain Ecophon (Hyllinge, Sweden).

After measuring all six samples, having reconstructed the effective properties of the material, and having performed the minimization, we obtain the following resulting JCAL parameters and Euler angles for the glasswool Ecophon as presented in Table 3.

Table 1. Reconstructed JCAL parameters and Euler angles for every principal direction. The JCAL parameters are namely the open porosity ϕ, thermal characteristic length Λ', thermo-static permeability k', high-frequency limit of tortuosity τ^∞, viscous characteristic length Λ, and visco-static permeability k^0.

	ϕ	Λ'	k'	τ_i^∞	Λ_i	k_i^0	θ
	-	µm	$10^{-9}\,\mathrm{m}^2$	-	µm	$10^{-9}\,\mathrm{m}^2$	deg
Ω	1	378.4	4.2				
$\mathbf{e}_{\mathrm{I}}$				1.0	132.7	1.9	66
$\mathbf{e}_{\mathrm{II}}$				1.0	118.3	1.9	51
$\mathbf{e}_{\mathrm{III}}$				1.0	165.8	3.1	0

From these results in Table 3, we can conclude that the material is orthotropic, which is in agreement with the literature on the glasswool acoustic properties (see [6] or [7] for example).

4. CONCLUSION

An experimental procedure has been proposed for retrieving the bulk modulus and all six components of the density tensor of an anisotropic porous material. The procedure relies on measuring the reflection and transmission coefficients in an impedance tube for different samples taken from a material at different orientations.

5. ACKNOWLEDGMENTS

The authors would like to thank Saint-Gobain Ecophon (Hyllinge, Sweden) for providing the samples. This work is funded by the Independent Research Fund Denmark, under DFF-Individual International Postdoctoral Grant 1031-00012. L.M, M.G. V.R.-G. and J.-P. G. gratefully acknowledged the support of the ANR-RGC METARoom project (ANR-18-CE08-0021).

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