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SIMILITUDE OF A DAMPED VIBRATING COMPOSITE PLATE

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ABSTRACT

The concept of similitude is now widely applied in the engineering vibroacoustics applications since it is particularly useful when applied to the testing of models with increasing complexity. The similitude schemes allow predicting the dynamic response of an original system by using the information obtained from a similar one (defined as avatar or replica according to the partial or complete degree of similitude). In addition to geometric similarity, the focus on damping similitude has increased in recent years: this because the damping represents a highly relevant property during the design phase of any structure. Every material has an intrinsic damping: for example, generally, the damping in metal structures, as aluminum alloys, is lower and this results in high resonances of vibrations. Moreover, the damping evaluation for composite structures is even more complicated because it depends on concomitant factors: lay-up sequence, the type of fibres and resin and the manufacturing process. In this view, it could be helpful to develop a criterion of design evaluating the damping variation from a structure to another and verifying if this relative damping variation can validate with experimental tests.

Keywords: *composite material, damping, similitude*

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1. INTRODUCTION

Vibration and noise reduction are crucial in maintaining high performance level and prolonging the useful life of a machinery, automobiles, aerodynamic and spacecraft structures. This is the reason why modern structures need to be investigated with different approaches: analytical, numerical and experimental approach. The experimental investigation is crucial to analyze and predict the behavior of the structure but presents some limitation imposed, basically, by the size of the structure itself. Dealing with large experimental structures can be time-consuming and expensive. In this context, the similitude theory has a very important role since it allows to predict the dynamic response of an original system by using the information obtained from a similar one. In recent years, the focus has been on the study of similarity structures that differ in the damping factor. This study has proved to be very complex as the phenomenon of energy dissipation of a structure turns out to be the result of different and concomitant factors: structural damping, boundary conditions, hysteresis dissipation phenomena, internal friction and so on. The problem of the dissipation of energy in structures is a very important feature in mechanical design: The role of the damping is crucial because it depends on this dynamic property of the structure how vibrations vanish over time. The paper will be divided in different sections: in the section 2 a general description of the damping property of a composite plate will be given, the section 3 focuses on the methods for the damping prediction and one of the different methods will be chosen for the theoretical calculation of the loss factor of the composite plate under test, the section 4 reports the characteristics and information about the study case, the 5th section presents the numerical analysis of the same plate and a comparison of the numerical results in terms of mode shapes and harmonic analysis with an aluminum plate, keeping the geometry. The 6th section describes the experimental tests and their results. Finally, in the section 7 conclusions about the total study carried out, will be given.

2. DAMPING IN COMPOSITE MATERIALS

A structural composite is a material system consisting of two or more phases on a macroscopic scale whose mechanical performance and properties are designed to be superior to those of the constituent materials acting independently. One of these phases is stiffer and stronger and is called reinforcement and the less stiff and weaker phase is known as matrix: the illustration of a composite plate is shown in the Figure 1 [1], where the reference system (O,x,y,z) is the reference system of the laminate, (O,1,2,3) is the local reference system of the laminate based on the arrangement of the fibres at θ angle to the reference system of the laminate, a, b, t are, respectively, the length, width and thickness of the plate. Damping in fiber-reinforced composite materials can be

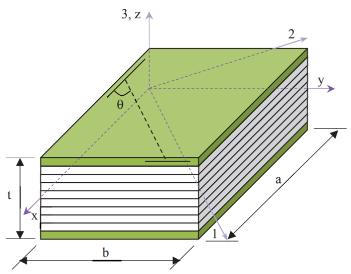

Figure 1. Composite plate - reference systems for the lamina and the laminate

incorporated as desirable by components. In general, the evaluation of this dynamic property results to be really complex since the composites are anisotropic and nonuniform bodies and a description of damping processes in these materials needs new developments in the theory of damping. There are many sources of energy dissipation in fiber-reinforced composite material: the viscoelastic nature of its components, presence of damages like a crack in matrix (as discussed in [2] damping is more sensitive than stiffness due to the damage in composite materials), broken fiber, slip between fiber and matrix interface [3]. Two additional contributes have to be also considered: (1) Damping due to the viscoplasticity, i.e. fiber-reinforced composite material exhibit nonlinear damping due to the presence of high stress and strain at large amplitudes of vibration, and (2) damping due to the thermoelasticity, *i.e.* in the case of cyclic loading materials get heated and energy is dissipated [4]. Composite materials can be studied with the use of the *classic laminate theory* with the definition of the matrices [A], [B] and [D] which define the relationships between the fundamental stresses in terms of normal stress $\{\mathrm{N}\}$ and moment $\{\mathrm{M}\}$ with the respective deformations: mean plane displacement $\{w_0\}$ and curvature $\{\mathrm{k}\}$. The cross effects are defined by the matrix [B] of coupling normal stress/curvature or bending moment/deformation in the plane. This matrix is a zero matrix when the laminate is symmetrical and balanced.

3. METHODS FOR DAMPING PREDICTION

Prediction of damping at the micromechanical, macromechanical and structural level based on the assumption of linear viscoelasticity are carried out by many analytical models. One of the methods used for the prediction of the total damping of a composite structure is the *strain energy method* [5]: with this method the loss factor η of the composite structure can be expressed as the ratio of summation of the product of the individual element loss factor and strain energy stored in each element to the total strain energy. In this paper the theoretical loss factor of the composite plate will be evaluated through an energetic method called *Theoretical Principle of Identifying Loss Factor of Fiber-Reinforced Composite Based on Complex Modulus Method* [6]. Assuming E_1^* and E_2^* as the complex elastic moduli of the layer parallel and perpendicular to the fiber, G_{12}^* as the complex shear modulus in the 1-2 surface, ν_{12}^* and ν_{21}^* as Poisson's ratios in the respective directions, ρ as the density of the composite plate, the complex elastic modulus in each fiber direction can be expressed as

$$E_1^* = E_1(1 + \mathrm{i}\eta_{11}), \quad (1)$$

$$E_2^* = E_2(1 + \mathrm{i}\eta_{22}), \quad (2)$$

$$G_{12}^* = G_{12}(1 + \mathrm{i}\eta_{12}), \quad (3)$$

where $\eta_{11}, \eta_{22}, \eta_{12}$ are the loss factors of the fiber in the longitudinal, transverse and shear direction. According to the classical laminate theory, the constitutive equation of the kth layer yields

$$\begin{Bmatrix} \sigma_1^* \\ \sigma_2^* \\ \sigma_3^* \\ \tau_{23}^* \\ \tau_{31}^* \\ \tau_{12}^* \end{Bmatrix}^k = [Q^*] \begin{Bmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \varepsilon_3^* \\ \gamma_{23}^* \\ \gamma_{31}^* \\ \gamma_{12}^* \end{Bmatrix}^k \tag{4}$$

The $[Q^*]$ matrix in the Equation (4) is the complex stiffness matrix for the composite plate can be explicitly written as:

$$[Q^*]^k = \begin{bmatrix} Q_{11}^* & Q_{12}^* & Q_{13}^* & 0 & 0 & 0 \\ Q_{12}^* & Q_{22}^* & Q_{23}^* & 0 & 0 & 0 \\ Q_{13}^* & Q_{23}^* & Q_{33}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66}^* \end{bmatrix}^k \tag{5}$$

each coefficient Q_{ij}^* is expressed as ratio of the complex mechanical characteristics as, for example,

$$Q_{11}^* = \frac{E_1^*}{1 - \nu_{12}\nu_{21}}. \tag{6}$$

The constitutive relationship between stress and strain expressed in the Equation (4) can be also written in the global reference system (O,x,y,z) considering the transformation matrix of sine and cosine functions of the fibre arrangement angle with respect to the global reference system as

$$\begin{Bmatrix} \sigma_x^* \\ \sigma_y^* \\ \sigma_z^* \\ \tau_{yz}^* \\ \tau_{zx}^* \\ \tau_{xy}^* \end{Bmatrix}^k = [\bar{Q}^*] \begin{Bmatrix} \varepsilon_x^* \\ \varepsilon_y^* \\ \varepsilon_z^3 \\ \gamma_{yz}^* \\ \gamma_{zx}^* \\ \gamma_{xy}^* \end{Bmatrix}^k \tag{7}$$

The coefficient of complex stiffness matrix includes the real and the imaginary part, therefore, they can be expressed in the following way:

$$Q_{ij}^* = Q'_{ij} + \mathrm{i}Q''_{ij}, \tag{8}$$

where Q'_{ij} and Q''_{ij} are the real and the imaginary part of the stiffness matrix coefficients: the real part of these coefficients can be associated to the total strain energy and the imaginary part to the dissipated energy. The strain energy U^k in the kth layer is given, in the general case, by the sum of three different contributes, the longitudinal U_1^k, transverse U_2^k and thickness U_6^k strain energy

$$U^k = U_1^k + U_2^k + U_6^k, \tag{9}$$

where

$$U_1^k = U_{11}^k + U_{12}^k, \tag{10}$$
$$U_2^k = U_{21}^k + U_{22}^k, \tag{11}$$
$$U_6^k = U_{66}^k. \tag{12}$$

Each contribute is given by:

$$U_{11}^k = \frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b}\int_{h_{k-1}}^{h_k} Q'_{11}(\varepsilon_1^*)^2 dxdydz \tag{13}$$
$$U_{12}^k = \frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b}\int_{h_{k-1}}^{h_k} Q'_{12}\varepsilon_1^*\varepsilon_2^* dxdydz \tag{14}$$
$$U_{21}^k = U_{12}^k \tag{15}$$
$$U_{22}^k = \frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b}\int_{h_{k-1}}^{h_k} Q'_{22}(\varepsilon_2^*)^2 dxdydz \tag{16}$$
$$U_{66}^k = \frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b}\int_{h_{k-1}}^{h_k} Q'_{66}(\gamma_{12}^*)^2 dxdydz. \tag{17}$$

The total longitudinal, transverse and in the thickness energy is given by the summation of the contributes in each layer k. If N is total number of layers, the total longitudinal, transverse and thickness strain energy yields

$$U_{11} = \sum_{k=1}^{N} U_{11}^k \tag{18}$$
$$U_{12} = \sum_{k=1}^{N} U_{12}^k \tag{19}$$
$$U_{21} = \sum_{k=1}^{N} U_{21}^k \tag{20}$$
$$U_{22} = \sum_{k=1}^{N} U_{22}^k \tag{21}$$
$$U_{66} = \sum_{k=1}^{N} U_{66}^k. \tag{22}$$

The total strain energy is expressed as:

$$U = U_{11} + 2U_{12} + U_{22} + U_{66} \tag{23}$$

An equivalent theoretical approach can be considered to calculate the total dissipated strain energy ΔU which is, again, the sum of different contributes, and reads

$$\Delta U = \Delta U_{11} + 2\Delta U_{12} + \Delta U_{22} + \Delta U_{66}. \tag{24}$$

The single contribute of the total dissipated strain energy is related to the imaginary part of the coefficient of the stiffness matrix, for example the longitudinal dissipated energy

$$\Delta U_{11} = \pi \sum_{k=1}^{N} \int_0^a \int_0^b \int_{h_{k-1}}^{h_k} Q''_{11}(\varepsilon_1^*) dx dy dz. \quad (25)$$

From the calculation of the total strain energy with the Equation (23) and the total dissipated strain energy with the Equation (24) is possible to evaluate the specific damping capacity ψ of fiber-reinforced composite plate as follows

$$\psi = \frac{\Delta U}{U}. \quad (26)$$

Generally, the relationship between the specific damping capacity ψ and the structural damping ratio ζ of the fiber-reinforced composite plate is given by

$$\zeta = \frac{\psi}{4\pi} = \frac{\Delta U_{11} + 2\Delta U_{12} + \Delta U_{22} + \Delta U_{66}}{4\pi(U_{11} + 2U_{12} + U_{22} + U_{66})}. \quad (27)$$

In the condition of *low damping* of the system [7], it is also possible to relate the loss factor η to the damping ratio or fraction of critical damping ζ as

$$\eta = 2\zeta. \quad (28)$$

4. STUDY CASE

The damping properties will be evaluated for the plate with the following dimensions: $a = 1$ m, $b = 1$ m and $t = 0.003$ m. The material is fabric with four layers and a cross-ply configuration $[0/90]_s$. The mechanical characteristics are listed in the Table 1.

Table 1. Mechanical Properties Fabric

Mechanical Properties Fabric	Value
Density ρ	1466.7 kg/m^3
Longitudinal Young Modulus E_1	38×10^9 Pa
Transversal Young Modulus E_2	38×10^9 Pa
Shear Modulus G_{12}	5×10^9 Pa

For the plate described in the Table 1 the method explained in previous section has been implemented assuming the following value for the damping coefficients along the direction $\{11\}$, $\{22\}$, $\{12\}$ in the laminate reference system: $\eta_{11} = 0.008$, $\eta_{22} = 0.08$, $\eta_{12} = 0.04$. With these assumptions the damping ratio for the composite plate results to be $\zeta \simeq 0.0249$, thus the loss factor will be $\eta = 2\zeta = 0.0498 \simeq 5\%$: this value has been assumed as input parameter for the numerical analysis with NX Nastran.

5. NUMERICAL STUDY

The numerical study has been performed with NX Nastran for the composite plate 1 m x 1 m x 0.003 m (Table 1). The same study has been carried out for the aluminum plate with the same geometry and with the mechanical properties in the Table 2, both with free boundary conditions at all edges. The plates are represented in the Figures 2 and 3. First mode shape is reported in the Figure 4.

Table 2. Mechanical Properties Aluminum Al 6069 T3

Mechanical Properties Aluminum	Value
Density ρ	2810 kg/m^3
Young Modulus E	69×10^9 Pa
Poisson Modulus ν	0.33
Structural damping η	2%

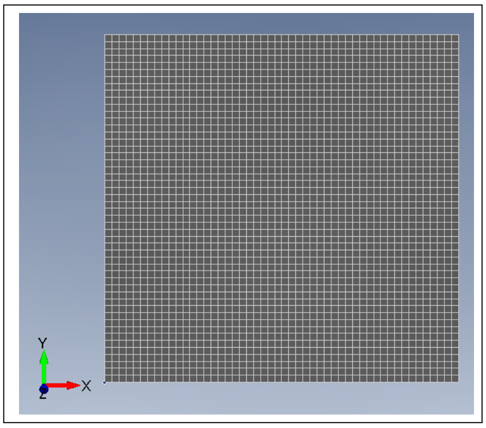

Figure 2. Aluminum Plate 1 x 1 x 0.003 m in Femap

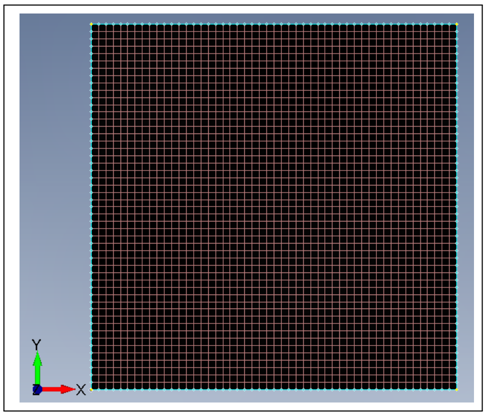

Figure 3. Composite Fabric Plate 1 x 1 x 0.003 m in Femap

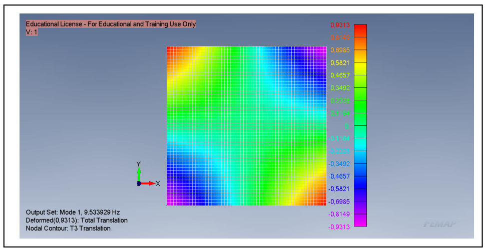

Figure 4. Aluminum Plate 1 x 1 x 0.003 m - 1st mode shape in Femap

The natural frequencies, both for the composite plate and the aluminum plate, for the first 7 elastic modes are shown in the Table 3. In the same table the scaling factor between the two plates for the natural frequencies has been also evaluated that is defined as

$$\Phi_{f_i} = \frac{f_{i,comp}}{f_{i,al}}. \tag{29}$$

Table 3. Natural frequencies composite and isotropic plates 1 m x 1 m x 0.003 m

Composite Plate	Isotropic Plate	Scaling Factor
f_{1st} = 6.30 Hz	f_{1st} = 9.53 Hz	$\Phi_{f_1} = 0.66$
f_{2nd} = 15.4 Hz	f_{2nd} = 13.9 Hz	$\Phi_{f_2} = 1.11$
f_{3rd} = 18.6 Hz	f_{3rd} = 17.6 Hz	$\Phi_{f_3} = 1.06$
f_{4th} = 21.1 Hz	f_{4th} = 24.7 Hz	$\Phi_{f_4} = 0.85$
f_{5th} = 21.1 Hz	f_{5th} = 24.7 Hz	$\Phi_{f_5} = 0.85$
f_{6th} = 36.0 Hz	f_{6th} = 44.0 Hz	$\Phi_{f_6} = 0.82$
f_{7th} = 47.1 Hz	f_{7th} = 44.0 Hz	$\Phi_{f_6} = 0.82$

The average mobility $\hat{H} = \frac{\hat{v}}{F}$ evaluated for the composite and the aluminum plate is plotted with a *loglog* scale in the Figure 5. In the Figure 6 the normalised mobility function is represented with a linear scale on both the axes and the difference in the damping value in frequency for both the plates is more visible, in particular increasing the frequency.

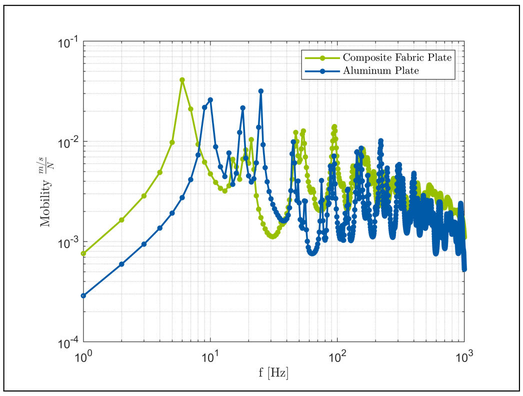

Figure 5. Numerical Mobility (Femap NX Nastran) - composite and aluminum plate

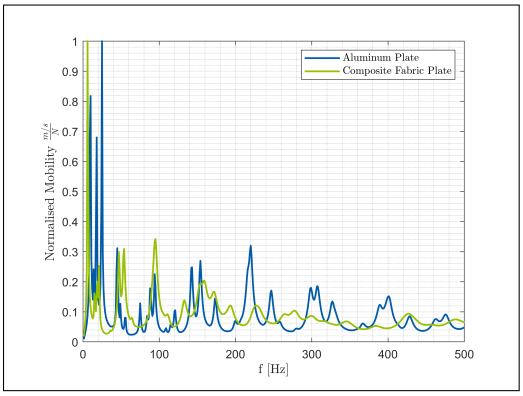

Figure 6. Numerical Normalised Mobility (Femap NX Nastran) - composite and aluminum plate

The numerical study allows to evaluate the modal damping η_i for the first six vibration modes for the composite and the aluminum plate, given the frequency response in terms of mobility.

The Table 4 shows, as done for the natural frequencies, the comparison between the two plates considering and defining the scaling factor for the modal damping that is defined as

$$\Phi_{\eta_i} = \frac{\eta_{i,comp}}{\eta_{i,al}}. \tag{30}$$

Table 4. Modal damping composite and isotropic plates 1 m x 1 m x 0.003 m

Composite Plate	Isotropic Plate	Scaling Factor
η_{1st} = 13.4 %	η_{1st} = 12.6 %	$\Phi_{\eta1} = 1.06$
η_{2nd} = 10.1 %	η_{2nd} = 7.01 %	$\Phi_{\eta2} = 1.44$
η_{3rd} = 4.84 %	η_{3rd} = 5.71 %	$\Phi_{\eta3} = 0.85$
η_{4th} = 5.19 %	η_{4th} = 4.79 %	$\Phi_{\eta4} = 1.08$
η_{5th} = 5.19 %	η_{5th} = 4.79 %	$\Phi_{\eta5} = 1.08$
η_{6th} = 2.78 %	η_{6th} = 3.43 %	$\Phi_{\eta6} = 0.81$

6. EXPERIMENTAL INVESTIGATION

The test specimen is the composite fabric plate with dimensions and mechanical characteristics described in the previous section. The plate is a cross-ply made of 4 different layers. The experimental setup consists in the composite plate with free boundary conditions, laser Doppler vibrometer, active shaker providing the excitation. The shaker is also connected to a power amplifier. The results are visualized with a laptop connected to the laser Doppler vibrometer.

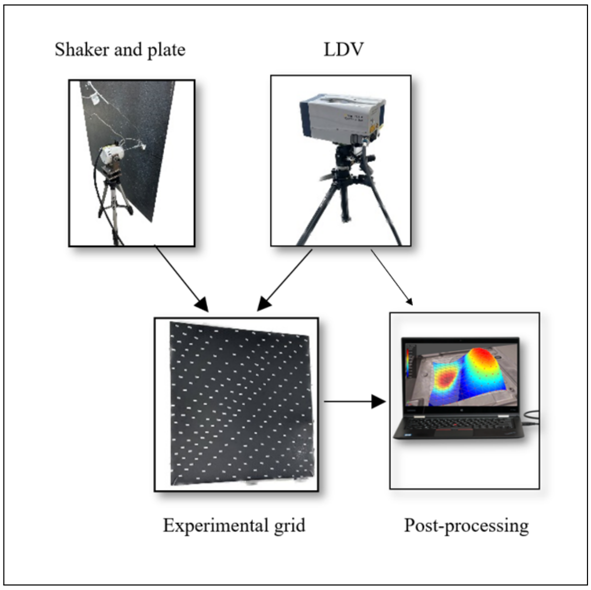

Figure 7. Experimental setup schematisation

Three different excitation forces have been considered to evaluate the experimental frequency response of the composite plate: white noise, sweep and periodic chirp.

The experimental mobility has been plotted in frequency and it is shown in the Figure 8

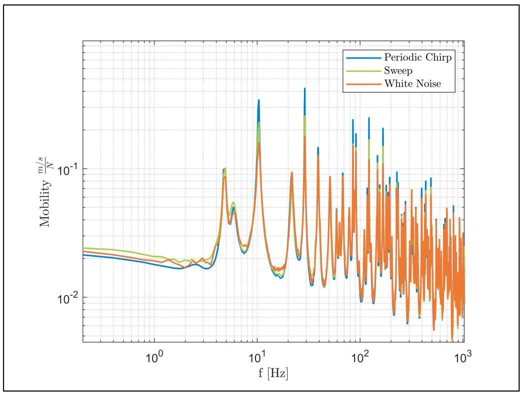

Figure 8. Experimental Mobility - Periodic chirp, sweep and white noise

In the Table 5 a comparison between the experimental and natural frequencies of the composite plate is listed.

Table 5. Natural frequencies composite and isotropic plates 1 m x 1 m x 0.003 m

Fabric Plate numerical	Fabric Plate experimental
f_{1st} = 6.30 Hz	f_{1st} = 4.90 Hz
f_{2nd} = 15.4 Hz	f_{2nd} = 10.4 Hz
f_{3rd} = 18.6 Hz	f_{3rd} = 21.6 Hz
f_{4th} = 21.1 Hz	f_{4th} = 29.0 Hz
f_{5th} = 21.1 Hz	f_{5th} = 29.0 Hz
f_{6th} = 36.0 Hz	f_{6th} = 39.0 Hz
f_{7th} = 47.1 Hz	f_{7th} = 50.0 Hz

Finally, the experimental mobility is compared with numerical mobility in the Figure 9. In fact, it is recalled that the damping factor entered within the "NX Nastran/Femap" software is a damping factor of 5 % obtained by implementing the "classical laminate theory" for the calculation of plate deformations and the evaluation of the total deformation energy U and the dissipated deformation energy ΔU by assuming values of damping along the reference system directions of the laminate $\eta_1, \eta_2, \eta_{12}$. From the comparison between the natural frequencies, from the third vibrational mode the experimental natural frequencies are bigger compared with the numerical ones and this might suggest that the overall loss factor of the composite structure is lower than theoretically estimated and this is demonstrated considering the normalised numerical and experimental mobility plotted in the Figure 9.

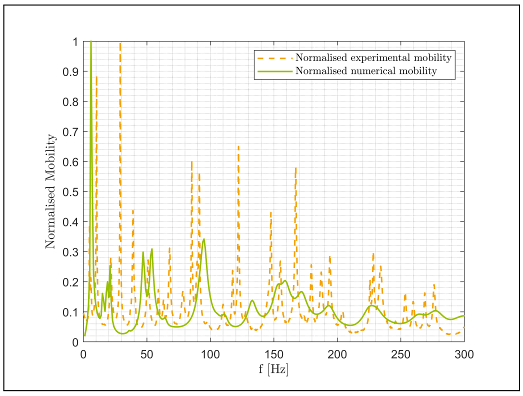

Figure 9. Numerical and experimental normalised mobility

7. CONCLUSIONS

This study shows the complexity in studying the phenomenon of damping in composite structures. The phenomenon of damping, which in itself is already complex for structures made of isotropic material, is of greater complexity when the structure is made of composite material as concomitant factors are added to those already present for isotropic materials such as the lay-up sequence and the percentage of fibres and matrix present. Adding to this complexity is the fact that the mechanical properties of composite structures are not always well known and the structural damping factors of the individual components, fibres and matrix, are not tabulated as is the case for composite materials whose range is at least known. Therefore, the theoretical determination of the damping factor, carried out in Section 3, may be incorrect if the input damping coefficients are wrong and this may result in an incorrect determination of the theoretical damping coefficient, considered as input to the numerical study. In future studies, the idea is to focus mainly on this point by combining it with a more in-depth study of square structures, whose dynamics are more complex due to the coupled and combined modes of vibration. In addition, the idea is to derive a similarity law linking the frequency responses of the composite plate and that in isotropic material, assuming a special orthotropy for the composite plate itself, for which the theoretical response exists. With this in mind, a big step forward in similarity theory could be made if a consistent relationship exists, in a general way, between structures in isotropic material and structures in composite material, including through spectral FEM methods [8] that guarantee the reliability of the damping.

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