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AN EFFICIENT NUMERICAL SIMULATION OF ULTRASONIC WAVE PROPAGATION IN SOLID MATERIALS FOR ADVANCED NON-DESTRUCTIVE TESTING

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ABSTRACT

This work proposes a new approach to modeling the normal incidence ultrasonic signal through a layered structure immersed in water. The response to the propagation of a longitudinal ultrasonic wave in a layered structure is studied analytically using the transfer matrix method (TMM), in which the layer is established as a quadrupole formalism combining stresses and velocities. The transfer matrix method enables us to determine the reflection coefficient of the layered structure, and the modeled backscattered ultrasound is represented in the time domain. The TMM method is used to verify the existence of a similar agreement to the results obtained experimentally for a transducer with a center frequency of 5MHz. The comparison showed perfect agreement between the modeling results for the time-domain representations of the backscattered signal in the plates. The numerical simulation method established in this work can be proposed as an effective complement to experiments and used to characterize structural materials by simulating ultrasonic responses for different thicknesses and frequencies, overcoming the problems associated with computational instability.

Keywords: *Transfer matrix method, Longitudinal wave, structure Materials, Numerical simulation, Ultrasonic responses.*

1. INTRODUCTION

The transfer matrix method (TMM) is a widely used technique in mathematics and physics, applied to systems represented as a sequence of interconnected subsystems using transfer matrices. While it finds extensive use in various fields like optics, acoustics, and electromagnetics, it is important to acknowledge its limitations. In acoustics, the TMM approach, as explored by researchers such as Thomson [1] and Haskell [2], can encounter stability issues for certain cases, particularly as the overall thickness of a layer or the frequency of waves increases. Additionally, the TMM becomes more complex and time-consuming. Thus, it is crucial to consider these limitations while applying the transfer matrix method to ensure accurate and reliable results [3,4].

The purpose of this paper is to contribute a new and accurate model for simulating ultrasonic signals backscattered by layered structures immersed in water at normal incidence, even in the presence of different types of layers, thicknesses, and frequencies. This model is based on the transfer matrix method (TMM). To address the numerical instability associated with thickness and frequency variations, the transfer matrix elements of the layer structures must be robust, simple, and independent [5].

Analytical calculations are excluded due to the complexity of the structures and propagation of modes, making purely numerical calculations necessary to simulate the control system, ultrasonic transmitter, fluid propagation, bulk or guided mode propagation in the structure, and receiver [6].

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In this paper, we present a numerical simulation methodology that analytically studies the response to the propagation of a longitudinal ultrasonic wave in a layered structure using the TMM [7,8]. The layered structure is represented as a quadrupole formalism combining stresses and velocities. By employing the TMM, we determine the reflection coefficient of the layered structure and model the backscattered ultrasound in the time domain, as well as assess its feasibility.

We verify the agreement between our modeling results and experimental data obtained using a 5 MHz center frequency transducer, demonstrating a perfect match in the time-domain representations of the backscattered signal in the plates. Additionally, we provide a 3D representation of the reflection coefficients in the (X, f) plane. Through an evaluation of the reflection coefficients as a function of frequency and thickness, we demonstrate the computational stability and robustness of our proposed method across a wide range of frequencies and thicknesses, overcoming issues associated with computational instability. This comprehensive analysis confirms the accuracy and effectiveness of our numerical simulation method, making a valuable contribution to estimating the ultrasonic parameters of solid layers and enhancing the field of ultrasonic nondestructive testing (NDT).

The remainder of the paper is organized as follows: Section 2 presents the mathematical model of the response of an ultrasonic wave in an immersed structural material. Section 4 presents the simulation results of the ultrasonic response of the structures in a time representation, compares the simulation results with experimental results, and finally, Section 5 concludes the paper and discusses future work.

2. THE MODEL

The theory and explanation of the proposed model are elaborated and visually represented in Figure 1. The figure presents an elastic layer structure immersed in water, highlighting the incident, reflected, and transmitted wave components.

Figure 1 illustrates the orientation and direction of a longitudinal wave incident on an infinitely long layered structure immersed in a fluid medium. The longitudinal wave is treated as a plane wave, perpendicular to the excitation surface and propagating along the positive abscissa axis. More precisely, the plane wave is simulated using a sinusoidal function multiplied by the Hanning window, centered at a frequency of 5 MHz. The layer is considered as a homogeneous and isotopic structure. Taking into account the solution of the propagation equation, the displacement field in x of a plane wave can be expressed by [9]:

$$u(x,t) = U e^{i(\omega t - \gamma_l x)} \tag{1}$$

Where U, ω and γ_l are the displacement amplitude, the angular frequency and the wave number of the plane wave.

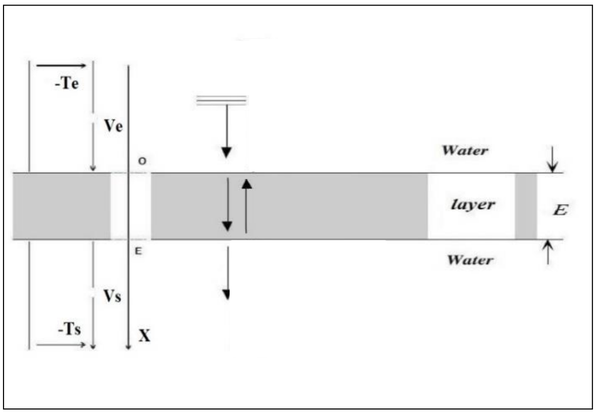

Figure 1. The schematic of the transfer matrix method for the layer structure. Ve: Input velocity, Te: Input stress, E: Thickness of layer, O: The origin (X=0), Vs: Output velocity, Ts: Output stress.

The displacement field in a layer is a combination of two ultrasonic waves (incident and reflected) which is written as Follows [9]:

$$u(x,t) = M e^{i(\omega t - \gamma_l x)} + N e^{i(\omega t + \gamma_l x)} \tag{2}$$

M and N are displacement amplitude.

In the presented model, the layer can be represented by a quadrupole. Adopting the quadrupole formalism and knowing M and N, we can derive Ve and −Te as a function of vs and −Ts as state variables.

The expression for stress and velocity can be written [9]:

$$v(x,t) = \frac{\partial u}{\partial t} \tag{3}$$

$$T(x,t) = c_l \frac{\partial u}{\partial x} \tag{4}$$

Where c_l is the longitudinal velocity in the layer.

At the interfaces between the fluid and the layer structure, the displacement field, stress and velocity at the interface must be continuous.
The continuity of the stress is:

$$Te = T(0,t);\ Ts = T(E,t) \tag{5}$$

The continuity of the velocity is:

$$Ve = V(0,t);\ Vs = Vs(E,t) \tag{6}$$

The relationship between the state variables on the input and output side of the layer structure can be written in the matrix form [9]:

$$\begin{bmatrix} v_e \\ -T_e \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_s \\ -T_s \end{bmatrix} = [T].\begin{bmatrix} v_s \\ T_s \end{bmatrix} \tag{7}$$

Where $[T]$ called transfer matrix, a_{ij} are parameters depending on the thickness and characteristic of elastic layer. The coefficient of transfer matrix has property as follows [9]:

$$a_{11} = a_{22} \tag{8}$$

$$a_{11}a_{22} - a_{21}a_{12} = 1 \tag{9}$$

Transfer Matrix Method (TMM) is indeed a valuable technique for analyzing the ultrasonic signal response of waves in layered structures immersed in a fluid:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \cos(\gamma_p E) & \dfrac{i\sin(\gamma_p E)}{Z_p} \\ i\, Z_p \sin(\gamma_p E) & \cos(\gamma_p E) \end{bmatrix} \tag{10}$$

By utilizing the TMM, one can solve a system of linear equations and determine the reflection coefficient. Considering the reflection coefficient as a function of frequency and elastic property is indeed crucial for accurately predicting the behavior of the backscattered wave [9].

$$R(\omega) = \frac{Z_f.a_{11} + Z_f.\, Z_f.a_{12} - a_{21} - Z_f.a_{22}}{Z_f.a_{11} + Z_f.\, Z_f.a_{12} + a_{21} + Z_f.a_{22}} \tag{11}$$

The acoustic impedance of water, denoted as Z_f water. In order to capture all possible echoes and obtain accurate results, long signals are typically employed in the analysis, to ensure that any potential echoes or reflections from the layered structures in the fluid are adequately captured and considered in the calculations.

3. RESULTS AND DISCUSION

Currently, a number of computer tools are available to support numerical simulations for modeling the ultrasonic signal response. In this study, a simulation of ultrasound wave propagation in layer structures is developed using MATLAB tools.
In our research, we focused on investigating and demonstrating the effectiveness of our proposed numerical simulation approach for different thicknesses of a layer structure immersed in water (Sound velocity = 1480 m/s and Density = 1000 Kg/m^3) at selected frequency (5 MHz in our case). To ensure reliable comparisons, we selected glass material, with sound velocity of 5790 m/s and density of 2300 Kg/m^3 for our experiments as we had access to specific experimental data for glass. This enabled us to directly compare the signals generated by our simulations with the corresponding experimental measurements. Significantly, our numerical simulation approach can be extended to analyze layer structures composed of any material, making it versatile and applicable to various scenarios [5,9,10].

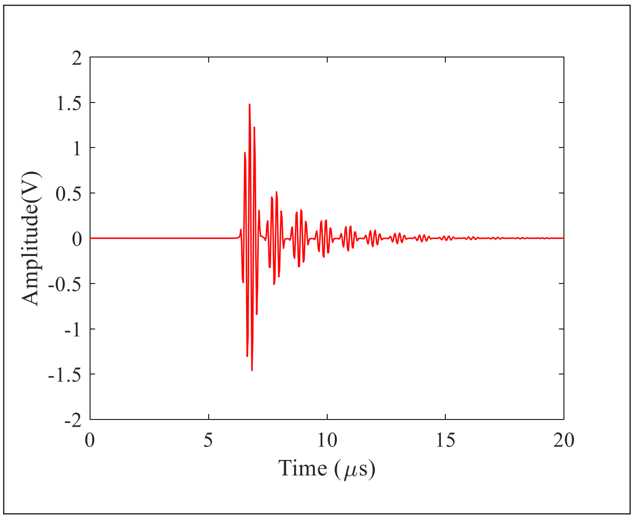

Figure 2. The simulated signal backscattered by 3 mm thick glass plate with 5 MHz of center frequency at normal incidence.

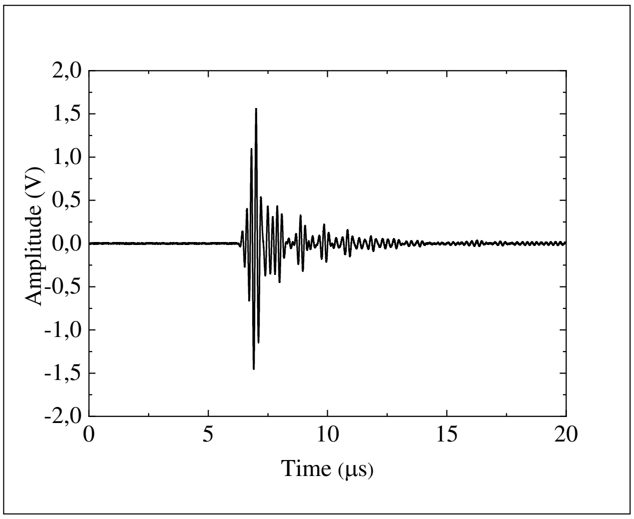

Figure 3. The experimental signal backscattered by 3 mm thick glass plate with 5 MHz of center frequency at normal incidence.

The availability of experimental data played a crucial role in our study. It served as a critical benchmark against which we evaluated the simulated signals. By comparing the simulated results with the experimental measurements, we validated the accuracy of our numerical simulation approach. This comparison allowed us to assess the reliability of our model and gain confidence in its predictive capabilities.

we present the results obtained from simulating the backscattered ultrasonic signals through 3 mm and 6 mm thick glass plates immersed in water at 5 MHz of central frequency. These results, illustrated in Figure 2 and Figure 4 respectively, reveal a distinct pattern of regularly spaced echoes in the signal. These echoes correspond to longitudinal waves bouncing back and forth inside the glass plate. In the time domain, these echoes are clearly visible as distinct peaks, indicating the presence of multiple reflections occurring within the glass plate.

Importantly, our proposed model demonstrates excellent agreement with the experimental results, as demonstrated in the comparison shown in Figure 3 and Figure 5. This confirms the accuracy and reliability of our adopted model in estimating the ultrasonic parameters of the glass plate. The agreement between the simulation and experimental results validates the reliability and accuracy of the adopted numerical simulation approach. Our model effectively estimates the ultrasonic parameters of the glass plate, providing valuable insights into its behavior and allowing for the characterization of the material.

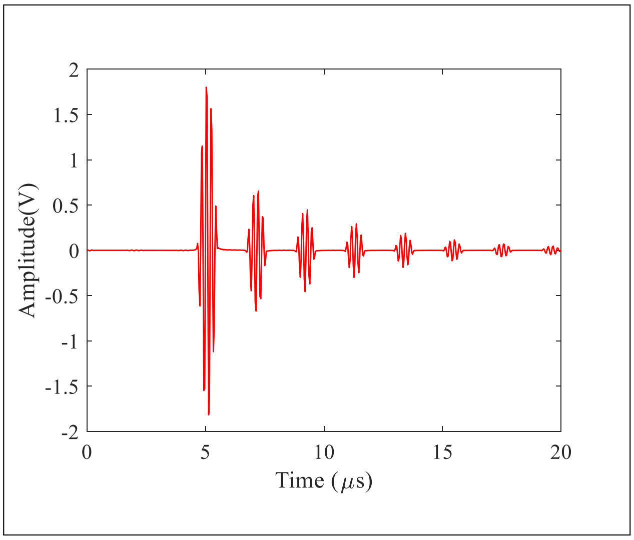

Figure 4. The simulated signal backscattered by 6 mm thick glass plate with 5 MHz of center frequency at normal incidence.

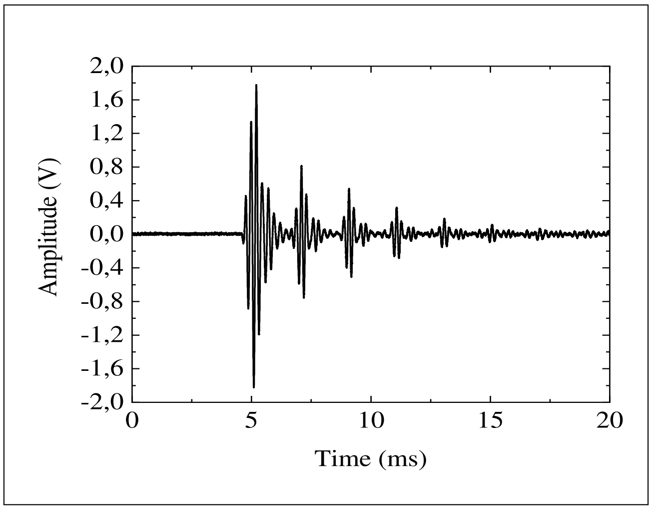

Figure 5. The experimental signal backscattered by 6 mm thick glass plate with 5 MHz of center frequency at normal incidence.

Furthermore, we provide a 3D representation of the reflection coefficients in the (X, f) plane, as depicted in Figure 6. This representation highlights the computational stability and robustness of our proposed method across a wide range of frequencies and thicknesses. Overcoming computational instability issues is crucial in ensuring reliable and accurate simulations. that this proposed method is computationally stable and robust over a wide range of different frequencies and thicknesses, which overcomes problems associated with computational instability.

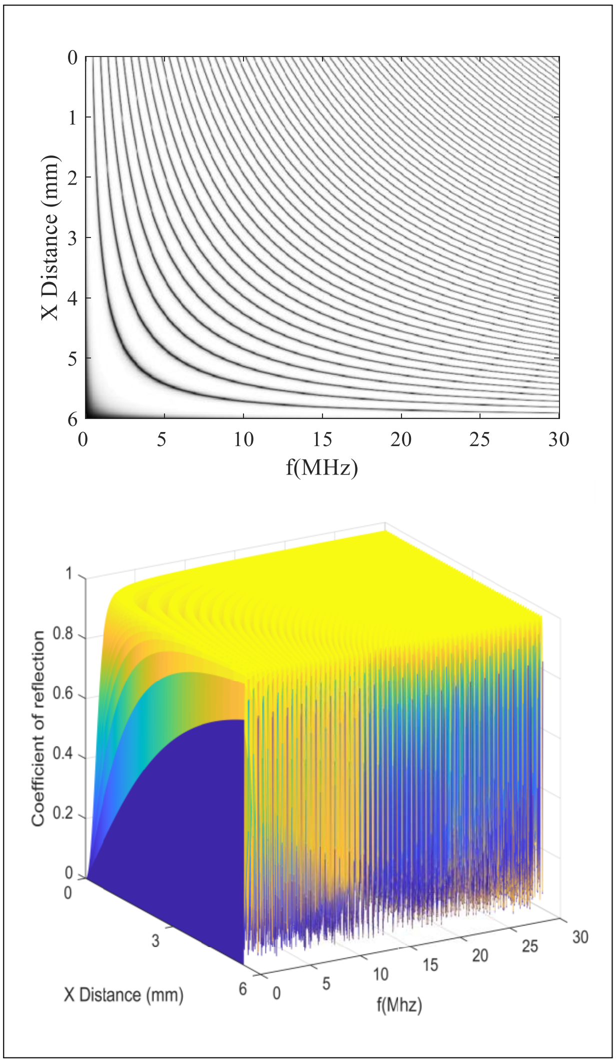

Figure 6. The 3D representations of the reflection coefficient in distance (thickness) versus frequency plane for a glass plate.

4. CONCLUSION

In conclusion, our numerical simulation methodology has successfully demonstrated its effectiveness in studying the ultrasonic response of layer structure immersed in water. The simulation results closely align with the experimental data, providing strong validation for the accuracy of our model in estimating the ultrasonic parameters of the glass plate. Furthermore, the 3D representation of the reflection coefficients showcases the computational stability and reliability of our method across different frequencies and thicknesses. These findings have significant implications for the field of ultrasonic nondestructive evaluation, offering valuable insights and practical applications for researchers and practitioners.

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