DOI: 10.61782/fa.2023.1056

forum acusticum 2023

ACOUSTIC LUMPED ELEMENT TECHNIQUES TO MEASURE THE LOW FREQUENCY RESPONSE OF POROUS MATERIALS

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ABSTRACT

The acoustic experimental characterization of the dynamic properties of a porous material is generally carried out through the standard three- and four- microphones techniques. These methods are wave decomposition based and allow to assess the transfer matrix of the specimen. Recently, two innovative acoustic measurement methodologies based on the lumped element approximation have been developed. It has been demonstrated their potentiality to overcome the difficulties of the standard techniques in the low frequency range. The bulk modulus measurement setup relies for a rigid-end back to the material, whereas the complex density measurements require an open-end. Furthermore, for each test two measures are needed: one with the sample, named full, and one where only air fulfil the volume, named empty. The empty test allows to consider any imperfect rigid condition in the bulk modulus measurement, and the effect of the radiation impedance on the open-end in the complex density. In particular, the measures of the viscous and the thermal dynamic properties of a tested sample are obtained from the acoustic pressures picked up by two microphones: one placed in the cavity behind the loudspeaker, and one in the cavity where the sample is placed.

Keywords: *complex bulk modulus, complex density, lumped element technique, porous material.*

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1. INTRODUCTION

Porous materials can be described by using the complex bulk modulus and complex density which consider the thermal and viscous acoustic behavior, respectively. Apart from theoretical model, experimental techniques should be used to assess these acoustical properties Three [1] and four [2] microphone techniques (ASTM E2611-09 [3]), for example, represent the most used methods. By using the wave decomposition methods, they characterize the transfer matrix of a porous material, from which it is also possible to extract the acoustic properties of the material such as complex bulk modulus and complex density. Upper and lower frequency limits are related to the separation distance between the couple of the adjacent microphones s. An increase in s lowers the upper limit, but it is advantageous for low frequency measurements [4]. Furthermore, upper frequency is also related to the first cutoff frequency of the duct, which is inversely proportional to the tube diameter. However, various other phenomena, mainly associated with viscosity and heat conduction, cause large errors at low frequencies (e.g., below 150–200Hz) and limit the possibility to use large separation distances. Moreover, these errors increase as the tube diameter decreases [4,5]. Therefore, improve the results at low frequency, maintaining the same measuring apparatus and the same sample is desirable. This paper aims to illustrate two recently published measurement techniques for assessing the bulk modulus [6] and complex density [7]. These techniques are based on the lumped element hypothesis and can be seamlessly integrated with a standard impedance tube setup. The subsequent sections will provide an explanation of the lumped element approach, the experimental setup, and the corresponding results.

2. LUMPED ELEMENT TECHNIQUES

A slab of the porous material, whose thickness d is much smaller than the wavelength λ (i.e. $d \ll \lambda$), can be modelled as a lumped acoustic element by means of an acoustic transmission line [8], as shown in Fig. 1.a. The impedance Z_ν and $1/Y_\kappa$ that consider the viscous and thermal effect can be obtained as:

$$Z_\nu = -i\omega\tilde{\rho}, \quad Y_\kappa = -\frac{i\omega}{\tilde{K}}, \tag{1}$$

and

$$\sqrt{\frac{Z_\nu d}{\frac{1}{Y_\kappa d}}} = -i\tilde{k}d, \quad \sqrt{\frac{Z_\nu}{Y_\kappa}} = \tilde{Z}_c, \tag{2}$$

where $\tilde{k}$ and $\tilde{Z}_c$ are the wave number and the characteristic impedance, respectively. The state variables are the acoustic pressure p and volume velocity U ($U = vA$, where A is the cross-sectional area of the sample and v is the acoustic particle velocity) and are linked, between the two ends of the material, from the following transfer matrix:

$$\begin{bmatrix} p \\ U \end{bmatrix}_d = \begin{bmatrix} \cos(\tilde{k}d) & -i\frac{\tilde{Z}}{A\varphi}\sin(\tilde{k}d) \\ -i\frac{A\varphi}{\tilde{Z}}\sin(\tilde{k}d) & \cos(\tilde{k}d) \end{bmatrix} \begin{bmatrix} p \\ U \end{bmatrix}_0 . \tag{3}$$

The subscripts 0 and d indicate the conditions upstream and downstream of the material and φ the open-porosity. Lumped element hypothesis, verified in the case that $\tilde{k}d < 0.5$ [6, 7], simplify the element of the matrix as:

$$\begin{bmatrix} p \\ U \end{bmatrix}_d = \begin{bmatrix} 1 & -i\frac{\tilde{Z}}{A\varphi}\tilde{k}d \\ -i\frac{A\varphi}{\tilde{Z}}\tilde{k}d & 1 \end{bmatrix} \begin{bmatrix} p \\ U \end{bmatrix}_0 . \tag{4}$$

From the Eq. 4, it follows that

$$p_d = p_0 - i\frac{\tilde{Z}_c}{A\varphi}\tilde{k}dU_0 = p_0 - i\frac{\omega\tilde{\rho}}{A\varphi}dU_0. \tag{5}$$

In the case of $Z_\nu \ll 1/Y_\kappa$ (see Fig. 1.b) and $Z_{rad} \to \infty$, the acoustic pressure can be considered uniform inside the porous slab $p_d \approx p_0$. This latter condition can be achieved with a rigid and impervious termination at one end of the material. In the same way, from Eq. 4, it follows that:

$$U_d = -i\frac{A\varphi}{\tilde{Z}_c}\tilde{k}dp_0 + U_0 = -i\frac{\omega}{\tilde{K}}dp_0 + U_0. \tag{6}$$

When $Z_\nu \gg 1/Y_\kappa$, it is possible to consider that the acoustic volume velocity across the porous material slab is almost uniform $U_d \approx U_0$. This case can be accomplished if a very low impedance radiation value Z_{rad} (ideally a short-circuit) is considered for example by considering an open-end condition.

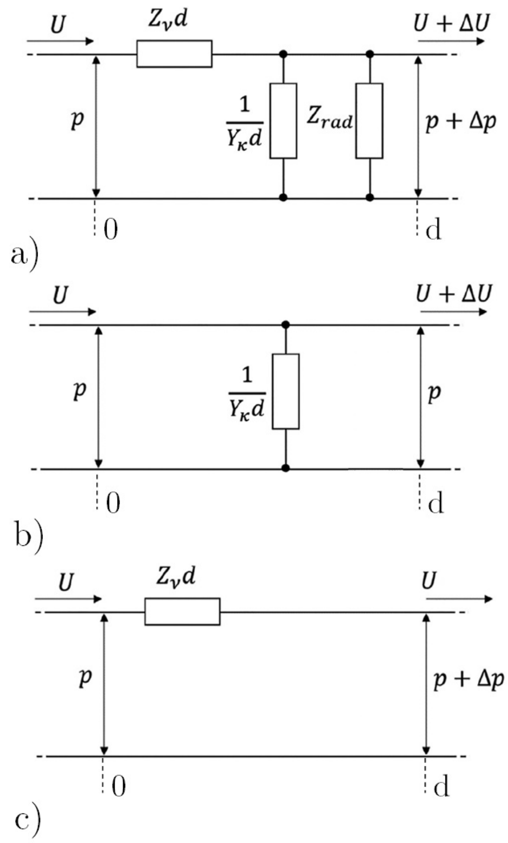


Figure 1. The acoustic transmission line showing (a) the complete thermo-viscous model; (b) the case of negligibility of transversal viscous impedance; (c) the case of negligibility of the acoustic impedance at the open-end section.

3. THE NEW MEASUREMENT METHOD

Measurement of acoustic properties of porous material can be enhanced at low frequency thanks to the new methodologies. It provides just a small improvement to the ordinary stationary wave tube that is utilized for the same measurements. It suffices to drill a hole in the box that houses the loudspeaker to accommodate a microphone (see Fig. 2.a). Indeed, under the hypothesis that the volume dimensions of box are small compared to the acoustic wavelength, the harmonic volume velocity U_0, very closed to the loudspeaker, can be provided by the sound pressure p_{LS} picked up right from this microphone as:

$$U_0 = -i\omega \frac{V_{LS}}{\gamma p_m} p_{LS}. \quad (7)$$

As will be shown in the next sections the proposed techniques require the use of a short new tube, called sample holder, where another microphone is inserted to pick up the pressure p_{SH} inside the chamber.

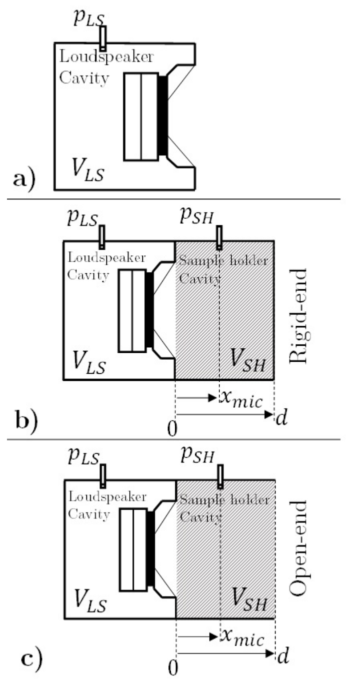


Figure 2. Sketchup of the experimental set up (a) loudspeaker cavity; (b) Sketchup of experimental set up for the bulk modulus measurement; (c) Sketchup of experimental set up for the complex density measurement.

3.1 Complex bulk modulus measurement

In this technique, the porous material is housed in the sample holder and hermetically sealed at one end ($x = d$) while the other end ($x = 0$) is placed above the loudspeaker (see Fig. 2.b). Measurements are repeated twice, the first time without material (*empty*) and then placing the material in the sample holder (*full*). Applying the Eqs. 6 and 7 it is possible to obtain:

$$\left(\frac{U_d}{p_{SH}}\right)_{full} = -i\omega \frac{V_{SH}\varphi}{\tilde{K}} - i\omega \frac{V_{LS}}{\gamma p_m}\left(\frac{p_{LS}}{p_{SH}}\right)_{full},$$
$$\left(\frac{U_d}{p_{SH}}\right)_{empty} = -i\omega \frac{V_{SH}}{\tilde{K}} - i\omega \frac{V_{LS}}{\gamma p_m}\left(\frac{p_{LS}}{p_{SH}}\right)_{empty}, \quad (8)$$

where in the case of the *empty* configuration, $Ad = V_{SH}$, whereas for the full configuration, $Ad = V_{SH}\varphi$. Please note that the microphone has not been placed right next to the loudspeaker as it is making use of the hypothesis that $p_d \approx p_0 \approx p_{SH}$. However, in-depth analysis of the influence of microphone position, as well as the presence of air gap between loudspeaker and material has been reported in Ref. [6]. The terms on the first member in Eqs. 8 are equal and follows that:

$$\tilde{K} = \frac{\gamma p_m}{\frac{1}{\varphi} - \frac{V_{LS}}{V_{SH}\varphi}\left[\left(\frac{p_{LS}}{p_{SH}}\right)_{full} - \left(\frac{p_{LS}}{p_{SH}}\right)_{empty}\right]}. \quad (9)$$

Numerical simulations [6] show that this equation is valid in the abovementioned hypothesis: $|\tilde{k}d| < 0.5$.

3.2 Complex density measurement

In this case, the porous material is housed in the sample holder and left open at one end ($x = d$) while the other end ($x = 0$) is placed above the loudspeaker (see Fig. 2.c). Measurements are repeated twice, the first time without material (*empty*) and then placing the material in the sample holder (*full*). Applying the Eqs. 5 and 7 it is possible to obtain:

$$\left(\frac{p_{SH}}{U_d}\right)_{full} = i\frac{\gamma p_m}{\omega V_{LS}}\left(\frac{p_{SH}}{p_{LS}}\right)_{full} - i\frac{\omega\tilde{\rho}}{A\varphi}(d - x_{mic}),$$
$$\left(\frac{p_{SH}}{U_d}\right)_{empty} = i\frac{\gamma p_m}{\omega V_{LS}}\left(\frac{p_{SH}}{p_{LS}}\right)_{empty} - i\frac{\omega\tilde{\rho}}{A}(d - x_{mic}). \quad (10)$$

In this case it is making use of the hypothesis that $U_d \approx U_0 \approx U_{x_{mic}}$. The terms on the first member represent the impedance radiations Z_{rad} which can be considered the same with or without the material inside the sample holder, therefore it follows that:

$$\tilde{\rho} = \varphi\left\{\rho_0 + \frac{\gamma p_m A}{\omega^2 V_{LS}(d - x_{mic})} \times \times\left[\left(\frac{p_{SH}}{p_{LS}}\right)_{full} - \left(\frac{p_{SH}}{p_{LS}}\right)_{empty}\right]\right\}. \quad (11)$$

Numerical simulation [7] shows that this equation is valid if the hypothesis $|\tilde{k}d| < 0.5$ is verified and if the radiation impedance can be considered negligible. In the latter case it is necessary to verify that $k_0R < 0.5$, where k_0 is the free-air wave number and R is sample holder radius. Please note that repeating the measurement twice avoids to assess the radiation impedance Z_{rad}.

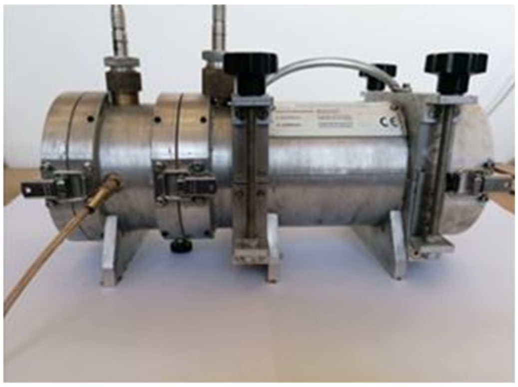
4. RESULTS

4.1 Experimental setup

The experimental apparatus for the measurement of the complex bulk modulus $\tilde{K}$ and complex density $\tilde{\rho}$ of the porous media, sketched in Fig. 2, are shown in Fig. 3. In this case the same specimen holder used for the measurements with *two* or *four* microphones' techniques, has been successfully adapted as a short tube. In theory, without removing the specimen it is possible to carry out $\tilde{K}$ (see Fig. 3.a) and $\tilde{\rho}$ (see Fig. 3.b) measurements as well as *two* and *four* microphone measurements (see Fig. 3.c). In particular, the driver must be sealed up to avoid air leakages. The same care is also required for the measurement of $\tilde{K}$ for the specimen holder. The first step is to provide accurate measurements of the volumes, V_{LS} and V_{SH}. An acoustic method for the evaluation of the volume cavities, i.e., acoustic compliance, is proposed in Ref. 8.

4.2 Experimental results

In this paper results on fibrous and foams are reported. Besides those that will be presented, the proposed techniques have been tested successfully on other numerous kinds of porous materials [6, 7]. The fibrous material is realized in polyester fibre with a thickness of 4.4 cm and an air-flow resistivity of 1800 [Pa s / m^2]. Results confirm the methods, when the lumped element hypothesis is verified, agrees with the theoretical model. Theoretical model chosen to verify the results is provided by Miki [9] because, for simplicity require only one parameter. Please note that, for the measurement of the bulk modulus, only the condition $|\tilde{k}d| < 0.5$ should be monitored (see the vertical dotted line in Fig. 4 and the related axis on the top of the graphic. Instead, in Fig. 5, it is required to examine both the $|\tilde{k}d| < 0.5$ (red continuous line) and $k_0R < 0.5$ (red dotted line) conditions and select the worse of the two for the rho measurement. In this case it is realized by $|\tilde{k}d| < 0.5$ at the frequency of about 480 Hz. Above this frequency the results are not true.



a)

b)

c)

Figure 3. Experimental set up for the bulk modulus measurement (a) and for the complex density measurement (b). (c) Entire standing wave tube.

The other porous material used in this paper is a plastic foam has a thickness of 4.8 cm and an air-flow resistivity of 2600 [Pa s / m^2]. From Fig. 6 it is possible to observe that the frequency over that the results depart from the theoretical prevision is a little bit lower than the previous case. This because the air flow resistivity and the thickess is higher than that the fiber porous material. Therefore, given a material, to enlarge the frequency range it is necessary to reduce the thickness. Looking at Fig. 6, it is possible to appreciate that also in this case, the $|\tilde{k}d| < 0.5$ condition (red continuous line) is more restrictive than the $k_0R < 0.5$ condition (red dotted line).

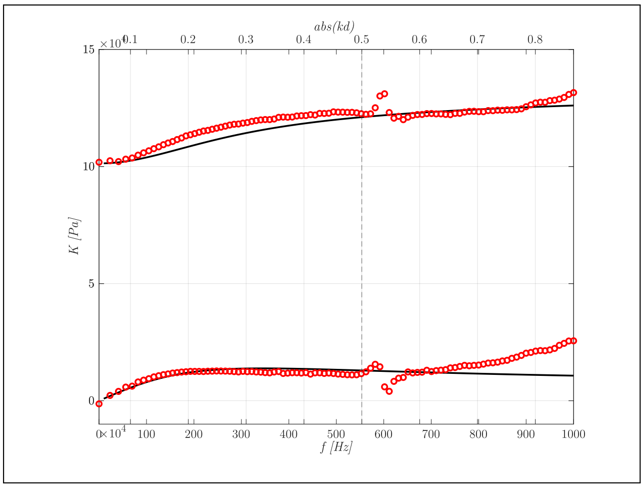


Figure 4. **Fiber material**; bulk modulus - Miki's model (—, continuous black line), experimental data (, red circle points). The black vertical dashed line (dashed lines) represents the limit value $|\tilde{k}d| < 0.5$.

5. CONCLUSION

In this paper, a new acoustic method for the low frequency measurement of the complex bulk modulus and complex density of a porous medium is proposed. This novel technique is based on a lumped element hypothesis, and it relies on the use of an experimental setup that require a simple modification of a ordinary standing wave tube. The measurements carried out on different types of porous materials, some of which are also reported in this work, have shown an excellent response to the theoretical predictions. Therefore it can be applied when there is a need to know the acoustic behavior of a material at very low frequencies. This can be the case for thermoacoustic applications or when a material to control the sound at very

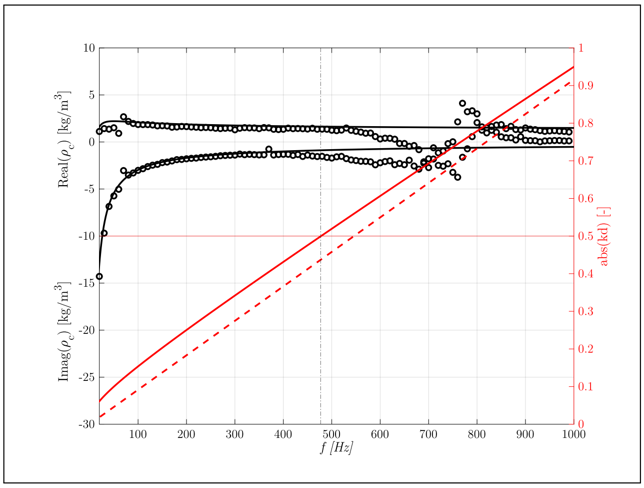


Figure 5. **Fiber material**; complex density - Miki's model (continuous black line) and the experimental data (lumped element technique) referred to as the left y axes. The parameter (continuous red line) and k_0R (dashed red line) referred to as the right y axes. The black vertical dashed-dotted line represents the cut-off frequency corresponding to $|\tilde{k}d| < 0.5$ and k_0R (horizontal red line).

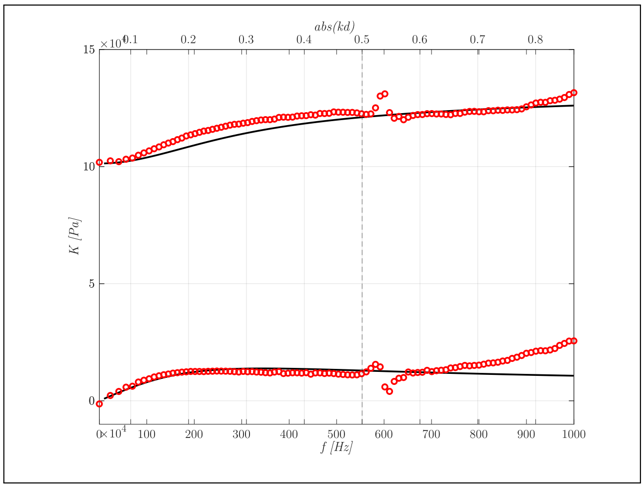


Figure 6. **Foam**; bulk modulus - Miki's model (—, continuous black line), experimental data (, red circle points). The black vertical dashed line (dashed lines) represents the limit value $|\tilde{k}d| < 0.5$.

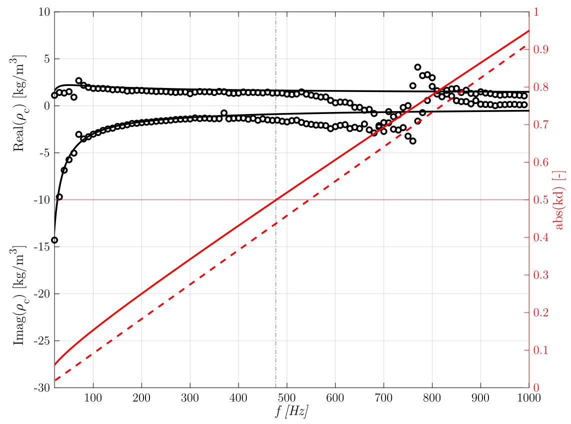


Figure 7. **Foam**; complex density - Miki's model (continuous black line) and the experimental data (lumped element technique) referred to as the left y axes. The parameter (continuous red line) and k_0R (dashed red line) referred to as the right y axes. The black vertical dashed-dotted line represents the cut-off frequency corresponding to $|\tilde{k}d| < 0.5$ and k_0R (horizontal red line).

high frequencies is required. The proposed measurement technique provides reliable results, when the lumped element hypothesis is satisfied (i.e. for $|\tilde{k}d| < 0.5$). Furthermore, since measurements with an open tube are required for complex density, the proposed measurement technique consider that the radiation impedance can be considered negligible (mathematically for $k_0R < 0.5$).

6. REFERENCES

[1] O. Doutres, Y. Salissou, N. Atalla, and R. Panneton, "Evaluation of the acoustic and non-acoustic properties of sound absorbing materials using a three-microphone impedance tube," *Applied Acoustics*, vol. 71, pp. 506–509, jun 2010.

[2] B. H. Song and J. S. Bolton, "A transfer-matrix approach for estimating the characteristic impedance and wave numbers of limp and rigid porous materials," *The Journal of the Acoustical Society of America*, vol. 107, p. 1131, feb 2000.

[3] American Society for Testing and Materials, "E2611 - 09 - Standard Test Method for Measurement of Normal Incidence Sound Transmission of Acoustical Materials Based on the Transfer Matrix Method," 2017.

[4] H. Bodén and M. Åbom, "Influence of errors on the two-microphone method for measuring acoustic properties in ducts," *The Journal of the Acoustical Society of America*, vol. 79, p. 541, aug 1998.

[5] J. Prisutova, K. Horoshenkov, J.-P. Groby, and B. Brouard, "A method to determine the acoustic reflection and absorption coefficients of porous media by using modal dispersion in a waveguide," *The Journal of the Acoustical Society of America*, vol. 136, p. 2947, dec 2014.

[6] M. Napolitano, E. Di Giulio, F. Auriemma, R. A. Romano, and R. Dragonetti, "Low frequency acoustic method to measure the complex bulk modulus of porous materials," *The Journal of the Acoustical Society of America*, vol. 151, pp. 1545–1556, oct 2022.

[7] E. Di Giulio, M. Napolitano, A. Di Meglio, R. A. Romano, and R. Dragonetti, "Low frequency acoustic method to measure the complex density of porous materials," *The Journal of the Acoustical Society of America*, vol. 152, p. 2220, oct 2022.

[8] G. A. Russell and E. D. Hjerpe, "On the analogy between the one-dimensional acoustic waveguide and the electrical transmission line," *Journal of the Acoustical Society of America*, vol. 94, no. 1, pp. 583–584, 1993.

[9] Y. Miki, "Acoustical properties of porous materials-Modifications of Delany-Bazley models-," *Journal of the Acoustical Society of Japan (E)*, vol. 11, no. 1, pp. 19–24, 1990.