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INVERSE IMPEDANCE IDENTIFICATION USING AN EFFICIENT BEM STRATEGY

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ABSTRACT

Limited accessibility often restricts the possibility of directly measuring the true acoustic properties of existing components. In that context, this paper investigates a hybrid approach for the inverse identification of the acoustic impedance of components, by combining microphone measurements with highly efficient acoustic Boundary Element Method (BEM) models. The modeling efficiency, achieved through a parametric model order reduction (pMOR) technique for BEM models [1], facilitates the fast transition among models with different impedance values and thus, greatly accelerates the inverse identification scheme. The effectiveness of the proposed technique is demonstrated with the well-studied example of the KU Leuven soundbox test setup [2].

Keywords: *BEM, parametric model order reduction, in situ impedance characterization.*

1. INTRODUCTION

Being a system level property, obtaining the acoustic behavior of components demands modeling the entire system to obtain a trustworthy simulation. However, this can result in a significant computational cost, which, particularly in case of inverse identification strategies, might become unacceptable. To enhance the efficiency of acoustic analyses, various Model Order Reduction (MOR) techniques have been developed accelerating for example vibro-acoustic Finite Element (FE) [3] simulations in [4, 5], or acoustic Boundary Element Method (BEM) [6] analyses in [7, 8]. Although such techniques are effective in solving acoustic systems with a single parameter, they are inadequate for dealing with systems containing multiple parameters, such as material parametrizations. Enabling model order reduction of multiparametric systems, several parametric MOR (pMOR) strategies have been developed within the field of acoustics, accommodating pMOR of FE models including topology [9], material [10] and boundary conditions parametrizations [11]. Respectively, in BE analyses only recently a technique for parametric MOR has been proposed, allowing for material, source position, and shape parametrizations [1].

This paper employs this recently developed technique for the inverse identification of the acoustic impedance of a porous layer placed within the KU Leuven soundbox. The approach of this paper follows a similar logic to the one presented within [12]. However, employing a parametric reduced order model as presented in [1] results in a more efficient approach allowing not only multiple iterations across different acoustic impedance values but also inverse identification for multiple frequencies, thus enabling complicated impedance functions. The paper exploits the reduced order model constructed in [13] and employs an inverse optimization scheme to retrieve the acoustic impedance of an acoustic lining patch, which is considered to be unknown. The paper starts by introducing the impedance parametrization in BEM systems and then briefly presents the parametric model order reduction strategy introduced in [1]. Finally, after introducing a basic inverse approach for the impedance identification the performance of the approach is investigated based on the virtual experiment of an interior acoustic cavity.

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2. IMPEDANCE PARAMETRIZATION IN BEM SYSTEMS

Inserting a piecewise constant approximation of the sound pressure $\psi(\mathbf{x})$ and the boundary admittance $Y(\mathbf{x})$ over the boundary into the system resulting from a discretization of the Helmholtz equation employing Boundary Elements [6] results in

$$\big(\mathbf{H}(\omega) - \mathbf{G}(\omega)\mathbf{Y}(\omega)\big)\mathbf{x}(\omega) = \mathbf{b}(\omega). \tag{1}$$

In (1) $\mathbf{H}, \mathbf{G} : \Psi \rightarrow \mathbb{C}^{N\times N}$ represent the contributions of each element to the double and single layer potentials and $\omega \in \Psi$ is the angular velocity defined on a set $\Psi := [\omega_{\min}, \omega_{\max}]$ by $\omega := 2\pi f$ with f representing the frequency. Additionally, $\mathbf{Y} : \Psi \rightarrow \mathbb{C}^{N\times N}$ is a diagonal admittance matrix, with each diagonal entry representing the respective admittance value of each element. Since typically several elements share identical boundary conditions, the discretization nodes can be grouped according to their admittance value in Ξ groups. Inserting also the necessary boundary conditions in equation (1) it is rewritten as

$$(\mathbf{A}(\omega) + \sum_{j=1}^{\Xi} y_j \mathbf{B}_j(\omega))\mathbf{x}(\omega, \mathbf{p}) = \mathbf{b}(\omega), \tag{2}$$

where y_j is the respective admittance value of the j$^{\text{th}}$ group, $\mathbf{B}_j : \Psi \rightarrow \mathbb{C}^{N\times N}$ represents the contributions of the impedance boundary condition on the system matrix $\mathbf{A} : \Psi \rightarrow \mathbb{C}^{N\times N}$ and $\mathbf{b} : \Psi \rightarrow \mathbb{C}^{N}$. As indicated in equation (2), the admittance value of each group y_j constitutes an affine parametrization of the system. In that way, having constructed matrices $\mathbf{A}(\omega)$ and $\mathbf{B}_j(\omega)$, it is possible to compute the acoustic response for varying admittance on the Ξ lining patches. It is noted that here the solution $\mathbf{x}$ is a function of both ω and $\mathbf{p} = [y_1, \dots, y_\Xi]$ and as such the system of (2) is a multi-parametric system.

3. INVERSE IMPEDANCE ESTIMATION THROUGH A PMOR FOR BEM SYSTEMS

In this section the parametric model order reduction technique employed on impedance parametrized BEM systems is introduced and subsequently the inverse problem is defined as an optimization procedure. More details about the techniques leveraged within this section can be found in references [1, 7, 14].

3.1 Approximation of the BEM system with Chebyshev polynomials

As indicated in section 2, although the impedance parametrization of the BEM system is affine, the system itself remains non-affinely dependent on angular frequency ω. An affine approximation of the system in equation (2) can be constructed selecting either a Chebyshev [15] or a Taylor approximation [7] for each entry of $\mathbf{A}(\omega), \mathbf{B}(\omega)$ and $\mathbf{b}(\omega)$ independently. For example, the system can be rewritten employing Chebyshev polynomials as

$$\Big(\sum_{i=0}^{M}{}' c_i(\omega)(\mathbf{T}_i^{\mathbf{A}} + \sum_{j=1}^{\Xi} y_j \mathbf{T}_{ji}^{\mathbf{B}_j})\Big)\boldsymbol{x}(\omega, \mathbf{p}) = \sum_{i=0}^{M}{}' c_i(\omega)\mathbf{q}_i, \tag{3}$$

where $c_i(\omega)$ are the Chebyshev polynomials of the first kind up to order M, with $i = 0 \dots M$, and the prime indicates that the first term is halved. Additionally, $\mathbf{T}^{\mathbf{A}} \in \mathbb{C}^{N\times N}$ and $\mathbf{T}^{\mathbf{B}_j} \in \mathbb{C}^{N\times N}$ represent the Chebyshev coefficients developed for the frequency dependent matrix $\mathbf{A}$ and $\mathbf{B}_j$, while $\mathbf{q}_i \in \mathbb{C}^{N}$ are the coefficients for $\mathbf{b}$. As indicated in [13] storing all coefficients of equation (3) might result in a tremendously high memory requirement, since $M \times (\Xi + 1)$ dense $N \times N$ matrices need to be stored. As a result, employing such a technique is enabled only when this is combined with an appropriate parametric model order reduction strategy.

3.2 Galerkin Model Order Reduction for impedance parametrized BEM systems

Imposing a Galerkin one-sided projection of the system in (3) enables the storage of the affinely approximated system but also leads to an efficient model order reduction strategy. Employing such a strategy, the true parameter dependent solution $\mathbf{x}(\omega, \mathbf{p})$ is approximated by a solution $\hat{\mathbf{x}}(\omega, \mathbf{p}) \in \mathscr{W}$, i.e. the solution vector is expressed as a linear combination of the ℓ column vectors of the reduced basis $\mathbf{W} \in \mathbb{C}^{N\times\ell}$, with $\mathscr{W} := \text{span}\{\mathbf{W}\}$, as

$$\mathbf{x}(\omega, \mathbf{p}) \approx \hat{\mathbf{x}}(\omega, \mathbf{p}) = \mathbf{W}\mathbf{x}_\ell(\omega, \mathbf{p}), \tag{4}$$

which reduces the dimension of the subspace within which the solution lies to ℓ. Imposing the Galerkin condition the system can be written as

$$\sum_{i=0}^{M}{}' c_i(\omega)\mathbf{W}^H(\mathbf{T}_i^{\mathbf{A}}+\sum_{j=1}^{\Xi} y_j\mathbf{T}_{ji}^{\mathbf{B}_j})\mathbf{W}\boldsymbol{x}_\ell(\omega,\mathbf{p}) \quad (5)$$

$$=\sum_{i=0}^{M}{}' c_i(\omega)\mathbf{W}^H\mathbf{q}_i, \quad (6)$$

where $\mathbf{W}^H$ represents the conjugate transpose of the projection basis $\mathbf{W}$, thus resulting in a series of reduced order matrices as

$$\sum_{i=0}^{M}{}' c_i(\omega)(\mathbf{T}_{r,i}^{\mathbf{A}}+\sum_{j=1}^{\Xi} y_j\mathbf{T}_{r,ji}^{\mathbf{B}_j})\boldsymbol{x}_\ell(\omega,\mathbf{p}) = \sum_{i=0}^{M}{}' c_i(\omega)\mathbf{q}_{r,i}. \quad (7)$$

In the above expression (7) it holds $\mathbf{T}_{r,i}^{\mathbf{A}}, \mathbf{T}_{r,ji}^{\mathbf{B}_j} \in \mathbb{C}^{\ell\times\ell}$ and $\mathbf{q}_{r,i} \in \mathbb{C}^{\ell}$. The approximation error introduced by (4) is driven by the quality of the reduction basis $\mathbf{W}$, which can be guaranteed by automatic and adaptive construction techniques as the ones reported in [1, 14]. However, in order to achieve an optimal algorithmic procedure, this basis needs to be constructed before deploying the affine approximation in order to avoid storing the entire sequence of full order dense matrices [7].

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3.3 Constructing the reduction basis with a multi-parameter Krylov recycling

To construct a high quality reduction basis for the parametric system of (2) a multi-parameter Krylov subspaces recycling strategy can be employed. The goal of such a strategy is to find a basis $\mathbf{W}, \mathscr{W} := \text{span}\{\mathbf{W}\}$, such that the resulting residual satisfies

$$r(\omega,\mathbf{p}) \le r_{\text{tol}}, \quad \forall\omega \in \Phi \subset \Psi, \mathbf{p} \in S \subset P, \quad (8)$$

where Φ is a predefined grid of Ψ and S a predefined grid of P, with $P = P_1 \times \cdots \times P_\Xi$ being the parametric space containing the Ξ single dimensional parameter spaces $P_1, \cdots, P_\Xi$. Without loss of generality, the initial guess for the system solution is chosen as $\mathbf{x}_0(\omega,\mathbf{p}) := 0$ and therefore omitted.

To construct the basis, Krylov subspaces that are commonly employed for the iterative solution of single linear systems are collected. These subspaces follow the form of

$$\mathscr{K}_\ell(\mathbf{A},\mathbf{b}) = \text{span}\{\mathbf{b}, \mathbf{A}\mathbf{b}, \ldots, \mathbf{A}^{\ell-1}\mathbf{b}\}. \quad (9)$$

Combining multiple of those Krylov subspaces can yield a high quality reduction basis as demonstrated in [1,7,14]. The cumulative subspace can be written as

$$\mathscr{K}^{tot} = \bigcup_{i=1,j=1}^{L,q} \mathscr{K}_{\ell(\omega_i,\mathbf{p}_j)}^{\omega_i,\mathbf{p}_j}, \quad (10)$$

where $\Omega := [\omega_1, \ldots \omega_L]$ and $Q := [\mathbf{p}_1, \ldots, \mathbf{p}_q]$. In the above expression the superscript indicates the parameter configuration for which these Krylov subspaces have been constructed, while the subscript indicates the dimension of those. The spacing between two sampled configurations and the dimension of the subspaces can be selected to remain constant as proposed in [7] or an adaptive selection can be employed as introduced in [1, 14].

3.4 Impedance identification as an optimization problem

Inverse identification methods aim to determine the parameter values that lead to the most accurate match between the model predictions and experimental measurements. This can be achieved by formulating an optimization problem and selecting the objective function to be minimized as the difference between the numerical and experimental measurements, while the system parameters are the variables. However, in this paper the measurement data are replaced by the response of a system retrieved through a virtual experiment, assuming that the frequency response functions (FRFs) of a system are given but its impedance properties are unknown.

The optimization problem can be obtained by minimizing the objective function f that represents the discrepancy between the measured and simulated FRFs, which can be written as

$$f(\omega,\mathbf{p}) = \sum_{i=1}^{n_{\text{mic}}}(\psi_i(\omega,\mathbf{p}) - \tilde{\psi}_i(\omega))^2, \quad (11)$$

where n_{mic} is the number of response functions considered, ψ is the simulated pressure and $\tilde{\psi}$ is the measured response from the (virtual) experiment. Therefore, to retrieve the frequency dependent impedance function the optimization problem defined as

$$\arg\min_{\mathbf{p}} f(\omega,\mathbf{p}) \quad (12)$$

needs to be solved for each value of ω.

The solution of this optimization problem is performed using the steepest descent method and the finite difference approach to calculate the gradient of the cost function. Since following such an approach multiple solutions of the system are required to evaluate both the cost function and the gradient for each iteration, employing the above pMOR approach can significantly reduce the computational resources required for this task.

4. RESULTS

In this section, the performance of the proposed technique to identify the impedance combining in-situ measurements with a reduced order Boundary Element model is examined. As an initial proof of concept the simulated response of the KU Leuven soundbox [2] with known impedance boundary conditions is considered as the measurement response. The soundbox is considered as a good demonstration example due to the numerous internal resonances that occur within the acoustic cavity. Achieving good performance for this case, exterior radiation cases or less resonant geometries are expected to match the performance levels set by this example.

4.1 Problem definition

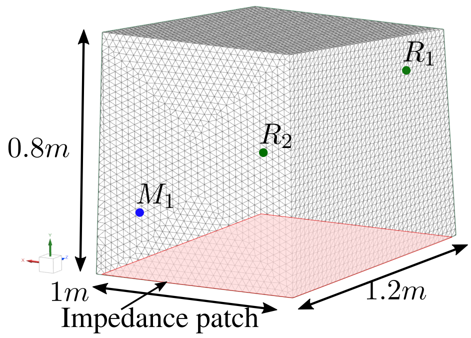


Figure 1: Soundbox model

The model of the soundbox considered in this paper is given in Figure 1. The model consists of 6503 linear elements and leads to a system of equations with 6503 Degrees of Freedom (DOFs) and is valid for the frequency range $\Psi := [0, 1000]$Hz following the rule of 6 elements per wavelength [16]. It contains an absorbing patch at the bottom side of the cavity, while all other walls are considered rigid, a monopole at the position $M_1 := [0.125, 0.05, 1.035]$ and two receivers $R_1 := [0.59, 0.52, 0.56]$, $R_2 := [0.78, 0.825, 1.08]$, which are located at the center and upper corner of the cavity, respectively. The acoustic admittance value y_1 of the patch is assumed to be described by the Delany-Bazley model [17]. The selected impedance function can be retrieved by assigning a static flow resistivity of $\sigma = 10000\text{Nsm}^{-4}$ and material thickness of $t = 1.4$cm. The response measured at the two receivers is given in Figure 2. These responses will be used as target for the inverse impedance identification performed based on section 3.4.

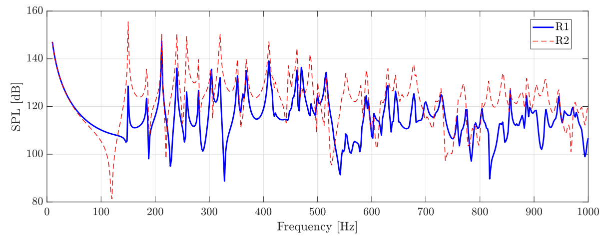


Figure 2: Measured response functions at R_1, R_2 for Delany-Bazley impedance on the porous layer.

4.2 Parametric MOR with Krylov recycling

As explained in section 3.4 the computational cost associated to such an inverse approach can be mitigated leveraging the technique elaborated in section 3.3. The parametric reduced order model constructed in this section is assumed to be valid for the frequency interval $\Psi := [0, 1000]$ and an impedance interval of the lining patch of $Z := [500, 2500] + [-10000, 0]j\text{Ns/m}^3$, where j is the imaginary unit. Employing the techniques introduced in section 3.3 a reduction basis is constructed aiming at an accuracy of $r_{\text{tol}} := 0.01$. The reduction basis consists of 530 basis vectors, implying that a reduced order model of the same size will be obtained upon a Galerkin projection of the full order system. To achieve a sufficiently accurate affine approximation of the system, 36 Chebyshev nodes are selected and thus, Chebyshev polynomials up to an equal order are used to ensure an accurate reconstruction for the largest distance of two geometry points, i.e. two opposite corners of the soundbox. For more information about the selection procedure, one is referred to [18].

Plotting the residual of the approximated solutions of equation (2) in Figure 3, it is possible to assess the quality of the parametric reduced order model for variations of the real part of the impedance patch. Observing that the global residual falls in general below the threshold of r_{tol}, it is concluded that any real impedance value within $\Re(Z)$ can be modeled by the constructed reduced order model,

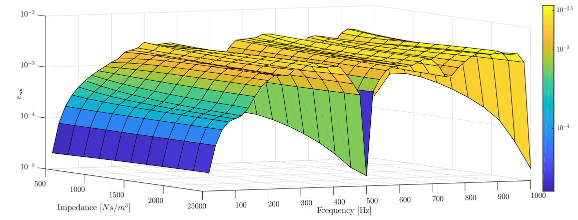


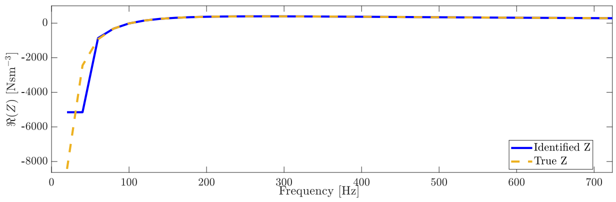
Figure 3: Residual of solutions for grid of impedance values for $\Re(Z) := [500, 2500]$Ns/m^3 using the reduced order model.

while similar conclusions can be made for the imaginary part. Thus, employing this reduced order model it is implied that calculation of the response using multiple different materials, including the one imposed in the virtual experiment can be efficiently approximated in the design phase or for inverse characterization of acoustic components. More information about the reduced order model can be found in [13].

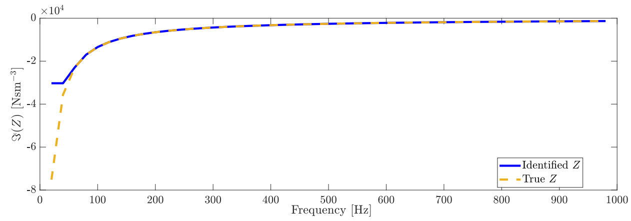
4.3 Inverse identification of patch impedance

Having built the above reduced order model, the acoustic response of the soundbox employing any impedance value lying within Z can be efficiently reconstructed. Deploying the procedure described within section 3.4, the impedance of the patch (see Figure 1) can be inversely identified as demonstrated in Figure 4. It is observed that the patch impedance is well reconstructed for all the frequencies above 50Hz both for its real and its imaginary part, while for higher frequencies, the optimization works well resulting in a practically negligible error or even to exact impedance value. The discrepancy at the lower frequencies can be explained by the reduced impact of the impedance to the acoustic response. As a result, the boundary condition is in close proximity to being fully rigid and thus, the optimization procedure can be trapped into potential local minima existing in that region.

It has to be noted that zero noise is considered in this inverse procedure, thus leading to the ideal response functions in Figure 2 and a very good reconstruction in Figures 4a and 4b. However, typical experimental procedures involve noise due to uncertainties in the model and experimental setup, in the material properties etc. Nevertheless, including noise would exceed the scope of this article, which is to provide an initial proof of concept for the inverse strategy.



(a) Real part



(b) Imaginary part

Figure 4: Comparison of identified impedance with reference

5. CONCLUSIONS

This paper proposes an inverse approach for the in-situ identification of the acoustic impedance of trim materials. The approach is based on a parametric model order reduction technique for constructing a reduced order model and an optimization scheme that retrieves the impedance value minimizing the difference between the measured and simulated impedance. To demonstrate the validity of the proposed approach, the challenging case of the soundbox interior model is examined. The measured response is collected using a virtual experiment, i.e., the response of the model with a known impedance function. The impedance function is well reconstructed, especially for the higher frequency range.

6. ACKNOWLEDGMENTS

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