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EMERGENCE OF QUASIPERIODIC SOUND REGIMES IN FLUTE-LIKE MUSICAL INSTRUMENTS : ON THE INFLUENCE OF THE BORE INHARMONICITY

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ABSTRACT

Self-sustained musical instruments are nonlinear dynamical systems that can produce a wealth of sound regimes. These can be periodic but also quasiperiodic (oscillating with a strong beating component), and their characteristics depend sensitively on both the design and control parameters. We investigate here the emergence of non periodic sound regimes in two types of flute-like instruments: a recorder and a pre-hispanic Chilean flute. The specific geometry of the Chilean flutes, whose resonator are made of two cylinders of similar length but different cross-sections, has been shown to favor a strong inharmonicity between the acoustic resonance frequencies. It has been demonstrated recently that both instruments can produce quasiperiodic sounds, which are either avoided or played on purpose. Here, we adopt a dynamical system point of view to investigate the emergence of these quasiperiodic sound regimes. A bifurcation analysis of a physical model of the instruments unveils the key role played by both a design parameter - the resonator inharmonicity - and a control parameter - the pressure in the musician's mouth - on the emergence of stable quasiperiodic sound regimes.

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1. INTRODUCTION

The family of flute-like musical instruments gathers a diversity of instruments, from transverse flutes to ocarina or pan-like flutes. These have been shown in the literature to display a complex dynamics [1–3]. Indeed, as nonlinear dynamical systems, flutes can produce equilibrium regimes (where no sound is produced while the musician blows in the instrument), a wealth of periodic regimes with different acoustical features (including different oscillation frequencies), but also non-periodic regimes. Because they are less commonly used than periodic regimes in a musical context, quasiperiodic regimes have been only sparsely studied [2, 4–6].

Here, we investigate the emergence of non periodic sounds in flute-like musical instruments in general, and in both a recorder and a traditional pre-hispanic flute from central Chile in particular. This is motivated by recent experimental results that showed that both these instruments can produce stable quasiperiodic sound regimes [7]. This is illustrated in Figure 1 which shows the time series of quasiperiodic sounds produced by both a recorder and a Chilean flute. In recorders, these are considered as a defect of the instruments, and both the instrument maker and the musician tend to avoid it. On the other hand, in the prehispanic Chilean flutes referred to as *flautas de*

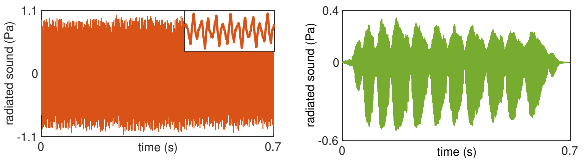

Figure 1. Quasiperiodic sound regimes produced by an alto recorder for a B fingering and a blowing pressure of 650 Pa (left) and by a Chilean flute for a blowing pressure of 1260 Pa. The inset shows an enlargement on the first 0.01 seconds of the time series.

Chinos [6], these quasiperiodic regimes featuring a strong beating component correspond to the desired sound [5].

In this article, the emergence of quasiperiodic sound regimes is investigated in a physical model of flute-like instruments. The influence of two parameters on the emergence of these sound regimes is studied : the blowing pressure which is the main control parameter for the musician on the one hand, and the inharmonicity of the resonator of the instrument on the other hand. Indeed, *flautas de Chinos* are made of two cylinders in series of similar length but with different cross-sections. This peculiar geometry has been shown to favor a strong inharmonicity [6]. Because non-periodic sound regimes are either wanted or avoided depending on the instrument and musical context, understanding how to favor or avoid them can be of particular interest for instrument making applications.

2. PHYSICAL MODEL OF FLUTE-LIKE INSTRUMENTS

Here, we consider the state-of-the-art physical model for flute-like instruments [8]. In all flute-like instruments, sound production results from the nonlinear interaction between an exciter constituted by a naturally unstable air jet (blown by the musician) oscillating around a sharp edge of the instrument called *labium* and an acoustical resonator made of the air column contained in the instrument.

The model itself is made of three coupled equations (the interested reader can refer to [8,9], for example, for a detailed description). A first equation describes the amplification and convection of perturbations (sustained by the acoustic field) along the naturally unstable air jet. More precisely, the transversal deflection η of the jet close to the labium is given by the following equation:

$$\eta(t) = \frac{h}{U_j} e^{\alpha_i W} v_{ac}(t-\tau). \tag{1}$$

Numerical values are given below in units of the international system. Here, $v_{ac}(t)$ is the acoustical velocity in the resonator close to the channel exit, $h = 10^{-3}$ is the height of the channel exit, U_j is the central velocity of the air jet, $\alpha_i = 0.4/h$ is the spatial amplification rate of perturbations along the jet and W is the length of the air jet, which is fixed here to 10^{-2} for the Chilean flute and to $4.25 \cdot 10^{-3}$ for the recorder. Equation (1) includes a delay term due to the convection time τ of perturbations along the jet, which is a key element of flute models. It is important to note that the value of τ is directly related to the blowing pressure which is the main control parameter for the musician.

Because of its unstable nature, the air jet oscillates around the so-called labium (a sharp edge of the instrument). This results in an alternate flow injection inside and outside the instrument. This is modelled as a dipolar source of pressure $\Delta p(t)$. The corresponding equation is written as follows:

$$\begin{aligned}\Delta p(t) = \frac{\rho b \delta_d U_j}{W} \frac{d}{dt} \left[tanh \left(\frac{\eta(t) - y_0}{b} \right) \right] \\ - \frac{\rho}{2\alpha_{vc}^2} v_{ac}(t) abs(v_{ac}(t)),\end{aligned} \tag{2}$$

with $\rho = 1.2$ the air density, $b = 2h/5$ the semi-half width of the air jet, $\delta_d = \frac{4}{\pi}\sqrt{2hW}$ the distance between the two flow sources, $\alpha_{vc} = 0.6$ a *vena contracta* factor [9] and y_0 the offset between the channel centerline and the labium, which is fixed here to $2 \cdot 10^{-4}$ for the Chilean flute and to 10^{-4} for the recorder.

The third equation describes, in the frequency domain, the acoustical response of the resonator through its input admittance $Y(\omega)$. Here, two different input amittances are considered, corresponding to the Chilean flute and to a recorder, respectively. The input admittance of the Chilean flute is measured experimentally, and the input admittance of the recorder is determined numerically from the pipe geometry of the recorder. In the model, these are approximated by a truncated sum of five resonance modes, as shown in Figure 2:

$$Y(\omega) = \frac{V_{ac}(\omega)}{\Delta P(\omega)} = \sum_{n=0}^{5} \frac{a_n j\omega}{\omega_n^2 - \omega^2 + j\omega\frac{\omega_n}{Q_n}}, \tag{3}$$

with a_n, ω_n and Q_n the modal amplitude, resonance angular frequency and quality factor of mode n, respectively.

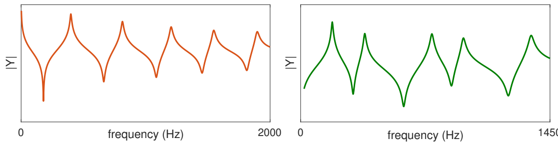

Figure 2. Modulus of the input admittance of a recorder (left) and of a Chilean flute (right), as used in the physical model of flute. The admittances are written as a sum of resonance modes.

It is worth noting that an additional mode is taken into account for the recorder to ensure convergence of the admittance at zero frequency [10]. This is related to the fact that the resonator of the recorder is open on both sides, while the one of the Chilean flute has one closed end. Overall, the model can be written in the time domain as a system of $2n$ (for the Chilean flute) or $2n+1$ (for the recorder) delay-differential equations of neutral type [11].

3. TIME-DOMAIN SIMULATIONS AND BIFURCATION ANALYSIS

The physical model of flute-like instruments is studied through time-domain simulations for two set of parameters corresponding to the two different instruments (recorder and Chilean flute). In both cases the model produces a quasiperiodic sound regime over a range of the blowing pressure, as illustrated in Figure 3. The acoustical features of these two regimes (*base* and modulation frequencies, depth of the amplitude modulation) are qualitatively different, in good agreement with the experimental observations on the real instruments [7].

The influence of the inharmonicity is investigated in a bifurcation analysis where both the time delay τ (directly related to the blowing pressure) and a global inharmonicity parameter are considered as bifurcation parameters. The global inharmonicity parameter Γ is defined to allow for a *morphing* (*i.e.* a continuous transition) from the experimentally measured input admittance of the Chilean flute as shown in Figure 2 (for $\Gamma = 1$) to the ideal case of a perfectly harmonic resonator (for $\Gamma = 0$) [12]. In more details, a parameter ξ_n defines the inharmonicity of each resonance mode of the Chilean flute:

$$\xi_n = \frac{\omega_n}{(2n-1)\omega_1} - 1. \tag{4}$$

From there, the resonance angular frequencies can be

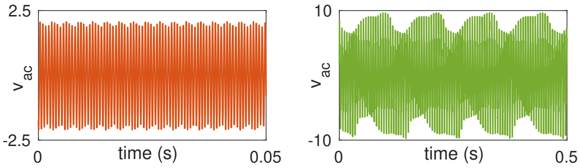

Figure 3. Time-domain simulations: dimensionless acoustical velocity with respect to time, obtained for the recorder parameters (left) and for the Chilean flute parameters (right), for fixed values of the dimensionless delay time τ corresponding to a blowing pressure of 4900 Pa for the recorder and 915 Pa for the Chilean flute.

changed in a continuous manner between the case of the Chilean flute and a perfectly harmonic case, by changing Γ:

$$\omega_{\Gamma_n} = (1 + \Gamma\xi_n)(2n-1)\omega_1. \tag{5}$$

The two-parameter bifurcation analysis is performed using DDE-Biftool [10, 13–15], a toolbox dedicated to numerical continuation for delay-differential equations. This allows to compute bifurcation curves that divide the parameter plane (τ, Γ) between regions corresponding to qualitatively different dynamics. A part of the obtained bifurcation set is shown in Figure 4, where the green region corresponds to the region in which a quasiperiodic regime exists. This shows that below a critical value of the global inharmonicity parameter $\Gamma = 0.58$, the quasiperiodic regime disappears. This demonstrates the crucial influence of inharmonicity of the resonator on the emergence of non periodic sound regimes.

4. CONCLUSION

We demonstrate here that a generic physical model of flute-like instruments is able to produce two kinds of quasiperiodic regimes produced by two different types of flutes - a recorder and a Chilean flute displaying a strong inharmonicity - in good agreement with the experimental observations. A bifurcation analysis performed using advanced numerical methods unveils the underlying mechanism for the emergence of quasiperiodic regimes in the physical model of flute-like instruments. This bifurcation analysis unveils the key role played by the inharmonicity of the resonator on the existence of quasiperiodic sounds, as well as on the range of blowing pressure on which they

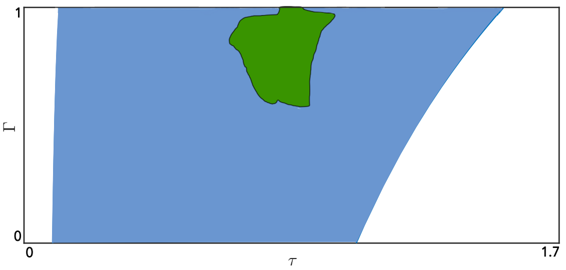

Figure 4. Bifurcation set of the flute model, in the plane (τ, Γ) of delay and inharmonicity. The blue region, bounded by curves of Hopf bifurcations, shows the region where the first register (periodic regime with an oscillation frequency close to the first resonance frequency) is stable. The green region, bounded by a torus bifurcation curve, shows the region of the parameter space in which a stable quasiperiodic regime is observed.

can be observed. The inharmonicity being directly related to the geometry of the instrument, these results suggest that the peculiar shape of the prehispanic Chilean flutes may have been selected to favor the emergence of non periodic sound regimes.

5. REFERENCES

[1] N. H. Fletcher, "Sound production by organ flue pipes," *The Journal of the Acoustical Society of America*, vol. 60, no. 4, pp. 926–936, 1976.

[2] J. W. Coltman, "Jet offset, harmonic content, and warble in the flute," *The Journal of the Acoustical Society of America*, vol. 120, no. 4, pp. 2312–2319, 2006.

[3] R. Auvray, B. Fabre, and P.-Y. Lagrée, "Regime change and oscillation thresholds in recorder-like instruments," *The Journal of the Acoustical Society of America*, vol. 131, no. 2, pp. 1574–1585, 2012.

[4] H. Wright and D. Campbell, "Analysis of the sound of chilean pifilca flutes," *The Galpin Society Journal*, vol. 51, pp. 51–63, 1998.

[5] J. P. De Arce, "Sonido rajado: The sacred sound of chilean pifilca flutes," *The Galpin Society Journal*, vol. 51, pp. 17–50, 1998.

[6] F. Blanc, P. De La Cuadra, B. Fabre, G. Castillo, and C. Vergez, "Acoustics of the *flautas de chinos*,," in *Proc. of 20th Int. Symposium on Music Acousitcs*, 2010.

[7] S. Terrien, C. Vergez, P. de La Cuadra, and B. Fabre, "Experimental analysis of non-periodic sound regimes in flute-like musical instruments," *The Journal of the Acoustical Society of America*, vol. 149, no. 3, pp. 2100–2108, 2021.

[8] B. Fabre and A. Hirschberg, "Physical modeling of flue instruments: A review of lumped models," *Acta Acustica united with Acustica*, vol. 86, no. 4, pp. 599–610, 2000.

[9] A. Chaigne and J. Kergomard, *Acoustics of musical instruments*. Springer, 2016.

[10] S. Terrien, C. Vergez, B. Fabre, and D. Barton, "Calculation of the steady-state oscillations of a flute model using the orthogonal collocation method," *Acta Acustica united with Acustica*, vol. 100, no. 4, pp. 690–704, 2014.

[11] S. Terrien, C. Vergez, and B. Fabre, "Flute-like musical instruments: a toy model investigated through numerical continuation," *Journal of sound and vibration*, vol. 332, no. 15, pp. 3833–3848, 2013.

[12] S. Terrien, B. Krauskopf, N. G. Broderick, V. A. Pammi, R. Braive, I. Sagnes, G. Beaudoin, K. Pantzas, and S. Barbay, "Merging and disconnecting resonance tongues in a pulsing excitable microlaser with delayed optical feedback," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 33, no. 2, p. 023142, 2023.

[13] K. Engelborghs, T. Luzyanina, G. Samaey, *et al.*, "DDE-Biftool: a matlab package for bifurcation analysis of delay differential equations," *TW Report*, vol. 305, no. 1, 2000.

[14] J. Sieber, K. Engelborghs, T. Luzyanina, G. Samaey, and D. Roose, "Dde-biftool manual-bifurcation analysis of delay differential equations," *arXiv preprint arXiv:1406.7144*, 2014.

[15] D. A. Barton, B. Krauskopf, and R. E. Wilson, "Collocation schemes for periodic solutions of neutral delay differential equations," *Journal of Difference Equations and Applications*, vol. 12, no. 11, pp. 1087–1101, 2006.