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PREDICTION OF RADIATED STRUCTURAL NOISE USING DECOMPOSED VIBRO-ACOUSTIC FREQUENCY RESPONSE FUNCTIONS

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ABSTRACT

The manner in which a system radiates noise is dependent upon the resonant behavior and the forced vibration of the structure. It has been shown previously that the modes of such a system can be identified by performing a singular value decomposition on the vibro-acoustic frequency response functions (FRFS) measured reciprocally between the structure and a remote response position. In this work, a volume velocity source is used to excite the structure, and reciprocally measured FRFs are used in conjunction with blocked forces, or pressures, to predict the radiated noise from the structure and the radiation modes are separated using a singular value decomposition (SVD). From the singular values, the contribution of each mode, or set of modes, can be applied to the blocked forces noise prediction. Specific singular values are extracted from the transfer functions between the remote excitation point and the structure, which are then rebuilt into transfer functions containing only the contribution of that singular value, which should be associated to a single mode if the surface of the structure is spatially sampled with a sufficient resolution. This single-singular value transfer function is then used to determine the contribution to the radiated noise from that individual mode.

Keywords: *Modal Analysis, Vibro-acoustics, Frequency Response Functions, Structural Analysis*

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1. INTRODUCTION

A volume velocity source is an acoustic source which creates a sound wave, with a known volume velocity, which is defined as the volume of air displaced by the sound wave per unit time [1][2]. The volume velocity source is used to measure the transfer functions for the radiation of the source to a receiver. This is done reciprocally, with the volume velocity source excitation at the receiver position and the responses to the excitation measured on the source structure. This transfer function is used in conjunction with the blocked forces of the system to make a prediction of the radiated noise of the system when it is excited by an operational source [3]. The blocked forces, when combined with the vibro-acoustic FRF measured using the volume velocity source, now translate to the blocked pressures on the surface of the system. The vibro-acoustic FRFs can be decomposed using an SVD, to separate the signal into contributions from specific singular values, which correspond to groups of radiation modes. In this paper, an experiment is detailed in which an aluminium plate, excited by a shaker, is measured in an anechoic chamber. The vibro-acoustic FRFs of the plate are measured directly, by using an impact hammer to excite at positions on the plate and the response measured at a number of microphones, and reciprocally, using a volume velocity source to make excitations at the microphone positions with the responses measured on the surface of the plate. The FRFs of the volume velocity source are then decomposed using SVD and transfer functions containing only one singular value are reconstructed and used in the blocked forces equation to make predictions of the contribution to the overall radiated noise attributed to the modal frequencies described by the chosen singular value.

2. THEORY

Using the blocked forces approach, a plate ‘B’ with shaker source ‘A’ can be considered a coupled assembly ‘C’. predictions are made for remote response ‘d’.

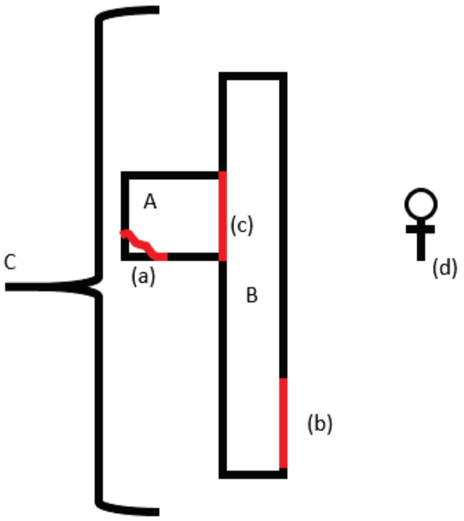

Figure 1: Schematic of components showing a plate (B) with a vibration source (A) and a remote response position (d)

A matrix of frequency response functions is constructed, containing FRFs of acceleration due to force at each accelerometer. The Source ‘A’ is then made operational, and the blocked forces of the plate due to excitation from the source A can be described as:

$$\bar{\dot{f}}_{A,c} = Y^{-1}_{C,cc}\,\dot{v}_{C,c} \tag{1}$$

Where $\bar{\dot{F}}_{A,c}$ is the blocked forces, $Y^{-1}_{C,cc}$ is the inverted mobility FRF matrix of the coupled assembly ‘C’, $\dot{v}_{C,c}$ is a vector of responses on the plate due to excitation from the vibration source ‘A’ and:

$$\bar{\dot{f}}_{A,c} = -\dot{f}_{A,c}\big|_{\dot{v}_{A,c}=0} \tag{2}$$

And the prediction of radiated noise is given by:

$$\dot{P}_{C,d} = H_{C,db}\bar{\dot{F}}_{A,c} \tag{3}$$

Where $\dot{P}_{C,d}$ is the predicted pressure at the response microphone ‘d’ due to radiated noise from the coupled assembly ‘C’, and $H_{C,db}$ is the vibro-acoustic transfer function between the accelerometer positions and the response microphone. This transfer function is measured both reciprocally and directly. The direct transfer function is measured using an impact at each accelerometer position on the plate surface, with the response measured at the remote microphone positions.

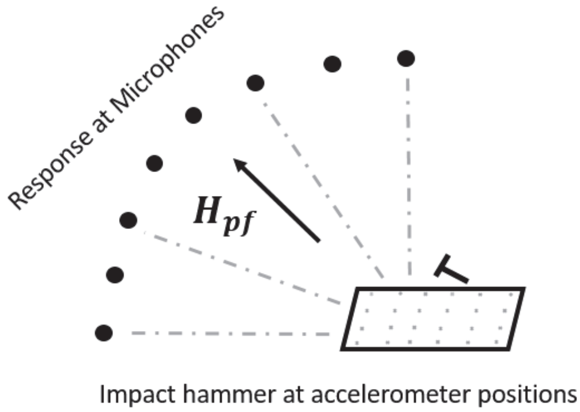

Figure 2: Illustration depicting the measurement of the direct vibro-acoustic transfer function using a force hammer

Which can be rendered as:

$$\dot{P}_{C,d} = H_{pf}\bar{\dot{F}}_{A,c} \tag{4}$$

The reciprocally measured version of the transfer function is measured using the volume velocity source, with excitations at the microphone positions and responses measured at the accelerometers on the plate. A reference microphone is used to calculate the transfer function. The equation for the transfer function is given by

$$H_{aq} = \frac{a}{q_1} = \frac{a}{P_2} \times \frac{j\rho_0 e^{j(\omega t - kr)}}{2\lambda r} \tag{5}$$

$$H_{vq} = \frac{H_{aq}}{j\omega} \tag{6}$$

Where $\frac{a}{q_1}$ is the transfer function between the accelerometers and the volume velocity source, P_2 is the sound pressure at the reference microphone, $\frac{a}{P_2}$ is a transfer function between the accelerometers and the pressure at the reference microphone, ρ_0 = air density, e = 2.71, ω = 2 * π * frequency, t = time = 1, k = wavenumber, r is the distance between the volume velocity source and the reference microphone, and λ = wavelength [4].

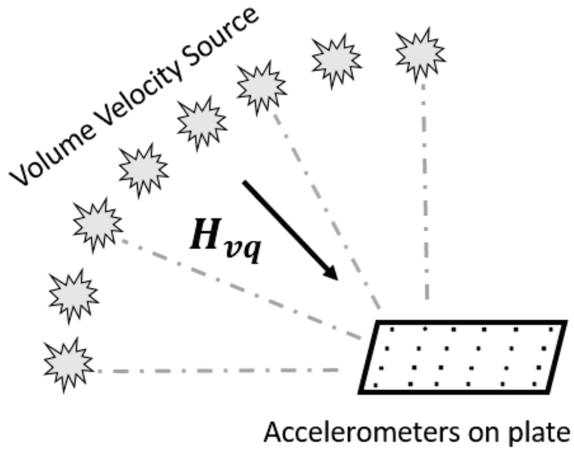

Figure 3: Illustration depicting the measurement of the reciprocal vibro-acoustic transfer functions using a volume velocity source.

This can then be rendered as:

$$\dot{P}_{C,d} = H_{vq}\bar{\dot{F}}_{A,c} \tag{7}$$

The decomposition of $\boldsymbol{H_{vq}}$ is conducted using an SVD in terms of location versus frequency, giving:

$$\boldsymbol{H_{vq}} = [\boldsymbol{\Psi}]\ [\boldsymbol{\Sigma}]\ [\boldsymbol{\Phi}]^T \tag{8}$$

Where $\boldsymbol{H_{vq}}$ is *n*x*m,* Ψ is the *m*x*m* matrix containing elements ψ which portray the mode shapes; Σ is the *m*x*n* diagonal matrix containing the singular values $\boldsymbol{\sigma}$ on the diagonal, and Φ^T is *n*x*n* and contains the frequency response of the modes $\boldsymbol{\phi}$.

By discarding all but a single singular value of $\boldsymbol{\sigma}$, a transfer function can be constructed which contains the contribution of only single value *n*:

$$\Xi_i = \boldsymbol{\Psi}\boldsymbol{\sigma}_n\boldsymbol{\Phi} \tag{9}$$

And a prediction of the contribution of that single singular value to the overall radiated noise can be made by multiplying this single singular value transfer function Ξ_i by the blocked forces of the plate:

$$P_{\Xi i} = \Xi_i \bar{\dot{F}}_{A,c} \tag{10}$$

3. METHODOLOGY

An experiment was conducted in an anechoic chamber using an aluminium plate of dimension 0.9m x 0.7m x 0.025m. The surface of the plate was discretized into a 4 x 6 grid of 24 accelerometers, and a small vibrating shaker was mounted to the rear of the plate. The shaker was excited using white noise, with a reference taken from the input voltage and a force transducer mounted between the shaker and the plate. The shaker was enclosed by a plywood baffle to reduce the direct transmission of the shaker to the receiver microphones, which were mounted on an arc to measure the responses of the plate between 0° and 90°. An instrumented force hammer was used to measure the FRFs of the plate.

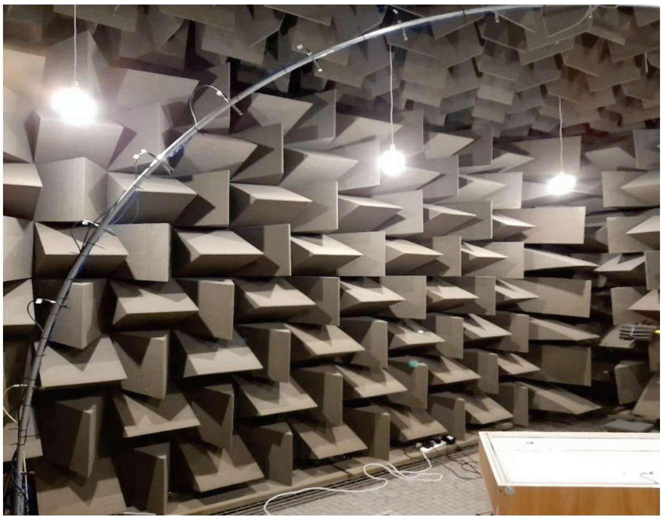

Figure 4: Microphone arc and plate in the anechoic chamber

The 0° microphone is located at the apex of the arc, pointing directly downwards at the surface of the plate. The 90° microphone is located towards the bottom of the arc, pointing directly at the side of the plate. The volume velocity source was activated at each of the microphone positions. These reciprocal vibro-acoustic FRFs are compared to the directly measured vibro-acoustic FRFs created using the hammer. The two types of FRF are used to make standard blocked forces radiated noise predictions, before the SVD is conducted to decompose the reciprocal FRFs into single singular value contributions for predictions of the specific singular value contribution to the overall radiated noise.

4. RESULTS

4.1 Comparison of H_{vq} and H_{pf}

For validation purposes, the two vibro-acoustic transfer functions are compared to determine their similarity. For the microphone at positions 0°, 30°, 60°, and 90°, and accelerometer number 14 located on the plate, the measured transfer functions are shown in dB:

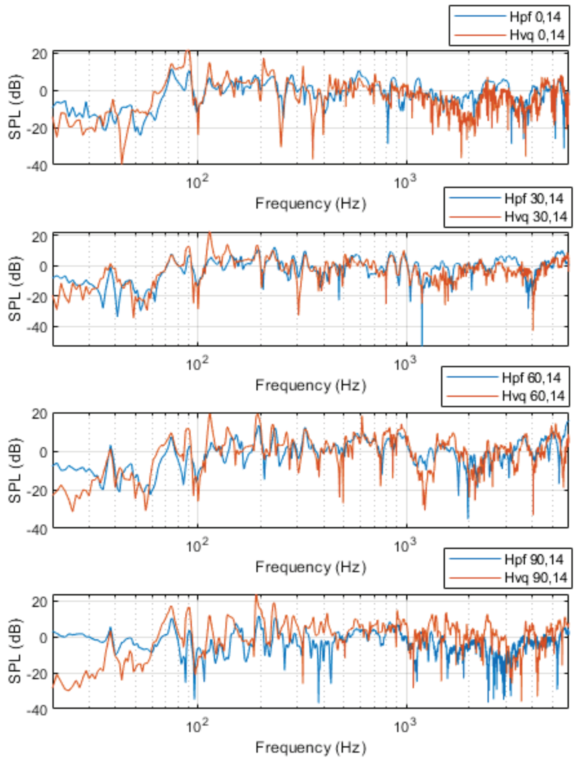

Figure 5: Comparison of H_{pf} and H_{vq} at accelerometer 14 on the plate, and microphones at positions 0°, 30°, 60°, and 90°.

4.2 Blocked Forces Predictions

The two vibro-acoustic transfer functions are used to make radiated noise predictions using the blocked forces, as described by Equations 4 and 7. Multiple measurement positions were used, the results for the predictions of radiated pressure in dB SPL are presented for the 0° microphone located perpendicular to the plate (above, at the apex of the arc), and the microphone located at 30° from the apex.

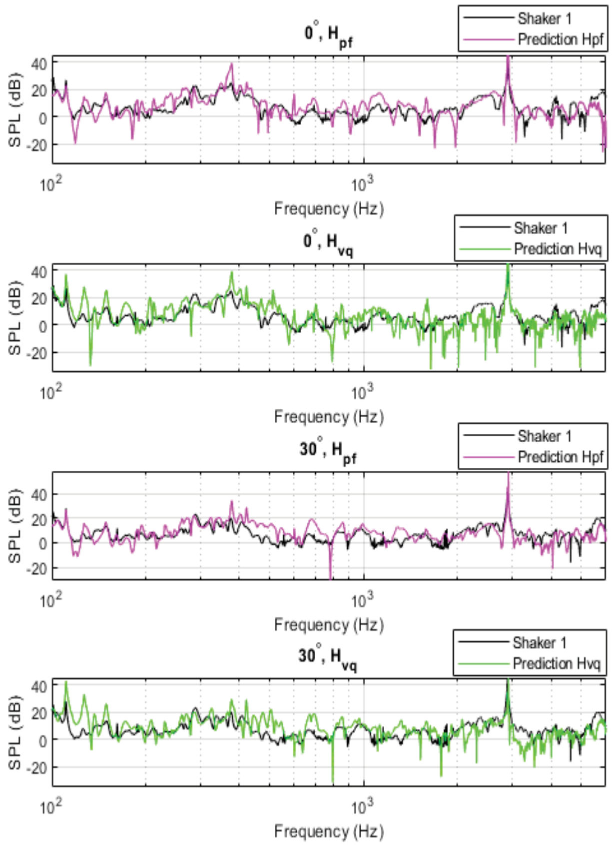

Figure 6: Prediction of radiated noise at response microphone at position of 0° and 30°. In black, the measured shaker response transfer function, in pink, the prediction using H_{pf} and in green the prediction calculated using H_{vq}. The x-axis is frequency between 100Hz and 6kHz. The y-axis is dB SPL between -20 and 50dBs.

Figure 6 depicts the prediction of radiated noise due to the excitation of the plate by a shaker at response microphones in positions of 0° and 30° relative to the plate. The black traces are the measured shaker response. The uppermost frame depicts the prediction for the 0° microphone calculated using the directly measured H_{pf} transfer function, and the second frame depicts the prediction for the same microphone position using the reciprocal transfer function H_{vq}. Frame 3 depicts the prediction of radiated noise of the plate at the microphone located at 30° using the direct H_{pf} transfer function in pink, and frame 4 depicts the same calculation using the reciprocal H_{vq} transfer function. Overall, there is relatively close agreement for both of the predictions when compared to the measured shaker response. There are errors, reaching as wide as 20 dB difference. However, the predicted frequency response is overall relatively close, with the large resonance at around 3 kHz described by both of the predictions.

4.3 Single Singular Value Transfer Functions Ξ_i

4.3.1 Plots of Σ for 0° and 30° microphones

The singular values, which represent the square root of the eigenvalues of the covariance matrix of the FRFs, are presented here for the vibro-acoustic FRFs between the microphones at 0° and 30° and the plate.

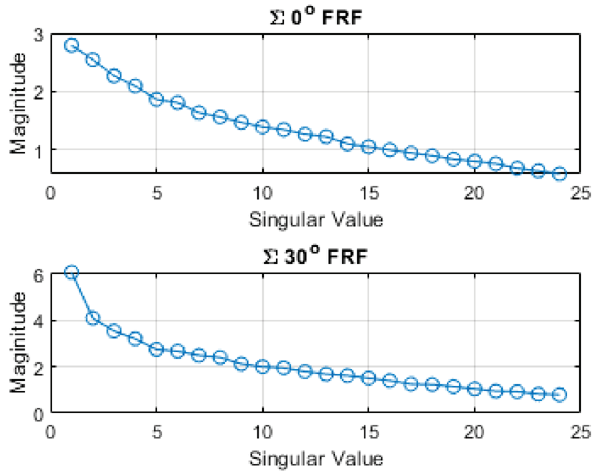

Figure 7: Diagonal of Σ for 0° microphone FRF and 30° microphone FRF.

If the surface is substantially sampled, there will be a knee in the curve of singular values, with some singular values of a substantially higher magnitude than the others. These singular values would indicate the significant modes of the system. This knee is not present in the singular values shown in Figure 7, indicating that the surface is not sufficiently sampled for the description of modes using the SVD.

4.3.2 Transfer functions Ξ_i

FRFs are constructed for each of the 24 singular values using equation 9. These are transfer functions scaled to each singular value, and if the surface was adequately spatially sampled, would contain the contribution of 1 or a small number of modes. As the plate in this experiment is under-sampled, the transfer functions contain the contribution for a number of modes.

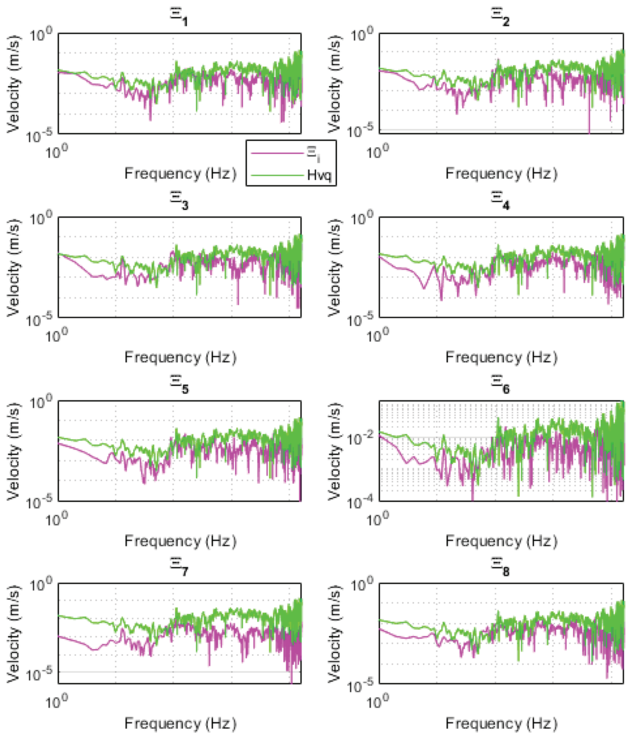

Figure 8: Ξ_1 to Ξ_8 transfer functions, shown in pink, and original undecomposed transfer function H_{vq} shown in green.

Figure 8 shows the transfer functions decomposed in terms of singular values, Ξ_i where i is the number of singular value, in pink. These are compared to the undecomposed transfer function H_{vq}. There is little differentiation apparent for the first 8 transfer functions. If the surface of the plate had been adequately sampled spatially, it would be expected that Ξ_1 would have a peak which corresponds to the highest peak in frequency in H_{vq}, with the rest of the frequencies at a lower magnitude than H_{vq}. Ξ_2 would correspond to the 2nd peak in the spectrum, and so on, until the singular values no longer describe modes and Ξ becomes of much lower magnitude than H_{vq}.

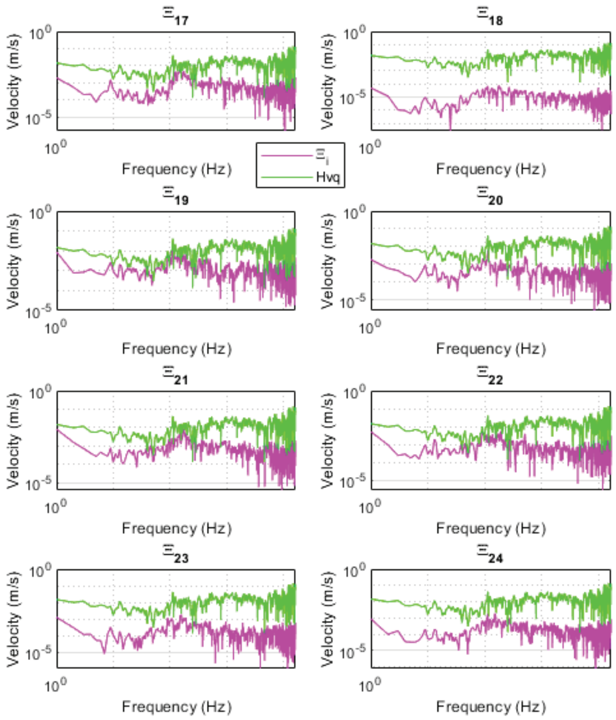

Figure 9: Ξ_{17} to Ξ_{24} transfer functions, shown in pink, and original undecomposed transfer function H_{vq} shown in green.

Shown in Figure 10 are the decomposed transfer functions for singular values σ 17 to 24. Interestingly, Ξ_{18} is of the lowest magnitude. It would be expected that Ξ_{24} would have the lowest magnitude, as the higher number singular values are the smallest. When compared to Figure 9, the decomposed transfer functions shown in Figure 10 indicate that the singular values are less significant as $i \rightarrow 24$.

4.3.3 Radiated noise prediction of contribution from Ξ_i

The decomposed transfer functions are used to make blocked forces predictions using Equation 10. The responses are predicted for the microphone at 30° to the plate.

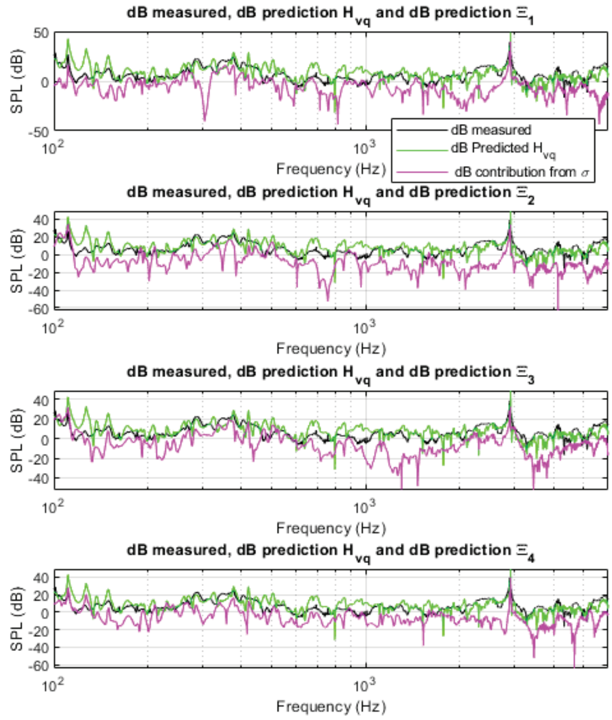

Figure 10: Prediction of radiated noise using transfer functions containing the contribution of single singular values 1 to 4, shown in pink, and prediction to radiated noise using undecomposed transfer function H_{vq} shown in green. Shaker response shown in black.

Figure 11 shows the predictions made using transfer functions containing only a single singular value for the first four singular values. The measured shaker response is shown in black, the blocked forces calculation using the original H_{vq} is shown in green, and the calculation of radiated noise contribution from the specific singular values is shown in pink. There is not a significant difference between the four predictions, which is to be expected for the case of an under-sampled surface. I the case of sufficient sampling resolution, the results would most likely show the dominant resonance predicted using the first singular value, i.e. the peak in the spectrum at 3 kHz, with little detail in the prediction across the rest of the frequency spectrum. The second prediction would likely show correspondence to the next highest peak in the spectrum, i.e. around 400 Hz.

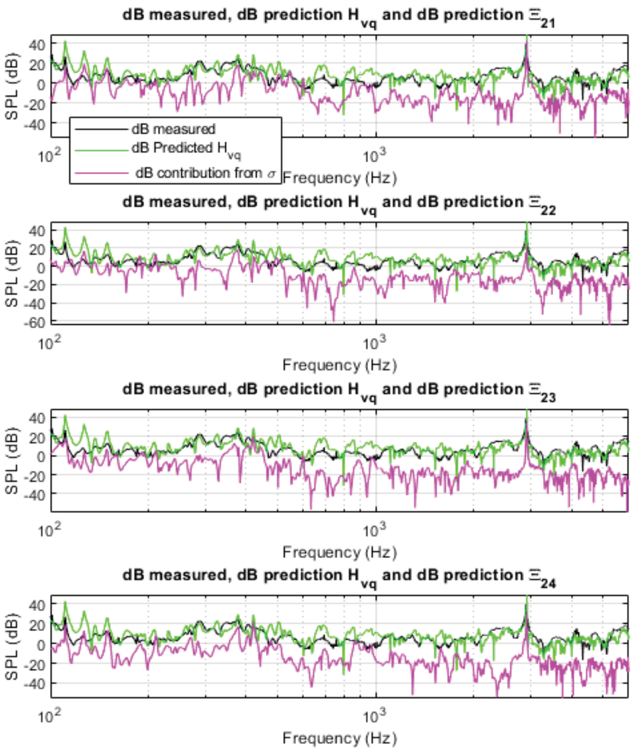

Figure 11: Prediction of radiated noise using transfer functions containing the contribution of single singular values 21 to 24, shown in pink, and prediction to radiated noise using undecomposed transfer function H_{vq} shown in green. Shaker response shown in black.

Shown in figure 12 are the predictions of radiated noise using decomposed transfer functions Ξ calculated using the 21st, 22nd, 23rd, and 24th elements of σ, which are the four smallest singular values, shown in pink. The measured shaker response at the 30° microphone position are shown in black, and the blocked forces prediction using H_{vq} is shown in green. It appears that the resonant peak at 3 kHz is described by every singular value, with considerable detail in each of the blocked forces predictions using the decomposed transfer functions. In a case where the plate had been sufficiently sampled, it would be assumed that the predictions made using the least significant singular values would not correspond as closely to the non-decomposed transfer function prediction made using H_{vq}. There are differences observable in terms of magnitude of the predictions made using Ξ_1 - Ξ_4, and Ξ_{21} - Ξ_{24}. The predictions for the 4 smallest singular values Ξ_{21} - Ξ_{24} correspond less to the measured noise and the blocked forces prediction made using H_{vq}.

5. DISCUSSION

The prediction of radiated noise using blocked forces using both the direct H_{pf} and reciprocal H_{vq} transfer functions yield similar results, which was pre-empted by the similarity between the transfer functions shown in Figure 5.

According to Ewins, the spatial sampling requirement for accurate description of a mode is given to be 5 to 10 points per wavelength, and to capture the complete spatial distribution of a mode, the distance between measurement points should be no greater than one-tenth of the wavelength of the mode [5]. The first mode for the baffled plate described is calculated as approximately 64 Hz. Assuming a sound speed of 6320 m/s in aluminum, the wavelength of a 64 Hz mode is approximately 98.75 mm. Therefore, the required sampling resolution is 6.42 mm / 98.75 mm = 0.065 or 6.5% of a wavelength. For the modes to be accurately described by the left singular matrix (Ψ), the sampling interval must be at least ½ λ. The measured plate does not meet these criteria for even the lowest mode, which indicates that there are multiple modes described by each σ, and therefore each Ξ. The decomposed Ξ transfer functions contain the contribution of the modes described by the single singular value σ, however with a sufficient sampling resolution, the transfer functions could describe the contribution of an individual mode.

6. CONCLUSIONS

The volume velocity source has been shown to be an effective alternative for reciprocally measuring vibro-acoustic FRFs instead of measuring the direct transfer functions with excitations on the surface of the plate. The volume velocity source offers advantages in terms ease of use and efficiency, as an alternative to using a force hammer for these excitations. In practice, the volume velocity source uses an excitation at the remote response position with the responses measured on the source. In the case of a heavily discretized surface, this allows the operator to measure data for the discretized positions of the surface with a single excitation at the remote response position, rather than exciting the surface at all measurement positions.

As the sampling intervals of the accelerometers discretizing the surface of the plate were insufficient to properly describe the modes of the system, the individual modes were not described by the singular values resulting from the SVD, meaning that the Ξ blocked forces calculation was unable to separate out the contributions from individual modes. Using a heavily discretized plate would allow further exploration of the method. This could improve the method by better sampling of the modes in terms of how many nodes of each mode are included. Variation of plate material would also render different results, due to different wavespeeds.

7. REFERENCES

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