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ON THE ROBUSTNESS OF INTERFACE TOPOLOGICAL MODES ON RODS

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ABSTRACT

Periodic structures present wave phenomena that can be used as wave attenuation in certain frequency bands. Usually, the periodic structures are divided into metamaterials (locally resonant structures) and phononic crystals. More recently, topological metamaterials have emerged as a possible application to overcome conventional metamaterials and phononic crystals. These structures can support interface states by combining two periodic structures with distinct topological invariants. The wave propagation path (or localization) is robust against variability and defects, which is the so-called topological protection. In the current study, we investigate the robustness of interface topological modes in a rod structure and show that this robustness has different effects on the mode shapes and the natural frequency of the topological and defect modes.

Keywords: *Topological metamaterial, Variability, Robustness, Vibration localization*

1. INTRODUCTION

The use of topological invariants to group physical objects with substantial similarities in groups that differ from the other groups began in 1980's. The topological invariant is the parameter used to classify the objects inside the groups. These mathematical tools applied to physics made possible the development of superconductors and powerful insulators, as well as the realization of topological phenomena in classical mechanical systems leading to more efficient energy harvesters [1–3]. These topological phenomena are considered robust against variability and smooth defects, i.e., the defect or variability needs to be severe enough to close and reopen the band gap, making the transition to a different topological invariant. Thus, the objective of the present study is to verify the robustness of topological modes, namely the interface edge modes.

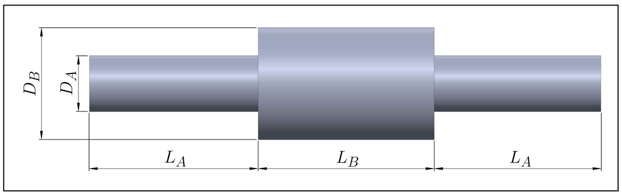

Figure 1: Unit cell made of two segments of length L_A and diameter D_A and a segment with length L_B and diameter D_B in the center. The structure is made of nylon with mass density (ρ) of 1,200 kg/m^3 and Young's modulus (E) of 4 GPa.

2. THE DETERMINISTIC MODELS

The spectral element method (SEM) and the finite element method (FEM) [4, 5] are used in the current investigation for modeling the periodic rod structure. FEM allows the

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computation of the natural frequencies and mode shapes after checking mesh refinement convergence [6]. On the other hand, the dispersion diagrams and the topological invariant, namely the Zak phase, are computed using the transfer matrix method together with the SEM model [7]. The unit cell of the rod phononic crystal is shown in Fig. 1 and consists of a thicker segment in the center, of length L_B and diameter D_B, with two symmetrically placed segments at the edges, of length LA and diameter D_A. The parameter $\Delta_L = \frac{L_A - L_B}{2}$, with $-L/2 \leq \Delta_L \leq L/2$ defines a topological transition point where a band gap is closed on the reciprocal space [3, 8].

The proposed structure is first assumed without geometry and mechanical properties variability. FEM was used to compute the natural frequencies and mode shapes and SEM was used to compute the dispersion diagrams and the wave modes that were used for the Zak phase computation.

The Zak phase is computed using the wave modes in a discrete way in frequency and space. The wave modes can be defined as u_{n,k_j,x_t} associated with the n$^{\text{th}}$ pass band and j$^{\text{th}}$ wavenumber and at spatial position x_t [9]. Thus, the Zak phase can be computed with [8]

$$\Theta_n^{\text{Zak}} = -\text{Im}\left\{\sum_{j=1}^{N} \ln\left[\sum_{t=1}^{S} u^*_{n,k_j,x_t} u_{n,k_{j+1},x_t} \Delta x\right]\right\}, \tag{1}$$

where N is the discretization size in the IBZ (wavenumber domain) and S is the discretization size in the unit cell (spatial domain).

Fig. 2 can be obtained by plotting the absolute value of the imaginary part of the wavenumber for different values of Δ_L. The red and green lines indicate where the Zak phase equals π or 0 respectively for the passbands above the colored line.

Assuming $L = 1$, $D_A = 10$ and $D_B = 20$, the forced response in Fig. 3 is obtained from excitation and measurement at the center (interface) of a metastructure consisting of 10 unit cells with $\Delta_L = -0.41$ and 10 unit cells with $\Delta_L = 0.41$. It is possible to notice a topological mode in the second band gap as it was designed to be. The shape of this mode with a natural frequency of 1,799.70 Hz is shown in Fig. 4.

In order to create a metastructure with a defect mode, an increase of 78% in the diameter of the segment (33.33 mm) at the center of the metastructure, which consists of 20 unit cells with $\Delta_L = 0.41$, was made. As can be seen in the forced response, shown in Fig. 5, a defect mode shows up at 1,933.4 Hz. The corresponding mode shape caused by the defect is visualized in Fig. 6.

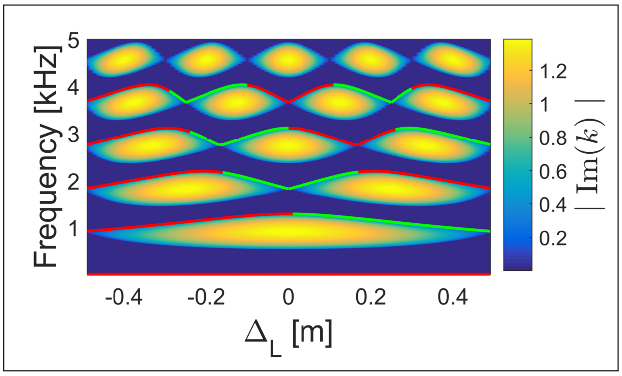

Figure 2: Imaginary part of the wavenumber over the frequency range and different values of the geometrical parameter Δ_L (length difference). Red and green lines indicate that the pass band over the line has a Zak phase of π and 0, respectively.

3. STOCHASTIC MODELS

Next, the mechanical properties ρ and E were assumed as stochastic fields, as done in [10], using a correlation length equal half of the unit cell length. The stochastic field is assumed as a Gaussian process with a mean equal to the assumed deterministic values and a correlation of variation equal to 2. A Monte Carlo method with 300 samples was used for the stochastic analyses.

When analyzing the variations in the natural frequency and mode shape of the proposed topological mode under the proposed variability, some metrics need to be defined. The coefficient of variation and the correlation coefficient are used as metrics of variation for the wave mode and natural frequency, respectively. The correlation coefficient values were 99.97% and 98.14% respectively. Some samples of the stochastic mode shapes are shown in Fig. 8 and Fig. 10 for the stochastic topological mode and defect mode, respectively. However, when analyzing the coefficient of variation of the stochastic natural frequency - six samples of the forced responses are shown Fig. 7 and Fig. 9 -, the stochastic defect mode with a coefficient of variation of 0.0053 and standard deviation of 11.53 is more robust than the topological mode with a coefficient of variation of 0.0077 and standard deviation of 13.53.

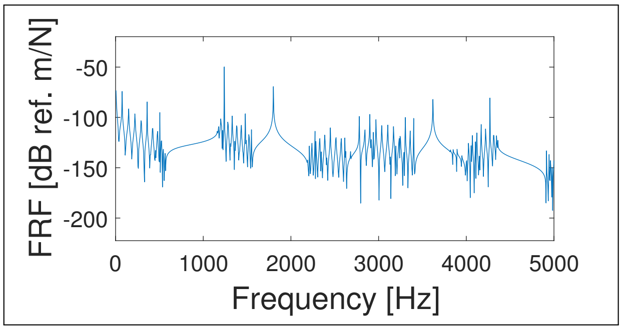

Figure 3: FRF with excitation and measurement at the center of a metastructure, at the interface of two crystals, one consisting of 10 unit cells with $\Delta_L = -0.41$ and the other consisting of 10 unit cells with $\Delta_L = 0.41$.

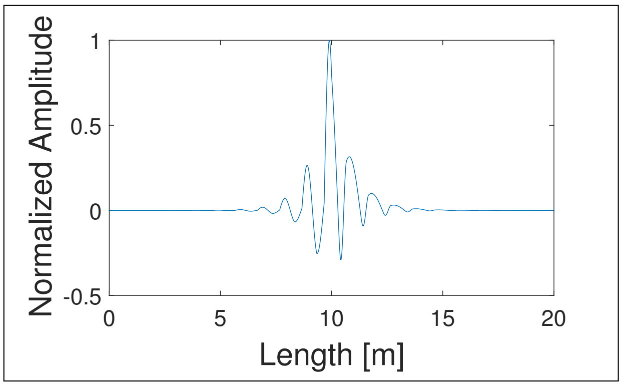

Figure 4: Mode shape corresponding to the peak in Fig. 3 at 1,799.70 Hz, the natural frequency of the topological mode.

4. CONCLUDING REMARKS

In the current research, using the Zak phase as a tool, we designed and simulated two metastructures, one having a topological mode and another having a defect mode. Those two modes have close frequencies and the metastructures were designed to be similar, except for the presence of the topological or defect modes. Under a small variability of the mechanical properties, the topological mode is more robust when considering the mode shape whereas the defect mode is more robust when considering the natural frequency of the mode.

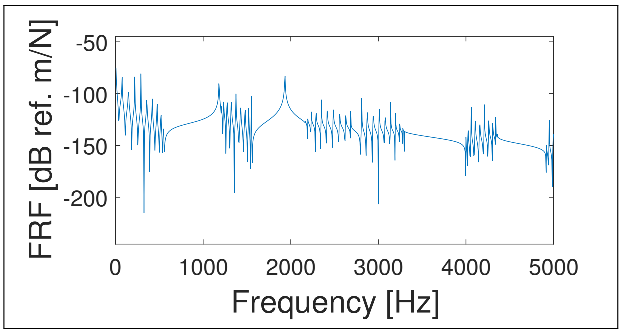

Figure 5: Forced response for excitation and measurement at the center of a metastructure with a defect made of 20 unit cells with $\Delta_L = 0.41$.

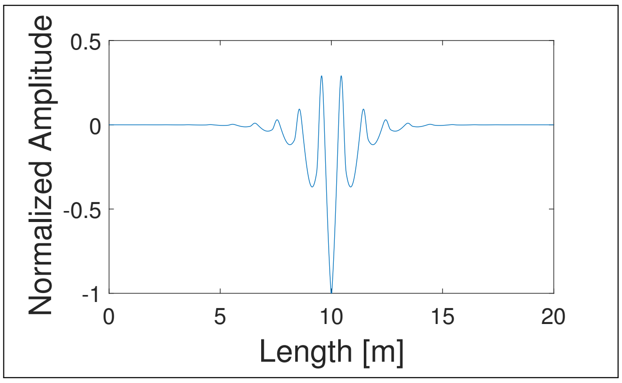

Figure 6: Mode shape corresponding to the peak at 1.933.4 Hz in Fig. 5 (defect mode).

5. ACKNOWLEDGMENTS

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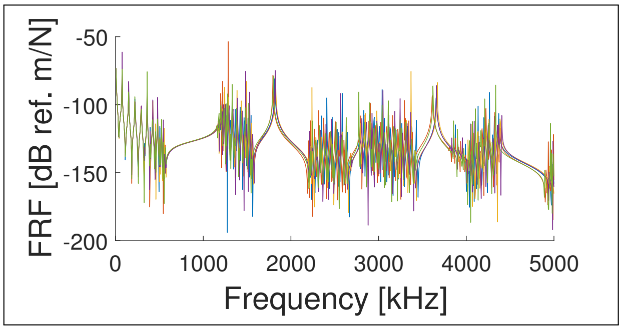

Figure 7: Six samples of the stochastic forced response for the metastructure with topological mode, with a coefficient of variation of 0.0077 and a standard deviation of 13.96 Hz.

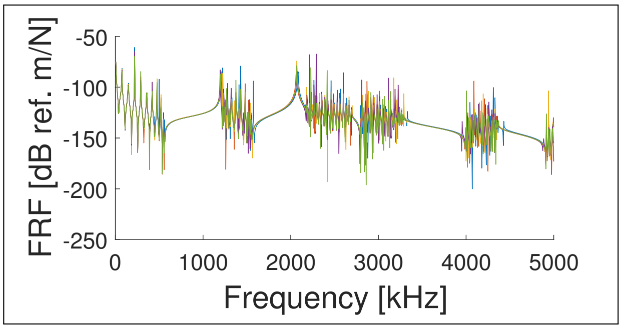

Figure 9: Six samples of the stochastic forced response for the structure with defect mode, with a coefficient of variation of 0.0058 and a standard deviation of 11.53 Hz.

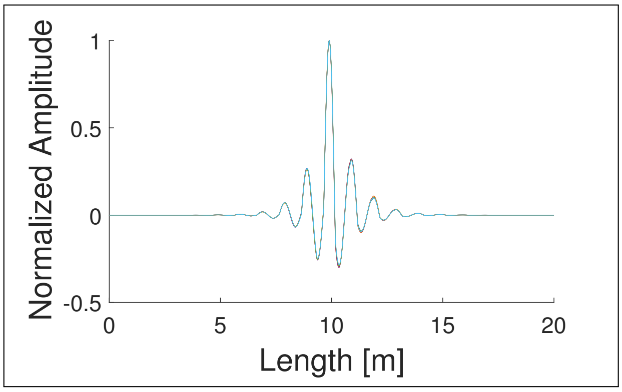

Figure 8: Six samples of the simulated stochastic topological mode shape. The correlation coefficient between them is 99.97%.

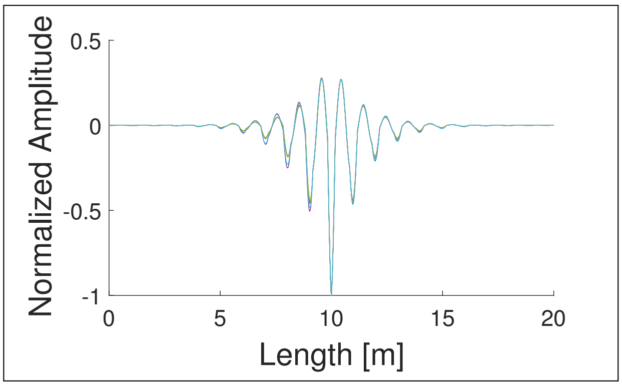

Figure 10: Six samples of the mode shapes of the simulated stochastic defect mode. The correlation coefficient between them is 98.14%.

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