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A STATISTICAL INVERSE METHOD FOR THE RECONSTRUCTION OF ROUGH SURFACES FROM ACOUSTIC SCATTERING

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ABSTRACT

A model inversion framework is proposed for the recovery of the depth profile of a rough surface. A broadband sound source is placed above the surface of interest and the scattered sound pressure is measured at a microphone array. The problem is modelled analytically using the Kirchhoff approximation, which provides a computationally efficient forward model, with reasonable accuracy in the far field. The inverse problem is formulated in a statistical sense within the Bayesian framework and sampled using a Markov chain Monte Carlo algorithm. In order to shorten the burn-in sampling phase, an initial solution obtained by deterministic optimisation is used. Special attention is devoted to modelling the smoothness of the surface using a prior probability distribution. The procedure is demonstrated experimentally on a surface with one-dimensional roughness.

Keywords: *Inverse scattering, surface reconstruction, Bayesian inference*

1. SCATTERING MODEL

The goal of the present work is to estimate the roughness profile of a surface indirectly from scattered acoustic pressure acquired at a microphone array. A schematic of the experimental setup is given in Fig. 1. The surface is insonified by means of a directive source, which ensures that the microphone array is outside of its direct field. Thus, the sound pressure at a microphone located at a position $\mathbf{r}$ is solely given by the scattered field from the surface.

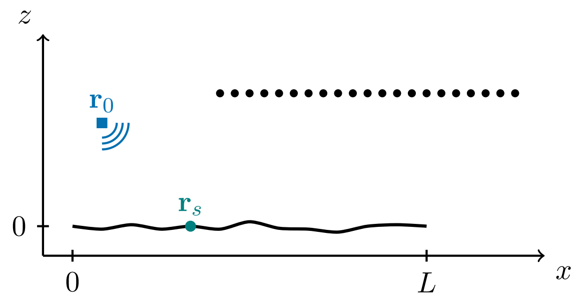

Figure 1. Scattering problem. — Rough surface of length L; ■ source; • microphones; • a point on the surface.

In this work, the Kirchhoff approximation [7] is used, yielding an explicit expression of the sound pressure p at a microphone position $\mathbf{r}$ as a function of the source position, wavenumber, and surface elevation profile $\zeta(x)$ [1, 2, 4],

$$p(\mathbf{r}, \mathbf{k}, \zeta) \simeq e^{i\pi/4}\sqrt{\frac{2\pi}{k}}\int_{x_s=0}^{x_s=L}\sqrt{\frac{|\mathbf{r}_s-\mathbf{r}_0||\mathbf{r}-\mathbf{r}_s|}{|\mathbf{r}_s-\mathbf{r}_0|+|\mathbf{r}-\mathbf{r}_s|}}$$
$$G(\mathbf{r}, \mathbf{r}_s)G(\mathbf{r}_s, \mathbf{r}_0)\left(\kappa(\mathbf{r}-\mathbf{r}_s)-\kappa(\mathbf{r}_s-\mathbf{r}_0)\right)dx_s \quad (1)$$

where k is the wavenumber in air, G is the Green function in free field, $\mathbf{r}_0$ denotes the source position, $\mathbf{r}_s = (x_s, \zeta)$ is a point along the surface, and $\kappa(\mathbf{r}) = (ik - 1/|\mathbf{r}|)\cos(|\mathbf{r}|, \mathbf{n}_s)$, with $\mathbf{n}_s$ the outgoing normal to the surface at $\mathbf{r}_s$. The surface can be discretised along x, yielding a finite number of unknown elevation points, as $\mathbf{z} = \begin{bmatrix}\zeta(x_1) & \zeta(x_2) & \cdots & \zeta(x_s) & \cdots & \zeta(x_N)\end{bmatrix}^{\mathrm{T}}$.

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2. INVERSE METHOD

The measured sound pressure $p^{\text{(meas)}}$ can be written as

$$p^{\text{(meas)}} = p(\mathbf{z}) + \varepsilon, \tag{2}$$

where $p(\mathbf{z})$ is the model as a function of the surface elevation, and ε is a modelling and measurement error, assumed zero-mean Gaussian with standard deviation σ_ε. The extent of the error is not known a priori, and therefore σ_ε is treated as an unknown.

The inverse problem is here formulated in a statistical sense within the Bayesian framework [5]. This yields the posterior probability density of the unknowns given a measurement p^{meas},

$$P(\mathbf{z}, \sigma_\varepsilon, \sigma_{\text{pr}} | p^{\text{(meas)}}) \propto P(p^{\text{(meas)}} | \mathbf{z}, \sigma_\varepsilon) P(\mathbf{z} | \sigma_{\text{pr}}), \tag{3}$$

where $P(p^{\text{(meas)}} | \mathbf{z}, \sigma_\varepsilon)$ is the likelihood of an experiment outcome for a given surface and $P(\mathbf{z}|\sigma_{\text{pr}})$ is the prior knowledge on the surface elevation profile. The degree of smoothness of the surface is incorporated by writing $P(\mathbf{z}|\sigma_{\text{pr}})$ as a Gaussian smoothness prior [5], which introduces a correlation between nearby points on the surface and where σ_{pr} is a hyperparameter considered unknown.

The statistical inversion is carried out using an adaptive Metropolis sampling scheme [3], and initialised at a solution obtained by deterministic optimisation [6].

3. RESULTS

The proposed method is here applied to the measured data acquired in Ref. [2] using a broadband source and a 34-microphone array. 21 frequency lines between 16 kHz and 21 kHz are used, thus resulting in $34 \times 21 = 714$ complex sound pressure experimental values.

The surface is discretised into $N = 150$ points, yielding a total of 152 unknowns. The sampling procedure consists of 120000 samples, where 20000 are discarded as burn-in. The solution of the inverse problem is presented as a maximum a posteriori (MAP) estimate and the 95% credible interval at each point of the surface. This is shown in Figure 2 together with the known elevation profile used to manufacture the surface specimen.

4. DISCUSSION

The result shows overall accurate reconstruction of the surface, with low uncertainty in the central area, where the source amplitude is larger and where specular reflection is recovered at the microphones. A large uncertainty can be observed at $x > 0.5$ m, which could be overcome by increasing the number of samples.

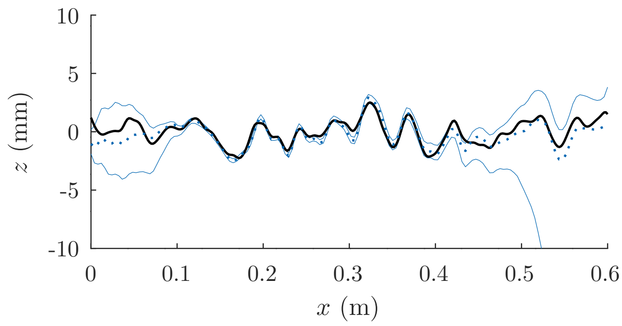

Figure 2. Estimated surface elevation. ··· MAP estimate; — 95% credible interval; — known target.

5. REFERENCES

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