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PROGRAMMABLE TOPOLOGICAL INTERFACE STATES IN LOCALLY RESONANT PIEZOELECTRIC METAMATERIAL BEAMS

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ABSTRACT

The literature about locally resonant metamaterials based on a fixed periodic scheme is well-established as the mechanisms behind wave control through band gaps are rigorously comprehended. However, they lack robustness and flexibility in waveform shaping and localization. In this context, the recent development of topological phononics, which describes analogs of quantum phenomena in elasticity and acoustics via topological concepts such as geometric phases, is paving new fascinating forms of manipulating elastic and acoustic waves with robustness against defects at the cost of breaking space and time symmetries. This study proposes inducing band inversion and interface modes in a 1D locally resonant piezoelectric material by tuning the shunt's inductance. We verify energy localization around the fixed interface in the harmonic response. Afterward, we demonstrate a flexible wave localization scheme by modulating the topological interface in space and time.

Keywords: *piezoelectricity, programmable metamaterials, topological interface states*

1. INTRODUCTION

The research of elastic/acoustic metamaterials (MMs) and phononic crystals (PCs) has gained new directions emerging from the link between the topological concepts in condensed matter physics and the classical wave theory [1]. Besides the newly discovered phenomena, topologically protected wave propagation immune to defects reveals novel mechanisms for robust wave manipulation [2].

In one-dimensional PCs, the existence of interface states can be demonstrated by combining two PCs with distinct Zak phases created by designing unit cells with different geometric parameters to induce band inversion [3–5]. Energy localization is observed for excitations at the interface mode's frequency. Including local resonators enables topological bandgaps in low frequencies and subwavelength topological interface states [6–9]. In this framework, present-day progress has shown that topological transitions in 1D systems can be obtained by only adjusting the local resonator properties without additional modifications in the host structure [10]. This result stimulates advances in programmable topological metamaterials with piezoelectric unit cells combined with digital shunt systems [11]. From this perspective and considering the recently proposed flexible localization in space-time of elastic waves in piezoelectric metastructures with trivial defects [12], this work presents a strategy for flexible localization of vibration energy by using programmable topological interfaces in a locally resonant electromechanical metamaterial beam.

2. LOCALLY RESONANT ELECTROMECHANICAL METAMATERIAL

The system comprises an aluminum substructure sandwiched by two piezoelectric layers with segmented electrodes, as shown in Fig. 1. Each electrode section (region delimited by red dashed lines in Fig 1) is connected to an inductive shunt circuit which forms a local resonator with

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the equivalent in-series piezoelectric capacitance. The combination of four consecutive sections results in the unit cell in our analyses. Band inversion and topological transitions are induced by perturbing the inductances of the unit cell ($L_1 = L_r(1+\delta)$ and $L_2 = L_r(1-\delta)$ in Fig. 1, where δ is the dimensionless perturbation parameter assumed to vary between -1 and 1 and L_r is the reference inductance to tune a periodic configuration. It has been used finite element modeling based on the Euler-Bernoulli beam element with an additional electrical degree-of-freedom for each piecewise continuous segment [13]. The space-time modulation of the interface depends on imposed time variation laws for the inductances. Therefore, a linear time-variant system must be treated numerically [12].

Each electrode segment in the unit cell has a length and width of 12.7 mm, whereas the metamaterial cross section has a substructure with a thickness of 0.167 mm and each PMN-PT layer with a thickness of 0.25 mm. The PMN-PT crystal presents an elastic modulus at constant electric field of 21.8 GPa; the effective piezoelectric stress is -14.1 C/m^2, the permittivity component at constant strain of 33 nF/m, and the mass density evaluated by 8120 kg/m^3. For the aluminum substructure, the elastic modulus and mass density are 69 GPa and 2700 kg/m^3, respectively.

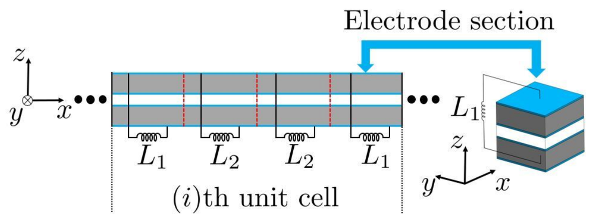


Figure 1: Scheme of the locally resonant electromechanical metamaterial, where the gray and white regions represent the piezoelectric and substructure materials, whereas the blue contours assume the role of the electrodes.

3. RESULTS AND DISCUSSIONS

The dispersion relations calculus involves a frequency eigenvalue problem with the Bloch-Floquet periodic boundary conditions applied to the degrees of freedom of the edge nodes [14]. Furthermore, we employ a qualitative procedure to determine the geometric phases by analyzing the wavemode shapes at the Brillouin zone limits, i.e., if both wave eigenmodes are symmetric or antisymmetric, it means the Zak phase is 0, otherwise is π [3, 4]. Fig. 2 depicts the dispersion diagrams when $\delta = 0$ and $\delta = -0.106$. For the periodic case ($\delta = 0$), Fig. 2(a) shows a local resonant bandgap with two transition points at $\kappa = \pi/\Delta$ (blue squares). When we analyze a nonzero δ, two additional bandgaps are opened at the transition points, as shown in Fig. 2(b). Although a band structure identical to the one of Fig. 2(b) is obtained for $\delta = 0.106$, their geometric phases, estimated from the wavemodes, indicate that they are topologically distinct.

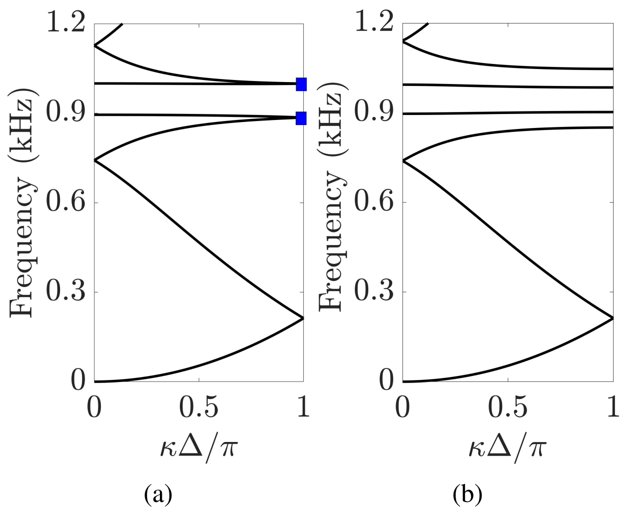
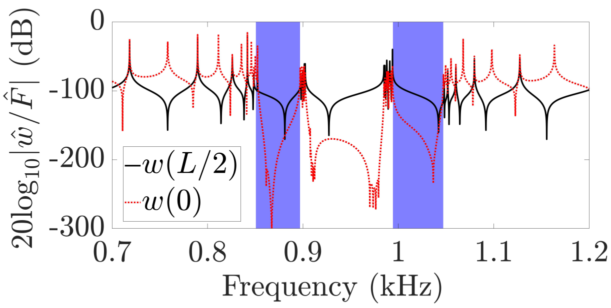
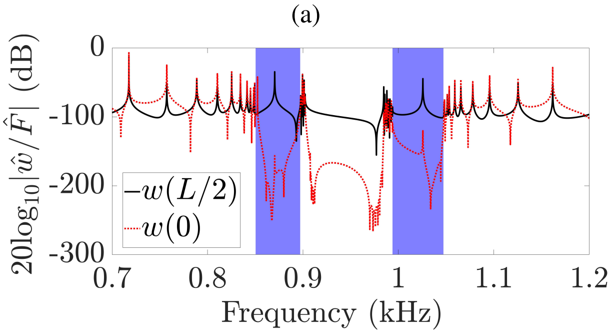


Figure 2: Dispersion diagrams for the unit cell tuned with $\delta = 0$ (a) and $\delta = -0.106$ (b).

After that, we demonstrate the existence of the interface mode by investigating the forced response in a finite topological metamaterial constructed with a connection of two topologically distinct periodic metamaterials. In this demonstration, the metastructure comprises ten unit cells, where the five cells to the left are tuned to $\delta = 0.106$ while the five to the right are adjusted with $\delta = -0.106$. Therefore, we create an interface at the center. In addition, we also investigate a periodic version formed by ten unit cells tuned to $\delta = 0.106$ to establish a comparison. The input consists of a harmonic moment applied around the interface with transverse forces at positions $x/L = 0.48$ and $x/L = 0.51$ (where L represents the total length of both metamaterials analyzed), and the transverse displacement is measured at $x/L = 0$ and $x/L = 0.5$. Fig. 3(a) shows the Frequency Response Functions (FRF) for the periodic metamaterial, where noticeable attenuation zones that correspond to the bandgaps regions shown in Fig. 2(b) are present at $x/L = 0$. The geometric phase analysis showed that the blue-colored bandgaps (i.e., the first and third bandgaps) are nontrivial for a half of the substructure and trivial for the other half. Consistently, the response of the tested metamaterial at the interface position ($w(L/2)$ at Fig. 3(b)) exhibits two peaks with high amplitude at 870.9 Hz and 1026.2 Hz within the highlighted bandgaps, which are absent in the periodic structure. In the response at $x/L = 0$, the respective attenuated peaks indicate that these isolated modes present high-vibration energy localization at the interface.



(a)



(b)

Figure 3: Harmonic response associated to the transverse displacements for the case without (a) and with (b) interface.

Finally, we exploit the space-time modulation of the interface state within the first bandgap of the same topological metamaterial (i.e., at 870.9 Hz). In the strategy proposed here, one of the unit cells adjacent to the interface is gradually converted to its topological opposite (considering the band inversion) while the others remain unchanged. Consequently, the interface can only shift to the immediate subsequent unit cell boundary in a single modulation process. For instance, if the interface is initially between unit cells i and $i+1$, a relocation of the wave localization pattern around the newly created interface between unit cells $i+1$ and $i+2$ is expected after the modulation. The topological transition in the unit cell $i+1$ is controlled by imposing a smooth and synchronized time variation in its inductances with cosine functions [12]. Then, the shunts of the outer electrode sections increase their inductance from L_2 to L_1 while those in the inner sections reduce their values from L_1 to L_2 simultaneously.

The previously discussed strategy's feasibility is illustrated through time-domain simulations executed with the Newmark method at a constant time step. The interface is initially assumed at the metastructure's center. A moment is again applied in the same positions, where the transverse force signals comprehend sinusoidal envelopes during the first 0.057 s. The envelope of the displacement field is illustrated in Fig. 4, where the white circles indicate the initial and final modulation instants. Before the modulation starts, the topology of the unit cells remains unchanged, and the vibration energy is localized around the initial interface in unit cells 5 and 6. During the modulation, the vibration energy gradually flows towards unit cell 7 while it escapes from unit cells 5 and 6. After 0.24 s, the energy significantly vanishes from unit cell 5 and concentrates predominantly in unit cells 6 and 7 to form the new interface state at $x/L = 0.6$. Negligible leakage is noticeable throughout the modulation, indicating a perfect energy transfer between the fifth and sixth unit cells.

4. CONCLUSIONS

Throughout this study, we demonstrated a workable strategy for conceiving flexible elastic wave localization by programming the topological interface in space and time. With local and synchronized control of the inductances, a relocation of the vibration concentration pattern was feasible without considerable leakage away from the interfaces. Though similar phenomena were found with localized defects, the topological states can provide interesting robustness features. The development of reconfigurable devices for energy harvesting, structural health sensing, and monitoring can benefit from the discoveries of this work.

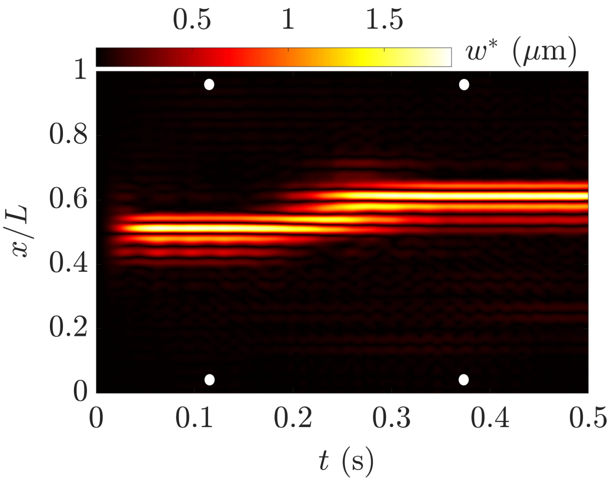


Figure 4: Envelope of the transverse displacement field that shows the interface modulation from $x/L = 0.5$ to $x/L = 0.6$.

5. ACKNOWLEDGMENTS

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