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# MODELING LOW-FREQUENCY NOISE IN DUCTED ATMOSPHERES: A MODAL APPROACH

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## ABSTRACT

Infrasound and low-frequency sound have gained significant attention recently due to their scientific and societal importance. For example, wind turbines and heat pumps can emit low-frequency noise, leading to increased noise-related complaints. Also, infrasound is used to locate violent sources such as earthquakes and volcanic eruptions, hence being an important indicator in early warning systems. Due to the large environmental scales, calculating low-frequency sound propagation is difficult and computationally expensive. Recently, normal mode approaches for low-frequency sound in the atmosphere have been adapted from those used in underwater acoustics, resulting in a more efficient method for low frequencies compared to other commonly used atmospheric propagation models. We show how the normal mode approach can be used, for example, to model and measure the low-frequency noise radiated downstream of wind turbines.

**Keywords:** *numerical acoustic, scattering, rotating frames,*

## 1. INTRODUCTION

Infrasound and low-frequency acoustic atmospheric waves gained attention, especially in connection with the renewable energy sectors, where the technologies used can emit low-frequency noise. The noise is also one of the reasons for an increasing number of complaints from residents, leading to a deceleration in commissioning processes for renewable energy converters. The topic also triggered interest in research on the health effects of low-frequency noise [1].
Predicting exact levels of low-frequency noise in residential areas is challenging as the levels depend very much on the atmospheric conditions, i.e. wind, temperature, and humidity profiles, and the exact measurement locations. Low-frequency sound propagation models that consider meteorological data and measurement position could provide a more comprehensive and objective evaluation of noise exposure.

Herein, we investigate the applicability of normal mode approaches for low-frequency sound propagation in the lower atmosphere. For long-range problems, normal mode approaches have been adapted from underwater acoustics, providing a possibly more efficient and accurate method for low-frequency sound propagation compared to other atmospheric propagation models, such as ray tracing and parabolic equations [2,3]. Normal modes develop for certain atmospheric conditions inside atmospheric waveguides ("atmospheric ducts"). In general, atmospheric ducts, formed by the downward refraction of acoustic waves due to temperature inversion and winds, allow low-frequency sound to travel efficiently over long distances [4,5]. It should be noted that normal mode approaches require a range-independent propagation, which is usually not true for long-range problems.

However, also winds in the lower atmosphere can create atmospheric ducts with a few hundred meters of height [6]. For example, the infrasonic blade tone and the low-frequency gear noise of wind turbines can propagate several kilometers downstream through those ducts, leading to large potentially noise-affected areas. In many cases, range independence can be assumed for relevant distances considered in those problems. The sound levels perceived in those areas can be spatially modulated, making it challenging to take conclusive measurements at single locations. **Figure 1** shows a two-dimensional simulation of a harmonic sound field at 15 Hz (without cylindrical spreading), emitted from a point source at 100 m height. Here, a linear-harmonic form

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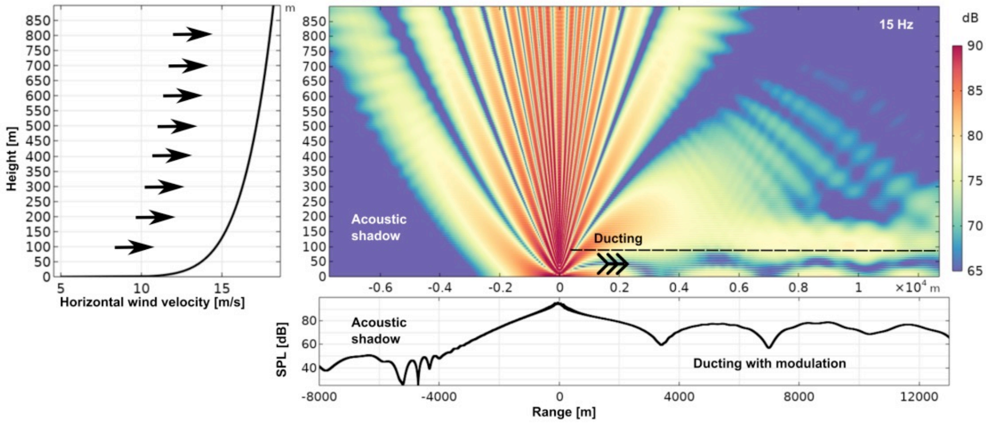

**Figure 1**. Ducting of a 15 Hz harmonic sound under refracting wind conditions. Two-dimensional simulation that does not include cylindrical spreading (Linearized Euler Equations in COMSOL Multiphysics).

of the Euler equation was solved for a logarithmic wind profile and constant temperature. The results demonstrated the ducting effect in the down-wind direction and the modulation of the acoustic pressure at 1 m height above the ground.

## 2. APPROACH

Under ducted atmospheric conditions, the linearized equations of fluid motion that describe the acoustic state variables can be separated in height and range dimensions, and an eigenvalue problem can be formulated in the vertical direction [7]. The solutions to this eigenvalue problem are the shapes of the atmospheric modes and their wave numbers. Solutions can be found for simplified versions of the vertical eigenvalue problem, using iterative root-finding algorithms in the complex plane [4]. Without simplifying the eigenvalue problem, only numerical solutions can be found, e.g. with the method described in Ref. [3].

The modes can be used to describe the acoustic far field efficiently and accurately, providing that enough modes are used in the computation. The pressure at any point outside a harmonic source region can be described in down-wind direction as

$$p(x,h) = \sum_{i\to\infty} \hat{p}_i \, \varphi_i \, e^{-ik_i\omega x}, \qquad (1)$$

with the range coordinate x, and the height coordinate h. The wave number and the mode shape of the i-th mode are $k_i$ and $\varphi_i$, and $\hat{p}_i$ is its complex-valued amplitude. Atmospheric damping can be included as an imaginary part of the wave number. Cylindrical and Spherical spreading can be included through correction factors as described in Ref. [3].

Although there is an infinite number of modes, in practice only a finite number of modes can be included in Eq, (1) with reasonable computational effort. This limits the method to low frequencies at which only a few modes are ducted to the acoustic far field. It is the main reason why the approach has been used mostly for ground-to-ground propagation modeling of violent sources (volcano eruptions, explosions) for ultra-low frequencies (<1 Hz) in the large atmospheric ducts (heights of several kilometers) formed by the jet streams in the Stratopause [2,3,8].

Here, we propose to use the modal approach to describe the near-ground propagation in smaller atmospheric ducts (several hundred meters height), which are relevant, for example, in the down-wind direction of wind parks. The physics governing the smaller ducts is the same as for large ducts, although the conditions for close-to-ground atmospheric ducting rely on the wind conditions, which are much less stable than the jet streams. Smaller ducts allow us to compute the far-field propagation for higher frequencies, including the transition range between infrasound and low-frequency noise (10 – 50 Hz), which is very important for the prediction of possible noise nuisance. For example, the lower-order harmonics of wind park's blade passing frequencies and the tones caused by the gearboxes of wind turbines are within this frequency range.

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