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A MESHFREE APPROACH FOR SOUND WAVE PROPAGATION IN PERIODIC MEDIA WITH MULTI-LAYER POROUS INCLUSIONS

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ABSTRACT

The application of periodic structures in acoustic problems covers a wide area, including noise barriers, cloaking devices or wave guiding systems. Many different methods have been used for this purpose, including numerical (FEM, FD, BEM, . . .) and analytical approaches. In the present paper, the authors address this problem extending previous works and proposing a general strategy to simulate infinite periodic media with multiple inclusions disposed periodically along one direction. The inclusions may be multi-layered and include porous materials, with each layer modelled as an equivalent fluid. In this paper, the authors make use of a 2D formulation based in the Method of Fundamental Solutions (MFS), using special Green's functions derived for periodic media. Within the scatterers, the multilayered structure is simulated by means of classical 2D Green's functions. Two types of periodic Green's functions are tested in terms of efficiency and accuracy. The MFS implementation is validated against a Finite Element model. A set of application examples are given to illustrate its applicability in the simulation of sonic crystal barriers with absorbing properties.

Keywords: *Periodic Media, Meshless Methods, Acoustic Waves.*

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1. INTRODUCTION

Periodic structures have been extensively investigated in the field of acoustics and vibrations, focusing different applications, such as acoustic barriers, cloaking devices or wave guiding systems. Different methods have been used for this purpose, including numerical and analytical approaches. A recent work has proposed the use of a meshfree formulation based on the Method of Fundamental Solutions (MFS) to tackle such problems in an infinite environment, defining fundamental solutions that may be used both in 2D and in 3D problems. In that work, the authors address the case of rigid scatterers in a 3D scenario, simulating a sonic crystal noise barrier [1]. Later, the authors extended the method for the case of scatterers with an absorbing layer made of porous material in a 2D environment [2]. In another earlier work, the case of an acoustic medium hosting a non-homogeneous inclusion has been addressed using a coupling between the Boundary Element Method (for the homogeneous host medium) and Kansa's method (for the inclusion) [3].

The present paper extends the previous works by proposing a general strategy based in the method of fundamental solutions (MFS) to simulate infinite periodic media with multiple inclusions with an internal layered structure. The host propagation medium is assumed to be a homogeneous acoustic medium, while the internal part of the scatterers is simulated by means of fluid-equivalent models, namely using the Horoshenkhov-Swift model [4]. Inside the scatterers, the classical 2D Green's functions are used, enforcing continuity of particle velocity and sound pressure between layers, while periodic fundamental solutions are used to simulate the homogeneous part of the domain. The method is validated against a Finite Element model, and an application example is given to illustrate its applicability. The proposed strategy can be quite efficient, accurate and elegant.

2. MATHEMATICAL FORMULATION

Consider an infinite acoustic medium, with density ρ and allowing a sound speed of c. The sound propagation in this medium in the frequency domain for a frequency ω, and considering a wavenumber k=ω/c, is governed by the Helmholtz equation, given as:

$$\Delta P + k^2 P = 0 \tag{1}$$

Green's function for this equation are well known for 2D, 2.5D and 3D problems. For the case of 2D periodicity, as illustrated in Fig. 1, the following periodic Green's function can be written following the work of [1]:

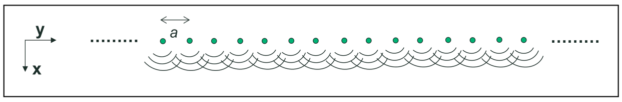

Figure 1. Periodically placed 2D sources, spaced a.

$$G_{per}(\mathbf{x}_0, \mathbf{x}) = \sum_{i=-\infty}^{+\infty} -i/4H_0^{(2)}(kr_i) \tag{2}$$

$$G_{per}(\mathbf{x}_0, \mathbf{x}) = -i/(2a) \sum_{n=-\infty}^{+\infty} e^{-ik_n(x-x_0)} e^{-i2\pi n(y-y_0)/a} \tag{3}$$

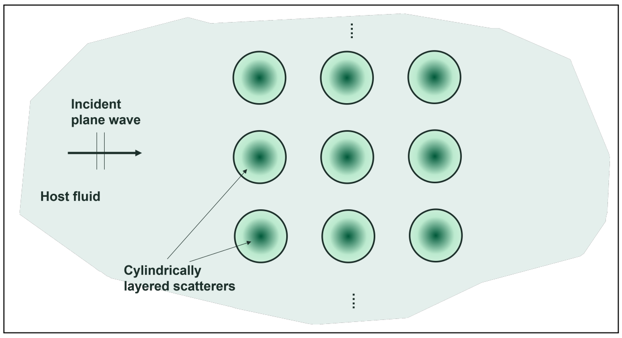

Figure 2. Global view of the problem, with a set of multilayer inclusions placed periodically along one direction within a homogeneous acoustic medium.

Fig. 3 illustrates the number of terms required for convergence of both Green's functions (Eqn. (2) in red, Eqn. (3) in gray), demonstrating that the second option has a clear advantage in terms of computational efficiency. Thus, it will be used in the presented examples, namely for the case shown in Fig. 2, in conjunction with the Method of Fundamental Solutions.

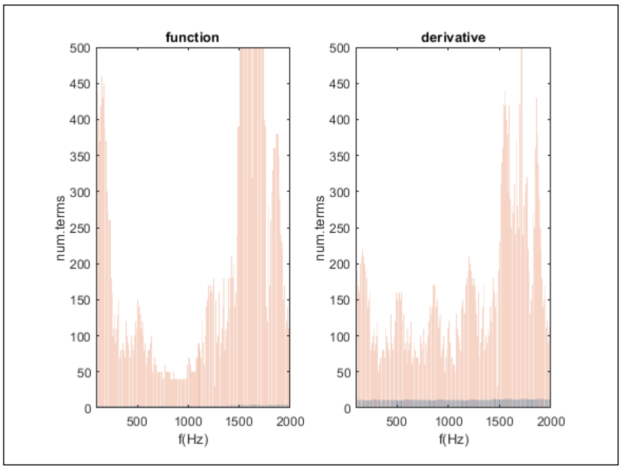

Figure 3. Number of terms required for convergence for the proposed Green's functions.

3. NUMERICAL EXAMPLES

To exemplify the application of the proposed strategy, consider a homogeneous medium with properties of air (c=$343 m/s$ and ρ=1.21 kg/m^3, hosting a periodically spaced cylinders with radius $6cm$ and a spacing $a = 17cm$. For this example, consider that the cylinders are made from a layer of porous material, with thickness $2cm$, described by the Horoshenkhov-Swift fluid-equivalent model, considering the parameters $\sigma = 3896N/m^4$, $\phi = 0.46$, $\alpha_{inf} = 1.89$ and $\sigma_d = 0.25$. The central part of the cylinders is a rigid core, with radius $4cm$. This system, made of 3 lines of scatterers, is subjected to the incidence of a plane wave, as depicted in Fig. 4.

This problem was solved using the proposed MFS formulation, with the periodic Green's functions, and the result was compared with that provided by a standard FEM formulation. Fig. 5 depicts a comparison between the Insertion Loss results provided by the two methods, clearly showing an excelent agreement between both.

An additional example is presented in Fig. 6, showing the results calculated when the scatterers have a porous

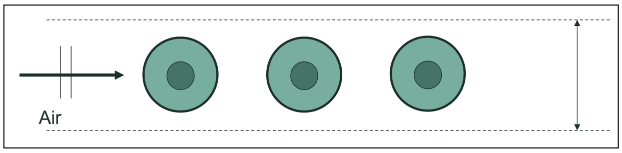

Figure 4. Geometry of the numerical example problem.

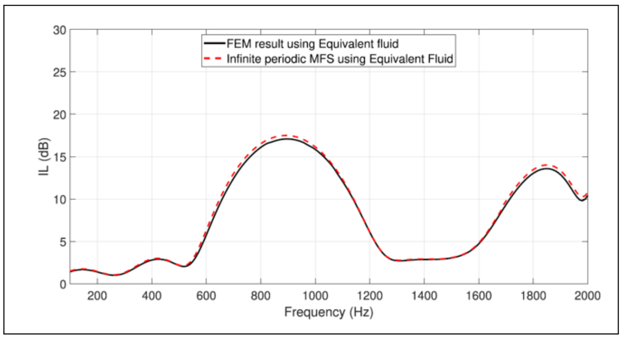

Figure 5. Numerical verification results comparing MFS and FEM.

material, with lower porosity and higher airflow resistivity ($\sigma = 7171 N/m^4$, $\phi = 0.36$, $\alpha_{inf} = 2.73$ and $\sigma_d = 0.41$) instead of the rigid core. For this example, 3 cases were analyzed, considering the inner part to have radii of $3cm$, $4cm$ and $5cm$.

For this case, it becomes clear that a marked change occurs in the behavior of the system, even when the core's radius is the same as in the verification results. Indeed, for this case, it seems that considering a porous core leads to a decrease in the bandgap amplitude, but accompanied by an increase of attenuation between $1.2kHz$ and $1.6kHz$.

4. FINAL REMARKS

In this short paper, 2D Sonic crystals embedded in a fluid medium were here analyzed using the MFS, considering periodic Green's functions. Two types of periodic Green's funcitons were used and analyzed, showing to be effective to tackle acoustic wave propagation in the presence of infinite periodic structures

The interior of the scatterers was assumed to have a layered structure, and representing a layered porous material modeled as an equivalent fluid. Sample results were shown illustrating the applicability of the strategy to study different types of structures Insertion Loss results. Numerical examples show a strong dependence on the in-

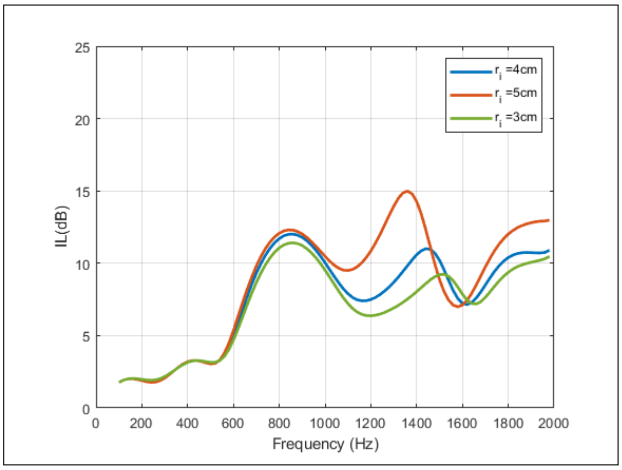

Figure 6. Numerical verification results comparing MFS and FEM.

ternal structure of the scatterer, significantly changing the registered Insertion Loss.

5. ACKNOWLEDGMENTS

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