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ACOUSTIC PROPAGATION THROUGH DIFFUSE INTERFACES

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ABSTRACT

We propose a novel approach for simulating acoustic wave propagation across multiphase media with a diffuse interface model. Extending Discontinuous Galerkin Spectral Element Methods (DGSEM) for the incompressible Navier-Stokes/Cahn-Hilliard systems, our approach incorporates a modified weakly compressible formulation that accommodates phase-dependent sound speeds. Numerical experiments demonstrate spectral convergence for Snell's law in 2D. This work aims to advance high-fidelity simulations of acoustic propagation in multiphase systems and has implications for marine aeroacoustics and related fields.

Keywords: *High order discontinuous Galerkin, Artificial compressibility, Multiphase, Diffuse interface, Navier-Stokes/Cahn-Hilliard*

1. INTRODUCTION

Modeling acoustic wave propagation in heterogeneous media has applications in medicine, aeroacoustics, hydroacoustics, and industry [1–8].

Diffuse interface phase-field models are garnering attention in multiphase modeling, offering an alternative to sharp interface methods like volume of fluid and level set. Initially designed for cases where interface thickness is comparable to physical scales, diffuse interfaces have recently demonstrated advantages in broader scenarios [9].

This work employs the high-order discontinuous Galerkin spectral element method (DGSEM), known for low numerical dissipation and provable stability across various equations. Traditional incompressible flow solvers rely on splitting schemes, which yield non-physical pressures, whereas weak compressibility formulations introduce an additional equation coupling velocity and pressure fields [10, 11]. Although initially a numerical tool, weak compressibility has been adapted for accurate acoustic wave propagation [12, 13]. However, its application in multiphase acoustics remains unexplored.

We extend our previous DGSEM work on incompressible Navier-Stokes and Cahn-Hilliard systems by introducing a weak compressibility-based approach for direct acoustic wave propagation across media. Our formulation allows for different sound speeds in each phase, separated by a diffuse interface. The method is validated against acoustic refraction following Snell's law, while also analyzing spectral convergence. This research is derived from the broader investigation outlined in [14].

2. MATHEMATICAL MODEL

We build over our previous work on a two-phase, entropy-stable DGSEM iNS/CH model with artificial compressibility, originally developed for high-density ratios [15], by modifying the artificial compressibility equation to accommodate variations in sound speed across phases.

The concentration $c \in [0, 1]$ of each phase is provided by the diffuse-interface Cahn-Hilliard equation:

$$\partial_t c + \nabla \cdot (c\boldsymbol{u}) = M_0 \nabla^2 \mu, \tag{1}$$

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μ is the chemical potential, given by:

$$\mu = \frac{df_0}{dc} - \frac{3}{2}\sigma\varepsilon\nabla^2 c, \quad f_0 = \frac{12\sigma}{\varepsilon}c^2(1-c)^2, \tag{2}$$

such that σ is the surface tension, ε is the diffuse interface width, and t_{CH} is an additional chemical characteristic time parameter that relate to the mobility parameter M_0 through:

$$M_0 = \frac{\varepsilon}{\sigma t_{ch}}. \tag{3}$$

The advective velocity $\boldsymbol{u}$ in equation (1) couples the Cahn-Hilliard equation with the Navier-Stokes equation:

$$\begin{aligned}\sqrt{\rho}\partial_t\left(\sqrt{\rho}\boldsymbol{u}\right) + \nabla\cdot\left(\frac{1}{2}\rho\boldsymbol{u}\boldsymbol{u}\right) + \frac{1}{2}\rho\boldsymbol{u}\cdot\nabla\boldsymbol{u} + c\nabla\mu \\ = -\nabla p + \nabla\cdot\left(\eta(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)\right) + \rho\boldsymbol{g},\end{aligned} \tag{4}$$

where p is the pressure, $\boldsymbol{g}$ represents gravitational acceleration, ρ is the density, and η is the viscosity. Previously, weak compressibility was used to enforce the incompressibility constraint; however, we now leverage it to propagate pressure with the appropriate acoustic speed. Assuming isentropic conditions and a low Mach number regime, the pressure equation is formulated as [16]:

$$\partial_t p + \rho a^2 \nabla\cdot\boldsymbol{u} = 0, \tag{5}$$

where a denotes the speed of sound.

The coupled system of equations (1), (4), and (5) defines the multiphase iNS/CH model and will be discretized with an entropy stable DGSEM formulation. Consult [15] for details on disciritization.

3. NUMERICAL EXPERIMENTS

The setup for the numerical experiments is given in Fig. 1 of an incident wave propagating from the left, incident on the diffuse interface slanted at an angle. The signal hits the interface with an angle of incidence θ_i. The analytical expression for the angle of transmission θ_t is given by Snell's law:

$$\frac{\sin(\theta_i)}{a_1} = \frac{\sin(\theta_t)}{a_2}. \tag{6}$$

The incident signal is generated by adding a source term to the pressure equation:

$$s = cos(2\pi f t)e^{\left(\frac{x-x_0}{b}\right)^2}; \tag{7}$$

where f is the frequency, x_0 is the position and b is the width of the forcing. A probe is placed on the right of the

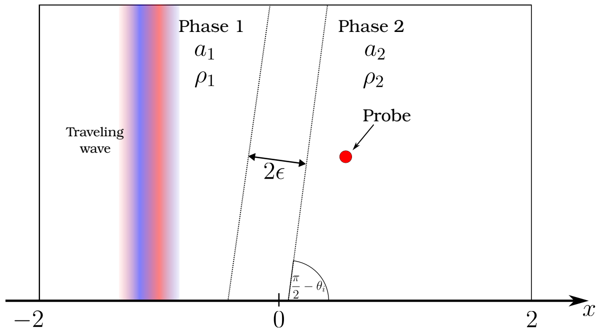

Figure 1. Setup

interface and is meant to monitor the incident wave and compute the numerical angle of transmission.

We ran on a number of meshes with varied polynomial order p and for an incident angle $\theta_i = 10°$ and frequency of 1 kHz. The simulation parameters are as follows. For Fluid 1, the density is $\rho_1 = 1\,\mathrm{kg/m^3}$, the speed of sound is $a_1 = 343\,\mathrm{m\,s^{-1}}$, and the viscosity is $\eta_1 = 10^{-16}\,\mathrm{Pa\,s}$. For Fluid 2, the density is $\rho_2 = 2\,\mathrm{kg/m^3}$, the speed of sound is $a_2 = 1481\,\mathrm{m\,s^{-1}}$, and the viscosity is $\eta_2 = 10^{-16}\,\mathrm{Pa\,s}$. The interface parameters are given by an interface thickness of $\varepsilon = 0.01\,\mathrm{m}$, a surface tension of $\sigma = 10^{-16}\,\mathrm{N\,m^{-1}}$, and a mobility coefficient of $M_0 = 0.01\,\mathrm{Pa^{-1}\,s^{-1}}$.

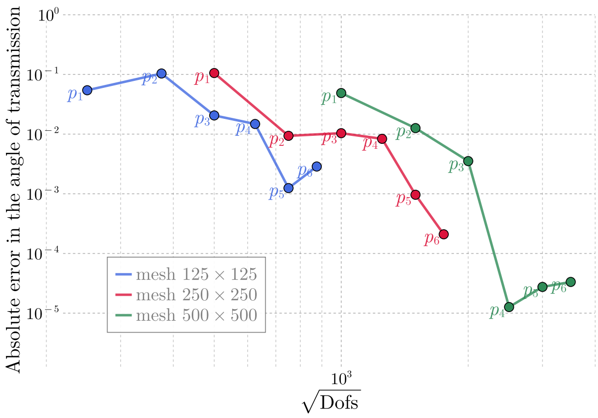

Figure 2. Errors for the angle of transmission for an incident angle $\theta_i = 10°$ and 1 kHz

Fig. 2 shows spectral convergence towards Snell's law. It is evident that for the same number of degrees of freedom, coarse meshes with high polynomial order yield lower errors than fine meshes with low polynomial order.

Fig. 3 shows the pressure field at the end of the simulation for the finest mesh. It shows qualitative agreement with Snell's law as the wave front changes direction.

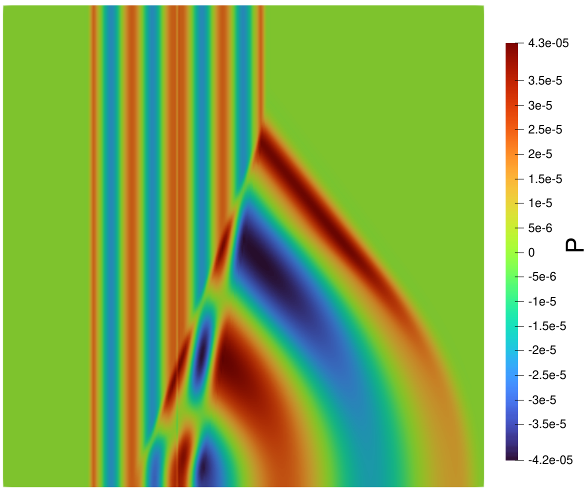

Figure 3. Pressure field at the end of the simulation, $\theta_i = 10^\circ$ and 1 kHz

4. CONCLUSIONS

We have developed a novel framework for simulating acoustic wave propagation through multiphase media using a diffuse interface approach. By extending the Discontinuous Galerkin Spectral Element Method (DGSEM) to incorporate weakly compressible formulations, we allow for phase-dependent sound speed variations. Our results validate the method against Snell's law in two dimensions, demonstrating spectral convergence and highlighting the potential of diffuse interfaces for high-fidelity acoustic simulations.

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