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APPLICATION OF CRAIG-BAMPTON REDUCTION IN VIBRO-ACOUSTIC COUPLING

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ABSTRACT

Vibro-acoustic coupling between structural and acoustic domains via the Finite Element Method can be tackled using different reduction techniques. This study compares three coupling approaches within the displacement-pressure formulation: standard modal coupling, modal coupling with static corrections, and Craig-Bampton vibro-acoustic coupling. A symmetrization procedure is introduced for all methods, facilitating the computation of coupled vibro-acoustic modes. Through a representative example, the methods are evaluated in terms of performance and computational efficiency, benchmarked against the physical, non-reduced coupling approach.

Keywords: *simulation, FEM, substructuring, vibro-acoustics*

1. INTRODUCTION

The Finite Element Method (FEM) has become a cornerstone in the study of vibro-acoustic problems, providing a flexible framework for modeling both structural and acoustic domains. A widely adopted approach involves representing the structure and the acoustic cavity through their respective uncoupled modal bases and subsequently coupling them in the modal domain. While this method offers computational advantages and conceptual simplicity, it often fails to capture the interaction accurately.

To address these limitations, pseudo-static corrections were introduced by Tournour [1] to account for the presence of the acoustic domain in computing the uncoupled structural modes and vice-versa, in terms of an added mass. An alternative strategy, proposed by Herrmann [2], involves computing internal modes for both the structural and acoustic domains while preserving the fluid-structure interface in its physical representation. This method is conceptually similar to the Craig-Bampton approach [3], traditionally used in dynamic substructuring, and enforces a more accurate treatment of the interface interactions.

Building upon these developments, the present work proposes a hybrid formulation wherein the structural domain is represented by its uncoupled in-vacuo modes, while the acoustic cavity is modeled using a Craig-Bampton-type reduction. In this formulation, the internal modes of the cavity are computed, and the degrees of freedom at the acoustic fluid-structure interface are kept in their physical form. This choice is motivated by the observation that the structural behavior remains relatively invariant whether it is in-vacuo or coupled with the cavity, whereas the acoustic field is significantly affected by the boundary conditions imposed at the interface.

Furthermore, a new formulation is introduced to transform the vibro-acoustic problem into a symmetric system representation. This reformulation improves the numerical manageability and stability of the problem, particularly when computing coupled vibro-acoustic modes. It also opens the door for more efficient eigensolvers and a clearer physical interpretation of the coupled system behavior. The proposed methods aim to strike a balance between model accuracy and computational efficiency, offering an improved framework for vibro-acoustic analysis.

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2. VIBRO-ACOUSTIC COUPLING METHODS

Coupling structural and acoustic models using the Finite Element Method is essential for simulating interactions between structures and their surrounding acoustic environments. This can be achieved through various methods, each balancing computational efficiency and accuracy. In this section different vibro-acoustic methods will be presented, as depicted in Figure 1.

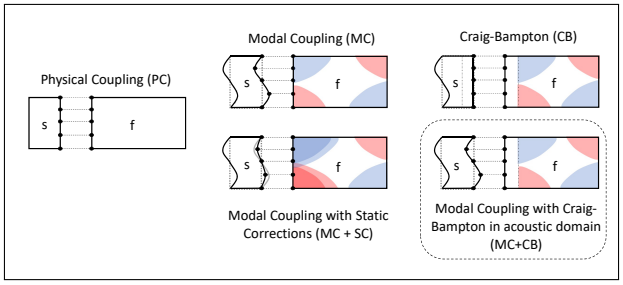

Figure 1. Comparison of different vibro-acoustic coupling methods using FEM. Structure and cavity are depicted as *s* and *f* respectively. The new proposed method is highlighted.

2.1 Physical coupling (PC)

The most simple way of connecting acoustic and structural domains using FEM is through physical coupling. It uses structural displacements and acoustic pressures to represent the interaction. A surface coupling matrix ensures exchange of forces across the interface. While accurate, this method can be computationally intensive for large systems due to the high dimensionality of the matrices.

The structural-acoustic coupling in time domain, in a Finite Element representation can be expressed in Equation (1) , using a displacement-pressure formulation [4]:

$$\begin{bmatrix} M_s & 0 \\ C_{sf}^T & M_f \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} K_s & -C_{sf} \\ 0 & K_f \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f_s \\ f_f \end{bmatrix} \quad (1)$$

With M_s, M_f, K_s, and K_f the structural and acoustic mass and stiffness matrices, C_{sf} the surface coupling matrix, f_s, f_f the structural and acoustic external loads and u, p the nodal displacement and pressure vectors of the structural and acoustic domains. The matrices M_f and K_f need to be divided by the density of the acoustic domain, so the acoustic equation is in volumetric basis. The surface coupling matrix can be obtained from the finite element discretization as:

$$C_{sf} = B_s^T N_f A_f B_f \quad (2)$$

with B_s and B_f matrices that select the DOF of the structural and acoustic domains at the fluid-structural interfaces, N_f a matrix containing the normal vectors pointing towards the structure of the corresponding acoustic nodes, and A_f a diagonal matrix containing the area of the corresponding acoustic nodes.

2.2 Modal coupling (MC)

Modal coupling analyzes the structure and acoustic cavity separately to extract their uncoupled modes. Structural modes are calculated *in vacuo*, and cavity modes assume rigid walls. The displacements and pressures can be approximated by a set of truncated modes:

$$\begin{aligned} u &\simeq \Phi_s u_m \\ p &\simeq \Phi_f p_m \end{aligned} \quad (3)$$

with Φ_s, Φ_f the truncated modal sets and u_m, p_m the modal participation factors of the structural and acoustic domains respectively. Take the Equation (1) that expresses the fluid-structure interaction in physical coupling. If both structural and acoustic domains are represented by their first modes, given by Equation (3), multiplication on the first line by Φ_s^T and the second line by Φ_f^T, the system in physical domain can be expressed by the modal participation factors u_m, p_m:

$$\begin{bmatrix} I_s & 0 \\ C_{sf,m}^T & I_f \end{bmatrix} \begin{bmatrix} \ddot{u}_m \\ \ddot{p}_m \end{bmatrix} + \begin{bmatrix} \Omega_s^2 & -C_{sf,m} \\ 0 & \Omega_f^2 \end{bmatrix} \begin{bmatrix} u_m \\ p_m \end{bmatrix} = \begin{bmatrix} \Phi_s^T f_s \\ \Phi_f^T f_f \end{bmatrix} \quad (4)$$

The coupling between structural and acoustic subdomains approximate the physical coupling if the number of modes of each domain is sufficient. However, since it involves the independent computation of modes, the equilibrium condition between structural and acoustic domains, given by the fluid-structure interaction, is poorly satisfied.

2.3 Modal coupling with pseudo-static corrections (MC+SC)

An improvement can be done in the modal coupling by adding pseudo-static corrections [1]. These corrections

use residual flexibilities at the interfaces, so the approximation of u and p in Equation (3) is enhanced by a static term:

$$\begin{aligned} u &\simeq \Phi_s u_m + G_{r,s}(f_s + C_{sf}p) \\ p &\simeq \Phi_f p_m + G_{r,f}(f_f - C_{sf}^T \ddot{u}) \end{aligned} \tag{5}$$

with $G_{r,s}$ and $G_{r,f}$ the residual flexibility matrices of the structural and acoustic domains, that can be obtained as:

$$\begin{aligned} G_{r,s} &= K_s^{-1} - \Phi_s \Omega_s^{-2} \Phi_s^T \\ G_{r,f} &= K_f^{-1} - \Phi_f \Omega_f^{-2} \Phi_f^T \end{aligned} \tag{6}$$

If K is singular, because there are not enough boundary conditions to prevent its rigid body motion, then K^{-1} is obtained by a procedure that constrains K to be invertible and suppresses the rigid body modes [4].

Inserting the transformation (5) in the coupled equation of motion in physical domain (1). the resultant system of equations is [4]:

$$\begin{aligned} &\begin{bmatrix} I_s + M_{s,am} & 0 \\ C_{sf,m}^T & I_f + M_{f,am} \end{bmatrix} \begin{bmatrix} \ddot{u}_m \\ \ddot{p}_m \end{bmatrix} \\ &+ \begin{bmatrix} \Omega_s^2 & -C_{sf,m} \\ 0 & \Omega_f^2 \end{bmatrix} \begin{bmatrix} u_m \\ p_m \end{bmatrix} = \begin{bmatrix} \Phi_s^T f_s + \Phi_s^T C_{sf} G_{r,f} f_f \\ \Phi_f^T f_f - \Phi_f^T C_{sf} G_{r,s} \ddot{f}_s \end{bmatrix} \end{aligned} \tag{7}$$

where $M_{s,am}$ and $M_{f,am}$ are the added masses in the structural and acoustic domains respectively due to the vibro-acoustic interaction.

$$\begin{aligned} M_{s,am} &= \Phi_s^T C_{sf} G_{r,f} C_{sf}^T \Phi_s \\ M_{f,am} &= \Phi_f^T C_{sf}^T G_{r,s} C_{sf} \Phi_f \end{aligned} \tag{8}$$

This method improves the natural frequencies of the vibro-acoustic coupled system [1]. However, the local behavior at the fluid-structure interface is is not ensured.

2.4 Craig-Bampton coupling (CB)

A different way to tackle the vibro-acoustic coupling is by using a Craig-Bampton [3] reduction. In [2] this type of coupling is applied. It reduces the complexity of the coupling by using internal and constraint modes for the structural and acoustic domains, reducing the size of the physical matrices while preserving fluid-structure interfaces in physical domain.

Consider a transformation of the physical DOF such that internal and interface DOF are explicitly separated:

$$u = L_s \begin{bmatrix} u_i \\ u_b \end{bmatrix}, \quad p = L_f \begin{bmatrix} p_i \\ p_b \end{bmatrix} \tag{9}$$

with the subindices i and b indicating internal and boundary interface, respectively. The localization matrices L_s and L_f are defined as:

$$L_s = \begin{bmatrix} B_{ii,s}^T & B_{bb,s}^T \end{bmatrix}, \quad L_f = \begin{bmatrix} B_{ii,f}^T & B_{bb,f}^T \end{bmatrix} \tag{10}$$

where $B_{ii,s}$ and $B_{ii,f}$ select the internal DOF and $B_{bb,s}$ and $B_{bb,f}$ select interface DOF from structural and acoustic domains, respectively. The Craig-Bampton transformation [5] in structural and acoustic domains can be achieved using a truncated set of internal modes Φ_i and a full set of constraint Ψ_{ib} modes:

$$\begin{aligned} \begin{bmatrix} u_i \\ u_b \end{bmatrix} &\simeq \begin{bmatrix} \Phi_{i,s} & \Psi_{ib,s} \\ 0 & I_b \end{bmatrix} \begin{bmatrix} u_{m,i} \\ u_b \end{bmatrix} = T_{CB,s} u_{CB}, \\ \begin{bmatrix} p_i \\ p_b \end{bmatrix} &\simeq \begin{bmatrix} \Phi_{i,f} & \Psi_{ib,f} \\ 0 & I_b \end{bmatrix} \begin{bmatrix} p_{m,i} \\ p_b \end{bmatrix} = T_{CB,f} p_{CB} \end{aligned} \tag{11}$$

with $p_{m,i}$ and $u_{m,i}$ the modal participation factors of the acoustic and structural internal domains. The constraint modes are defined as:

$$\begin{aligned} \Psi_{ib,s} &= -K_{ii,s}^{-1} K_{ib,s}, \\ \Psi_{ib,f} &= -K_{ii,f}^{-1} K_{ib,f} \end{aligned} \tag{12}$$

where $K_{ii,s}, K_{ii,f}$ are the stiffness matrices associated to the internal degrees-of-freedom, and $K_{ib,s}, K_{ib,f}$ are the stiffness matrices connected to internal-interface terms, for structural and acoustic domains respectively. The Craig-Bampton transformations (11) can be applied respectively to each side of the vibro-acoustic equation of motion (1), resulting in the following equation.

$$\begin{aligned} &\begin{bmatrix} M_{CB,s} & 0 \\ C_{sf,CB}^T & M_{CB,f} \end{bmatrix} \begin{bmatrix} \ddot{u}_{CB} \\ \ddot{p}_{CB} \end{bmatrix} \\ &+ \begin{bmatrix} K_{CB,s} & -C_{sf,CB} \\ 0 & K_{CB,f} \end{bmatrix} \begin{bmatrix} u_{CB} \\ p_{CB} \end{bmatrix} = \begin{bmatrix} f_{CB,s} \\ f_{CB,f} \end{bmatrix} \end{aligned} \tag{13}$$

This methodology involves a higher number of DOF, since the interfaces of both structural and acoustic domains are retained in physical domain. However, it attempts to improve the vibro-acoustic coupling.

3. NOVEL METHODS

3.1 Hybrid modal-Craig-Bampton vibro-acoustic method (MC+CB)

The equation of motion of the vibro-acoustic system after application of a Craig-Bampton reduction (13) may contain many DOF at the structural-acoustic interface. In or-

der to reduce the number of interface DOF, one could assume that only the acoustic domain is subject to an important modification of the pressure field at the fluid-structure interface when interacting with a structural domain. In this assumption, the structural modes are solved *in vacuo* while the acoustic domain keeps the interface representation in physical domain.

This reduction can be performed as a hybrid modal-Craig-Bampton transformation. Modal on the structural side, and Craig-Bampton on the acoustic side. The combined transformations can be summarized as:

$$\begin{aligned} u &\simeq \Phi_s u_m \\ p &\simeq L_f T_{CB,f} p_{CB} \end{aligned} \tag{14}$$

Inserting this transformation in equation (1), the resultant equation of motion of the vibro-acoustic assembly is:

$$\begin{bmatrix} I_s & 0 \\ C_{sf,m,CB}^T & M_{CB,f} \end{bmatrix} \begin{bmatrix} \ddot{u}_m \\ \ddot{p}_{CB} \end{bmatrix} + \begin{bmatrix} \Omega_s^2 & -C_{sf,m,CB} \\ 0 & K_{CB,f} \end{bmatrix} \begin{bmatrix} u_m \\ p_{CB} \end{bmatrix} = \begin{bmatrix} f_{m,s} \\ f_{CB,f} \end{bmatrix} \tag{15}$$

The number of DOF of the assembled system is reduced in the number of interface displacements u_b respect to the Craig-Bampton coupling (see section 2.4). At the same time, in situations when a structural domain is fully embedded in an acoustic subdomain, meaning that all structural DOF are connected to acoustic DOF, this method prevents the suppression of all internal structural DOF.

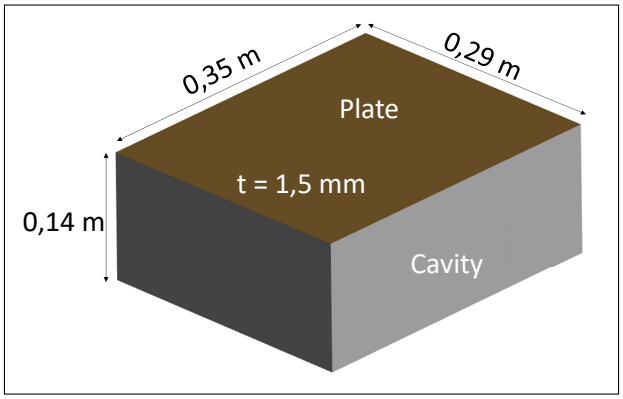

Figure 2. Cavity-plate example.

3.2 Symmetrization of vibro-acoustic systems

A new symmetrization method is proposed in this section, that intends to be general for all the presented vibro-acoustic coupling methods. When applying the vibro-acoustic interaction, the exhibited modes of the system may differ from the individual uncoupled structural and acoustic modes. For that reason, it is convenient to obtain the coupled vibro-acoustic modes for a better understanding of the global behavior.

In the previous sections, different reductions were performed over the structural and acoustic domains separately, so the vibro-acoustic equations of motion were obtained (1), (4), (7), (13) and (15). All of them are non-symmetric, which makes it more difficult to solve for the eigenvalues and eigenvectors of the system.

A method for turning the Equation of motion (4) symmetric was introduced in [6]. The full method is summarized in the book [4]. However, this method involves the inversion of the eigenvalue matrix of the acoustic domain, which may be singular due to acoustic static modes. The new symmetrization method avoids this inversion.

Consider the equation of motion in physical domain (1). This equation can be rendered symmetric by multiplication of T_{sym} on the left of the equation.

$$T_{\text{sym}} = \begin{bmatrix} K_s M_s^{-1} & 0 \\ -C_{sf}^T M_s^{-1} & I_f \end{bmatrix} \tag{16}$$

The resultant equation of motion is symmetric:

$$\begin{bmatrix} K_s & 0 \\ 0 & M_f \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} K_s M_s^{-1} K_s & -K_s M_s^{-1} C_{sf} \\ -C_{sf}^T M_s^{-1} K_s & C_{sf}^T M_s^{-1} C_{sf} + K_f \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} K_s M_s^{-1} f_s \\ -C_{sf}^T M_s^{-1} f_s + f_f \end{bmatrix} \tag{17}$$

This procedure can be applied to Equations (4), (7), (13) and (15), by using the following modifications on the symmetrization matrix (16).

$$T_{\text{sym,m}} = \begin{bmatrix} \Omega_s^2 & 0 \\ -C_{sf,m}^T & I_{m,f} \end{bmatrix} \tag{18}$$

$$T_{\text{sym,ms}} = \begin{bmatrix} \Omega_s^2 \left(I_{s,m} + M_{s,am}\right)^{-1} & 0 \\ -C_{sf,m}^T \left(I_{s,m} + M_{s,am}\right)^{-1} & I_{m,f} \end{bmatrix} \tag{19}$$

$$T_{\text{sym,CB}} = \begin{bmatrix} K_{CB,s} M_{CB,s}^{-1} & 0 \\ -C_{sf,CB}^T M_{CB,s}^{-1} & I_{CB,f} \end{bmatrix} \tag{20}$$

$$T_{\text{sym,mCB}} = \begin{bmatrix} \Omega_s^2 & 0 \\ -C_{sf,m,CB}^T & I_{CB,f} \end{bmatrix} \tag{21}$$

The modes of the symmetric equation can be obtained by solving the associated eigenvalue problem.

$$K_{\text{sym}} \begin{bmatrix} \phi_u \\ \phi_p \end{bmatrix} = \omega^2 M_{\text{sym}} \begin{bmatrix} \phi_u \\ \phi_p \end{bmatrix} \tag{22}$$

This methodology, when implemented, allows to compute the eigenvalues of a symmetric system of equations using a symmetric eigenvalue solver, which is more computationally efficient than an unsymmetric.

4. EXAMPLES AND DISCUSSION

4.1 Cavity-plate system

The example presented in this section is taken from [4], and it is depicted in Figure 2. This example exhibits the limitations of using the modal coupling method via uncoupled modes in vibro-acoustics. In the original problem, only the modal coupling (MC) and the modal coupling with static corrections (MC+SC) are compared respect to the physical coupling. Furthermore, the results are confronted using a global indicator (mean square pressure), excluding a comparative analysis of the local vibro-acoustic response.

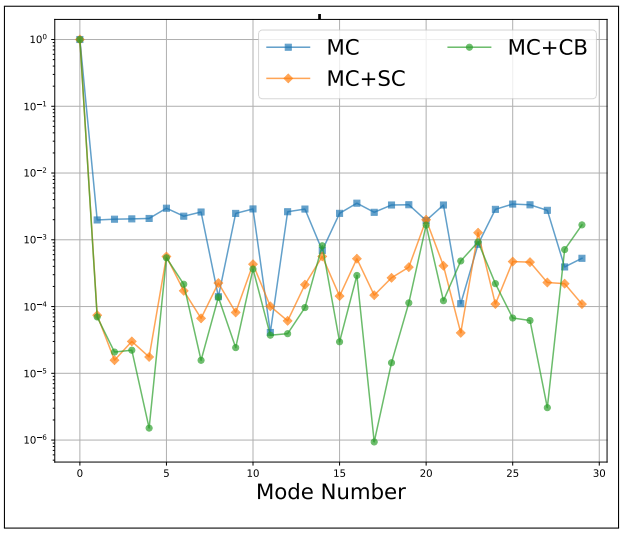

Figure 3. Relative error [-] in prediction of vibro-acoustic natural frequencies in the case of using air. Results are relative to the physical coupling.

The material and model properties can be found in Table 1. The example is showcased using two acoustic materials: air and water. In the case of air, the assumption of uncoupled *in vacuo* structural modes is reasonably good, while for the water case, it is not.

Table 1. Acoustic and structural model properties

	Cavity	Plate
Type	3D solid	3D shell
Nodes	19 x 16 x 4	19 x 16
ρ_{air} [kg/m^3]	1.225	2770
ρ_{water} [kg/m^3]	998.2	2770
c_{air} [m/s]	346.25	
c_{water} [m/s]	1482.13	
E [GPa]; ν		71; 0.33

The methods hereby compared are the modal coupling (MC), modal coupling with pseudo-static corrections (MC+SC) and the modal coupling with Craig-Bampton reduction in the acoustic side (MC+CB). All of them are compared with the physical coupling (PC). The reason why the Craig-Bampton method applied to both sides (CB) is not included, is because the structural domain is fully embedded in the acoustic domain, which makes the internal DOF set to be empty, making the example unsuitable to compute structural internal modes.

4.1.1 Acoustic material: air

In this example, the acoustic material used is air (see properties in Table 1). The natural frequency error is shown in Figure 3. As it was seen in the example from [4], the vibro-acoustic natural frequencies are well obtained using modal coupling (MC), exhibiting a relative error under 10^{-2}. The performance is however improved using the pseudo-static corrections (MC+SC), and even more using the modal coupling in structural and Craig-Bampton in acoustic side (MC+CB).

A detail on the CPU time and memory peak allocation is shown in Table 2. The physical coupling (PC) is taken as reference. As one can observe, MC is the most efficient one, but at the expense of exhibiting poorer results. In this specific study, the MC+CB coupling is more efficient than MC+SC. This may occur due to the inversion of the structural and acoustic stiffness matrices, a step that is needed in MC+SC, when in MC+BC it is only needed for the acoustic domain and with less DOF due to the inversion of the internal stiffness matrix. The processes included

in the computation of vibro-acoustic modes involve computing uncoupled modes, building and turning the vibro-acoustic matrices symmetric, and solving the symmetric vibro-acoustic eigenvalue problem.

Table 2. Memory and time consumption of different vibro-acoustic methods for vibro-acoustic mode computation. CPU time in seconds and RAM peak in MB.

	CPU	RAM	CPU [%]	RAM [%]
PC	18,26	250,83	100,00	100,00
MC	0,68	5,2	3,72	2,07
MC+SC	11,21	141,08	61,39	56,25
MC+CB	1,64	41,6	8,98	16,58

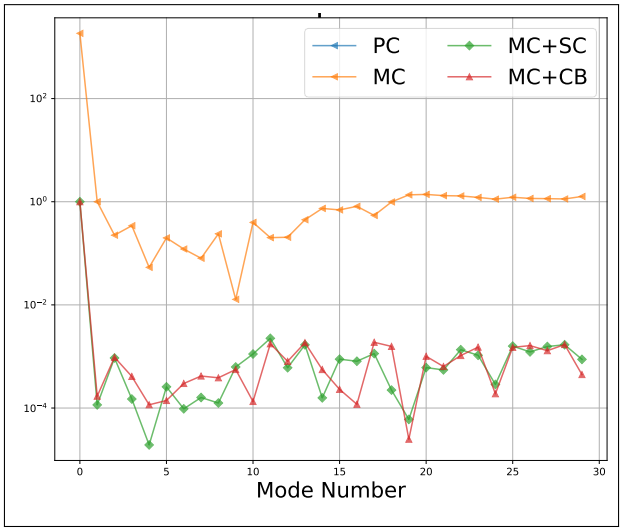

Figure 4. Relative error [-] in prediction of vibro-acoustic natural frequencies in the case of using water. Results are relative to the physical coupling.

4.1.2 Acoustic material: water

In this example, the fluid is substituted by water. The assumption of *in vacuo* structural modes and rigid walled acoustic modes is no longer valid, as it is shown in Figure 4, where the relative error of the MC method is two orders of magnitude greater than the MC+SC and MC+CB. The relative error of the latter two is lower than 10^{-2}.

The Modal Assurance Criterion (MAC) [7] is used in the acoustic DOF, once the vibro-acoustic modes are solved, to show the accuracy of the different methods in terms of acoustic modal shapes. In Figure 5 the MC and MC+SC methods cannot accurately describe the corresponding modal vectors of the acoustic domain in the vibro-acoustic environment, when the MC+CB method succeeds. The MAC matrices of the different methods are compared to the MAC matrix of the physical coupling (PC), which exhibits the reference MAC values.

The vibro-acoustic Mode 15 is depicted in Figure 6. Only the MC+CB method (right) is able to predict the shapes of the structural and acoustic models as the PC. The MC+SC method fails in predicting the acoustic shape, especially at the fluid-structure interface.

5. CONCLUSIONS

Several reduction methods for computing vibro-acoustic modes have been presented, including a novel approach derived from the previously developed vibro-acoustic Craig-Bampton method. This new method retains only the structural components in the modal domain while applying reduction, keeping the acoustic interfaces in physical representation. Additionally, a symmetrization technique has been introduced to reformulate the inherently unsymmetric vibro-acoustic equations into a symmetric form. The performance of these methods has been evaluated using a cavity-plate system, considering two fluid loading scenarios: air and water.

The results show that modal coupling using uncoupled modes is accurate in predicting natural frequencies when light fluids like air are used, but it is less accurate for denser fluids such as water. Pseudo-static corrections improve the accuracy of vibro-acoustic natural frequencies but fail to capture modal shapes accurately. In these scenarios, the modal coupling with Craig-Bampton reduction in the acoustic domain proves to be effective.

The methods were also compared in terms of CPU and RAM requirements for calculating vibro-acoustic modes. Modal Coupling (MC) is the most efficient but the least accurate. Modal Coupling with Static Corrections (MC+SC) requires more computational resources, offering better accuracy in natural frequencies. The Modal Coupling with Craig-Bampton reduction (MC+CB) provides a balanced approach, offering better computational performance than MC+SC while maintaining higher ac-

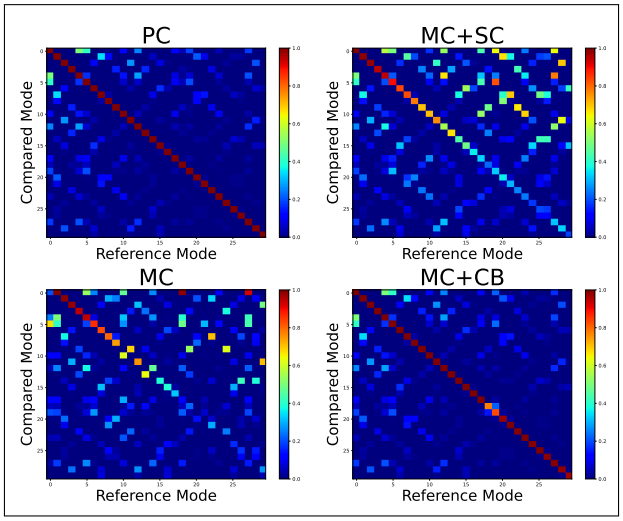

Figure 5. MAC values of the acoustic DOF within the vibro-acoustic modes. Water is the fluid.

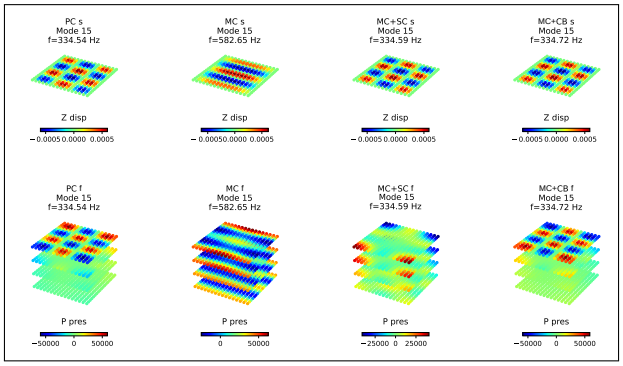

Figure 6. Vibro-acoustic mode 15. Case of water as acoustic medium. On the first row, the vibro-acoustic modes corresponding to the structural domain. On the second row, the acoustic side. Each method is presented: Physical Coupling (PC), Modal Coupling (MC), MC+static corrections (MC+SC), MC+Craig-Bampton (MC+CB).

curacy.

6. ACKNOWLEDGMENTS

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