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ASSESSING THE CONSISTENCY OF NUMERICAL MODELS FOR WIND INSTRUMENT IMPEDANCE

Augustin Ernoult[1*] **Jérémy Cabaret**[2] **Juliette Chabassier**[3]
Tom Colinot[4] **Jean-Pierre Dalmont**[5] **Jean-Baptiste Doc**[6]
Vincent Fréour[7] **Romain Viala**[8]

[1] Sorbonne Université and Inria Paris, CNRS, Institut Jean Le Rond d'Alembert, LAM team, 75005 Paris, France
[2] Institut Technologique Européen des Métiers de la Musique, ITEMM, 71 avenue Olivier MESSIAEN, 72000 Le Mans, France
[3] Modartt, 9 avenue de l'Europe, Bâtiment B18, 31520 Ramonville Saint Agne, France
[4] Buffet Crampon, 5 Rue Maurice Berteaux, 78711 Mantes-la-Ville, France
[5] Laboratoire d'Acoustique de l'Université du Mans UMR CNRS 6613, Le Mans Université, 72085 Le Mans, France
[6] Conservatoire National des Arts et Métiers, 292 rue Saint-Martin, 75141 Paris Cedex 03, France
[7] YAMAHA Corporation, Research and Development Division, 10-1 Nakazawacho, Hamamatsu, 430-8650 Shizuoka
[8] Department of Applied Mechanics, Université de Franche-Comté, CNRS, Institut FEMTO-ST, F-25000 Besançon, France

ABSTRACT

Numerical simulation of the input impedance of wind instruments is the basis of much work in music acoustics. However, the accuracy and reproducibility of such simulations across different contributors remain little documented. This study aims to benchmark the reliability of numerical simulations for pipe impedance, focusing on simple geometries such as cylindrical and conical pipes, and to identify key factors influencing simulation variability. A collaborative approach was adopted, involving multiple contributors employing various numerical models, including TMM, FEM (1D or 3D), etc., and eventually different physical models for the propagation, the boundary conditions, and the thermo-viscous effects. Significant variability in simulation results was observed due to differences in implementation, numerical precision, and boundary condition modelling. Incomplete convergence in some methods also contributed to discrepancies. Despite these challenges, the collaborative work and exchanges reduced variability and improved consistency across contributors. The study highlights the importance of such benchmark and the interest to adopt good practices including verification of the numerical tools through comparative analysis and identification of convergence criteria. This work provides a framework for improving the reproducibility an accuracy of numerical simulations in musical acoustics context.

Keywords: *wind instrument, impedance, simulation benchmark, linear acoustics*

*__Corresponding author__: augustin.ernoult@inria.fr.

1. INTRODUCTION

The input impedance of wind instruments is a fundamental quantity in musical acoustics. It serves as a basis for most physical modeling approaches, including sound synthesis, time-domain simulations, automatic design, etc. In practice, researchers often implement their own numerical models to compute this quantity, adapting them to the specific constraints of the studied objects or their research objectives.

However, the development of a new numerical model can introduce several sources of inaccuracy: the omission of physical phenomena that may significantly affect results (e.g. humidity rate), implementation errors, numerical approximations (e.g. rounding), and eventually artifacts due to spatial discretization. To address such issues, the scientific computing community has established the Verification and Validation (V&V) framework, particularly in fields such as Computational Fluid Dynamics (CFD) and Mechanical Engineering [1, 2].

Verification aims to ensure that the model accurately solves the underlying mathematical equations. This step typically involves comparisons with analytical solutions when available, or with results from independent numerical models in configurations where both are expected to be valid. Validation, on the other hand, focuses on assessing how well the model reproduces experimental data, accounting for uncertainties due to variability in geometry, material properties, boundary conditions, and measurement procedures.

In musical acoustics it is common to compare simulations and experiments (e.g. [3, 4]). However, the verification step is less systematically applied. Often, studies focus on validating new developments in configurations where existing models fail (e.g. [5]), without first verifying consistency between models on simpler, well-understood test cases.

This study specifically addresses the *verification* step, by comparing impedance results computed by different researchers using independent implementations. The focus is on simple geometries—closed or open cylindrical and conical tubes—for which reference or analytical solutions exist and for which numerical models are expected to agree. By identifying and resolving discrepancies in this controlled context, we aim to assess the robustness of current modeling practices and contribute to improving the reliability and reproducibility of impedance computations in musical acoustics.

2. CONSTRUCTION OF THE DATA SET

2.1 Configurations

The cases studied are 180 mm long pipes. This relatively short length gives the possibility to use heavy simulation methods (e.g. 3D-FEM) with a relatively low computation cost even at “high” frequency (5 kHz). A cylindrical pipe with a 14 mm internal diameter has been simulated with three radiation conditions: closed, open with infinitly thin wall (unflanged) and open with 7 mm wide wall (circular flanged). A closed conical pipe with an internal diameter from 10 mm to 22.6 mm (half angle: 2°) have been also simulated. For the latter, contributors assumed either plane or spherical wavefronts, which can be seen as different shapes of boundary conditions (plate or spherical caps). The wall material is assumed perfectly smooth, rigid, non-porous and a perfect conductor of heat.

The values of the physical quantities (speed of sound, air density, etc) are imposed. They are calculated, from [6, Chap.5.5.2], for a temperature of 25°C. For the lightest calculation methods, the recommended frequency axis for simulation is set to [0.2 Hz, 20 kHz] with a 0.2 Hz step. However, for methods necessitating high computation cost, a multiple of the frequency step could be used. Only 5 arbitrary frequencies was imposed for comparison ($f_{obs} \in [20, 100, 500, 1000, 5000]$ Hz) (see Sec.2.3).

2.2 Numerical methods

The input impedances of this set of configuration have been simulated by seven contributors, named A to F, using their own software or implementation of the models. For each simulation, the resulting impedance $\tilde{Z}$ is shared.

To simulate the wave propagation, contributors A to D use the one-dimensional (1D) telegrapher’s equations including the visco-thermal losses with the Zwikker and Kosten expression (see e.g. [6, Chap.5]) or approximation proposed by Keefe [7]. The closed condition is modelled by an infinite impedance. For the open conditions, a radiation impedance computed with the expressions proposed by Silva [8] and Dalmont [9] are used. Contributors A, B and D used their own implementations of the Transfer Matrix Method (TMM). This technique combines solutions of the telegrapher’s equation for cylinders (exact solution) or cones (approximation). For cone it is possible to refine the solution, by subdividing the geometry in small cones. Contributor C uses the software Openwind [10] given the possibility to solve the equations with TMM or 1D-finite elements method (1D-FEM) [11], which converges to the exact solution if the mesh is refined enough.

Contributor E used its own implementation of the multi-modal (MM) approach [12]. The thermo-viscous effects are taken into account as a modification of each mode shapes through the expressions proposed by [13]. For open pipes, the radiation domain is modeled using a wider pipe surrounding the actual pipe studied [14].

Contributor F used the software Montjoie [15] to solve 3D (or 2D axi-symmetric when it is possible) acoustic propagation with FEM. The thermo-viscous effects are taken into account using a two steps simulation (sequential linearized Navier-Stokes model or SLNS) [16], or effective boundary conditions based on Cremer approach [17]. More detailed are given in [18]. For open pipes, a radiation domain bounded by perfectly-matched-layers condition (PML) is added.

In order to study the practical use of simulation, each contributor was responsible for choosing the spatial discretisation, making his/her own compromise between accuracy and computation time. This choice is strongly influenced by the scientific culture of the contributor (numerician, experimentalist, etc.) or by the intended application (model comparison, manufacturing application, etc.).

2.3 Distance quantification

In numerical studies, it is common to use the L2-norm to quantify the distance between two simulations. However, the presence of peaks in the impedance curve can lead to large deviations from small variations (e.g., a slight shift in a resonance peak). To mitigate this, the reflection function is used:

$$\mathcal{R} = \frac{Z-1}{Z+1}. \tag{1}$$

where $Z = \tilde{Z}S/(\rho c)$ is the input impedance scaled by the lossless characteristic impedance. The distance between two simulations (p, q) is then computed as the L2-norm at the five arbitrary imposed frequencies $f_{obs}^{(i)}$:

$$d_R^{(p,q)} = \frac{\sum_i^N |\mathcal{R}_p(f_{obs}^{(i)}) - \mathcal{R}_q(f_{obs}^{(i)})|^2}{\sum_i^N |\mathcal{R}_p(f_{obs}^{(i)})|^2}. \tag{2}$$

For two equivalent models, this distance should reach machine precision (here, 10^{-6}).

From a musical perspective, the "modal characteristics" are also analyzed for simulations with sufficiently fine frequency resolution. Both resonances and anti-resonances are characterized by their frequency and magnitude. For the i-th (anti-)resonance peak, the frequency $f^{(i)}$ is defined as the point where the impedance phase crosses zero, and the magnitude $a^{(i)}$ as the modulus of the impedance (or admittance) at that frequency. Simulations are compared through their deviation from a reference value, expressed in cents for frequencies:

$$d_f^{(i,p)} = 1200 \log_2 \left(\frac{f_p^{(i)}}{f_{ref}^{(i)}} \right) \tag{3}$$

and in decibels for amplitudes:

$$d_a^{(i,p)} = 20 \log_{10} \left(\frac{a_p^{(i)}}{a_{ref}^{(i)}} \right). \tag{4}$$

The reference values are taken from the analytical solution when available, or otherwise from the average across all simulations.

3. ANALYSIS

3.1 Closed cylinder

The first test case is a closed cylindrical tube. The "closed" condition is numerically defined by enforcing zero normal velocity at the cap wall, corresponding to zero flow in 1D, or equivalently, infinite acoustic impedance. This assumption neglects the thermo-viscous effects at the cap wall.

For this geometry, the telegrapher's equation admits an analytical solution including thermo-viscous losses. With the above boundary condition, the input impedance reads:

$$Z_{\text{closed cyl}} = Z_{cc} \coth(\Gamma L) \tag{5}$$

where L is the cylinder length, $Z_{cc} = \sqrt{Z_v/Y_t}$ is the complex characteristic impedance, and $\Gamma = \sqrt{Z_v Y_t}$ the complex wave number. This configuration is well suited for validation, as all TMM implementations should yield identical results, apart from the specific thermo-viscous formulations, since this method is formally similar to the analytical solution Eq. (5) in this configuration.

Contributors A, B, and C used the ZK formulation. C also tested the first- and second-order Keefe approximations (Keefe 1 and 2); D used Keefe 2. Contributor E implemented the multimodal (MM) model, and F used 3D FEM with 2D-axisymmetric SLNS resolution.

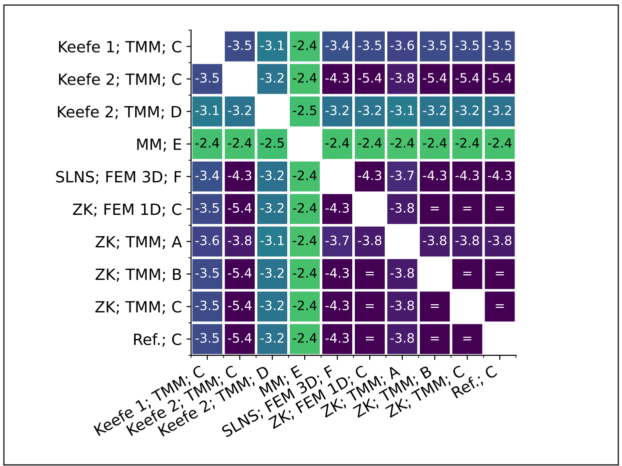

Figure 1. Closed cylinder. Relative distance matrix of the reflection function at reference frequencies ($f \in [20, 100, 500, 1000, 5000]$ Hz) in log scale ($\log_{10}(d_{\mathcal{R}})$). The symbol "=" denotes distances below machine precision (10^{-6}).

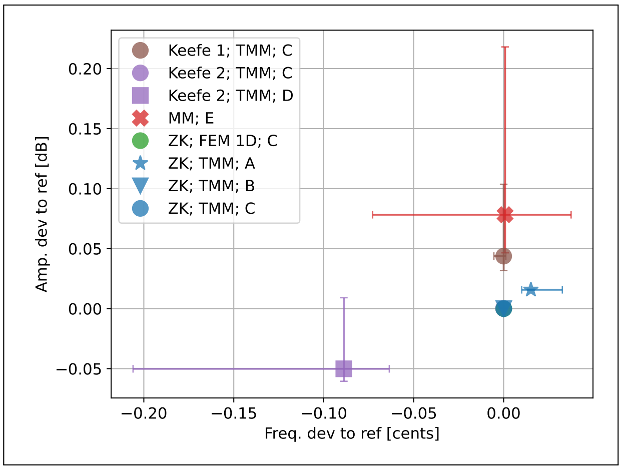

Figure 2. Closed cylinder. Deviation from the analytical modal characteristics (Eqs. 4, 3). Markers indicate median values; error bars denote the full range. Marker shapes identify contributors; colors indicate models and numerical methods.

Figure 1 presents the distance matrix for the reflection function (Eq. 2), and Figure 2 shows deviations in modal characteristics over 10 peaks (5 resonances, 5 antiresonances). In this second Figure, the 3D FEM results are omitted due to insufficient frequency resolution for modal fitting.

From a musical point of view, all simulations exhibit good overall agreement, with frequency deviations remaining within ± 0.2 cents and amplitude deviations within ± 0.2 dB. The results provided by contributors B and C (ZK-TMM, 1D FEM and analytical solutions) are consistent to within machine precision across all metrics. Similarly, the 3D FEM results from contributor F and the second-order Keefe approximation from contributor C are nearly indistinguishable, with reflection distances below 10^{-4}%, frequency deviations below 10^{-3} cents and amplitude below 10^{-3} dB. Contributor A shows slightly larger but still minor discrepancies, attributed to a numerical issue in the implementation of physical quantities, after the sharing of the data.

The impact of thermo-viscous modeling choices is clearly visible. The multimodal (MM) model introduces more noticeable deviations, in reflection distance ($d_{\mathcal{R}} \approx 4 \cdot 10^{-3}$), in frequency ($d_f \approx 0.08$ cents) and amplitude ($d_a \approx 0.2$ dB). The first-order Keefe approximation also leads to moderate differences. By contrast, contributor D, who used the second-order Keefe approximation, exhibits unexpected deviations (-0.05 dB, -0.1 cents, $d_{\mathcal{R}} \approx 7 \cdot 10^{-4}$) not observed in C's results with the same model, suggesting a probable but yet-unidentified implementation issue.

3.2 Other geometries

Similar figures and analyses can be produced for other test cases. However, for clarity, the agreement between simulations is here assessed through statistical indicators.

Table 1. Mean and maximal values of the logarithm of the distance $d_{\mathcal{R}}$ between simulations for each configuration.

Case	Mean $\log_{10}(d_{\mathcal{R}})$	Max $\log_{10}(d_{\mathcal{R}})$
Cyl. C.	-4.0	-2.4
Cyl. Unfl.	-3.0	-2.4
Cyl. Wall	-2.7	-2.2
Cone Pl.	-2.1	-1.7
Cone Sph.	-3.3	-3.0

Table 1 shows the mean and maximal values of

Table 2. Root mean square (RMS) and maximal values of the deviations d_f (in cents) and d_a (in dB) between simulations for each configuration.

Case	RMS d_f	Max d_f	RMS d_a	Max d_a
Cyl. C.	0.05	0.24	0.05	0.22
Cyl. Unfl.	0.07	0.62	0.05	0.18
Cyl. Wall	0.20	0.60	0.11	0.32
Cone Pl.	0.8	5.7	0.4	2.2
Cone Sph.	0.04	0.17	0.04	0.07

$\log_{10}(d_{\mathcal{R}})$ for each configuration. Table 2 provides the root mean square (RMS) and maximal deviations of the modal characteristics between simulations.

Agreement between results decreases as the simulated configuration becomes more complex. The best match is observed for the closed cylinder (Cyl. C.), followed by the unflanged open cylinder (Cyl. Unfl.) and the open cylinder with finite wall width (Cyl. Wall). The largest deviations occur for the closed cone with plane wavefronts (Cone Pl.), with errors reaching a few cents, a few dB, and around 1% for $\mathcal{R}$. Conversely, the closed cone simulated with spherical wavefronts (Cone Sph.) shows excellent agreement, comparable to the closed cylinder. This may be due to the fact that this case was only simulated by contributors using software that had already been verified.

4. CONCLUSIONS

Even though simulations are generally consistent from a musical perspective ($d_f < 1$ cent, $d_a < 1$ dB), this study shows that achieving strict numerical consistency remains challenging. Variations arise when different researchers use distinct physical models and implementations, even for simple cases.

The presented values already reflect extensive discussions, data comparisons, and issue resolution. For instance, deviations for the conical pipe reached 20 cents in the initial dataset. Despite these efforts, increasing model complexity leads to growing discrepancies, highlighting the sensitivity to boundary conditions and the importance of careful numerical implementation. The most consistent results were observed for simple cases and by contributors with strong numerical skills.

Beyond the analyses of the presented implementations, this study emphasizes the importance of verification when developing new models or tools. This includes comparing results with existing tools on simple cases for which both are valid, identifying sources of discrepancies (e.g., convergence, simplifications), and quantifying deviations. In simulations requiring discretization (e.g. sliced conical pipes in TMM), the convergence behavior should be systematically evaluated and reported.

Promoting such practices is key to improving reproducibility and accuracy in simulations. A practical solution for sharing the numerical results of this study is under consideration, which should allow anyone to *verify* their tools by comparing them with this dataset and then to carry out validation with experiments.

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