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## BOUNDARY ELEMENT METHOD WITH VISCO-THERMAL LOSSES: DEVELOPMENT, IMPROVEMENTS AND APPLICATIONS

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### ABSTRACT

Sound waves are affected by viscous and thermal losses, which manifest mainly inside a thin region close to the domain boundaries, the boundary layers [1,2]. Losses at boundaries cannot usually be neglected when modelling small or intricate devices, such as micro-transducers and metamaterials [3]. Over the last years, several numerical methods have been proposed and implemented to include viscothermal losses. Some methods assume restrictive hypotheses, as in the cases of the low reduced frequency (LRF) model by Beltman or the boundary layer impedance (BLI) reported by Berggren [4,5]. A full implementation using the Finite Element Method (FEM) was described by Malinen et al. and later included in commercial FEM software [6]. The Boundary Element Method (BEM) has also been adapted by Cutanda Henriquez and other authors as a full implementation of viscothermal losses, and it is employed as a research tool originally based on the open-source software OpenBEM [7,8,9]. This contribution summarizes the work done on the BEM with losses. Current work on the method will be described.

**Keywords:** *Boundary Element Method, Viscous and thermal losses.*

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### 1. INTRODUCTION

In the past two-three decades, the state of the art regarding the modeling of acoustics with viscous and thermal losses has seen a variety of new approaches, as described in the abstract. Several of these modeling techniques have been implemented in commercial software [10].

However, the user is left with a choice between methods with restrictive hypotheses such as the low reduced frequency model and the boundary layer impedance [4,5], or full methods with the only restrictions of linearity and no flow, existing in FEM and BEM [6,7,8,9]. The latter methods have the drawback of a heavy computational burden which places many relevant simulation problems almost or totally out of reach. Problems involving relatively large setups, high frequencies or intricate geometries with narrow passages cannot be treated with methods with restrictions [11,12].

### 2. BEM WITH LOSSES: REDUCED METHODS

There has been research in the past few years towards reducing the computational burden of the BEM with viscothermal losses [13-14]. The BEM has the advantage of being based on a discretization of the domain boundary, not the domain itself as in FEM. This means that the very thin boundary layers do not need to be represented with tiny elements as is the case in the full FEM with losses. However, the BEM presents full, frequency dependent matrices that thwart its initial reduced nature. Ref. [13] presents an improved version of the BEM with losses in [7,9], where the coordinate changes have been simplified and the system of equations reduced to the same size of the lossless BEM. Ref. [14] introduces a Model Order Reduction (MOR) technique to a BEM implementation





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with losses, this time based on the more simplified Boundary Layer Impedance.

New research is ongoing with the aim of extending MOR to full BEM with losses, possibly combining it with fast multipole methods.

### 3. CONCLUSIONS AND FUTURE TRENDS

There is a need for fast, reliable and not restricted modeling techniques that can deal with problems yet unreachable, such as intricate metalmaterials, minute transducers and acoustic devices such as hearing aids. Numerical optimization also needs such fast simulations to run and would open losses-controlled devices to numerical design.

The future probably belongs to a combination of advanced MOR methods and a yet more physics-oriented treatment of losses. Brute-force discretizations such as in the full FEM, or full BEM, with losses are inefficient. The BLI approach makes use of a simplified description of the boundary layers to reduce them to boundary condition. The LRF takes advantage of the behavior of the fluid within certain geometries. In this line, we need methods that cleverly employ the physical behavior of the fluid in an efficient description with less restrictive constraints [10].

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