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DIFFUSION EQUATION-BASED ESTIMATION OF SPATIALLY NON-UNIFORM SURFACE ABSORPTION COEFFICIENTS

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ABSTRACT

The acoustic diffusion equation provides a generalized statistical model of room acoustical fields, allowing for non-uniform distributions of sound energy and spatially varying surface absorption coefficients. Recent studies have shown that the diffusion equation can be used to estimate spatially uniform, frequency-dependent absorption coefficients from in situ measurements. In this work, we extend this approach to handle spatially non-uniform absorption by using the ratio of intensity to energy density. Numerical simulations demonstrate the feasibility of this method when used to estimate frequency-dependent, spatially non-uniform absorption coefficients in a room.

Keywords: *Acoustic diffusion equation, Non-uniform absorption coefficient estimation, Intensity - Energy density ratio, Field index.*

1. INTRODUCTION

Accurate descriptions of sound-absorbing materials are essential for reliable computer simulations of room acoustics [1]. While absorption coefficients can be measured in controlled environments, e.g., reverberation chambers [2], their applicability diminishes once materials are installed in a room. The angle-dependent nature of many sound-absorbing materials means that their absorption coefficients can depend on the room in which they are installed. Consequently, several in-situ measurement methods for measuring absorption coefficients can be found in the literature [3].

The diffusion equation has recently been used for in-situ estimation of spatially uniform, frequency-dependent absorption coefficients in a small room [4]. While the diffusion equation predicts a spatially uniform reverberation time (for an uncoupled room), it also predicts a non-uniform energy flux. The non-uniformity of the energy flux is utilized in this work to estimate spatially non-uniform absorption coefficients in a small room. The proposed inverse method employs eigenvalue analysis of a diffusion equation model of a room to estimate unknown surface absorption coefficients.

This paper is organized as follows: Section 2 provides background information relevant to the work. Section 3 presents the diffusion equation, a numerical implementation, and the post-processed solutions. The method for estimating the absorption coefficient is introduced in Sec. 4, and a simulated room with four different boundary conditions is presented as a test case in Sec. 5. The results of the absorption coefficient estimations are discussed in Sec. 6. Limitations and future studies of the proposed method are discussed in Sec 7, and Sec. 8 outlines the conclusions.

2. LITERATURE REVIEW

The normal incidence impedance of a material sample can be measured using an impedance tube [5], and a random incidence absorption coefficient can be computed from the normal impedance. However, measurements in an impedance tube preclude edge effects and may include unwanted installation effects. Thus, reverberation chambers are often used to measure the random incidence ab-

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sorption coefficients of large material samples [2]. Due to room and installation effects, the coefficients measured in a reverberation chamber may not reflect the actual absorption of a material placed in a room. Thus, in-situ methods can provide additional insight into the absorption characteristics of materials.

While many in-situ methods can be found in the literature [3], we limit our scope to those that consider intensity. Tijs and Druyvesteyn [6] propose and validate the use of several spatially distributed intensity measurements to determine the absorption coefficient of a material sample. Kuipers *et al.* [7] numerically verify a plane wave-based intensity method for measuring absorption coefficients of a material sample. Since the intensity over a sample can be non-uniform [8], intensity-based methods often require spatial averaging. Vigran [9] introduces a *field index*, which is the ratio of the intensity to the product of the energy density and the speed of sound. Wijnant *et al.* [10] note that the field index could be used as a measure of sound absorption. A review of the literature suggests that this observation has not been further explored.

Ollendorff [11] proposes a diffusion model of enclosed sound fields, and Picaut *et al.* [12] provide a mathematical derivation of the diffusion equation. Several researchers have developed and evaluated models based on the diffusion equation. For example, Foy *et al.* [13] compare predictions of reverberation time from two diffusion equation models to measured reverberation time data. Billon *et al.* [14] propose a boundary condition for the diffusion equation based on Eyring's absorption coefficient, and Jing and Xiang [15] propose a modified boundary condition for the diffusion equation. Prinn *et al.* [4] use the diffusion equation to estimate spatially uniform absorption coefficients of the surfaces of a measured room.

The diffusion equation allows for the incorporation of random incidence absorption coefficients that can vary spatially. By solving the diffusion equation, we can predict various parameters such as reverberation time, energy density, and intensity. This study utilizes the capability to model spatially non-uniform absorption along with predictions of reverberation time and field index to estimate absorption coefficients across multiple surfaces. The approach is demonstrated using low-frequency finite element simulations.

3. THEORY

In this section, we introduce the diffusion equation, a finite element method implementation, and the processing of the first eigenvalue and eigenvector of the diffusion equation.

3.1 Diffusion equation

Following the derivation by Picaut *et al.* [12], the homogeneous diffusion equation can be written as

$$\frac{\partial w(t,\mathbf{x})}{\partial t} - D\,\nabla^2 w(t,\mathbf{x}) = 0\,, \tag{1}$$

where $w(t,\mathbf{x})$ is the sound energy density, which depends on time t and spatial position $\mathbf{x}$. The diffusion coefficient $D = \bar{l}c/3$ is given in terms of the speed of sound c and mean free path $\bar{l}$. The mean free path is given by $\bar{l} = 4V/S$, where V is the room volume and S is the surface area. [1]

To complete the model, we make use of the sound absorption coefficient boundary condition proposed by Jing and Xiang [15]

$$D\,\frac{\partial w(t,\mathbf{x})}{\partial n} = \frac{-c\,\alpha}{2(2-\alpha)} w(t,\mathbf{x})\,, \tag{2}$$

where $\partial/\partial n$ is the normal derivative, and α is a random incidence absorption coefficient. Note that the absorption coefficient can be spatially dependent, a fact that forms the basis of this work.

Choosing a solution that decays exponentially, we write the sound energy density as [16, Eq. (5.6)]

$$w(t,\mathbf{x}) = \phi(\mathbf{x})\mathrm{e}^{-\lambda t}\,, \tag{3}$$

with amplitude (eigenvector) ϕ and damping constant (eigenvalue) λ.

3.2 Numerical implementation

Equations (2) and (3) are used in combination with the finite element method to reformulate Eq. (1), resulting in the eigenvalue problem

$$\left[D\,\mathbf{K} + \sum_{j=0}^{J} \frac{c\,\alpha_j}{2(2-\alpha_j)}\mathbf{C}_j - \lambda\mathbf{M}\right]\boldsymbol{\phi} = \mathbf{0}\,, \tag{4}$$

where $\mathbf{K}$, $\mathbf{C}$, and $\mathbf{M}$ are stiffness, damping, and mass matrices, respectively, j identifies an absorbing surface, λ is an eigenvalue, and $\boldsymbol{\phi}$ is a discretized eigenvector. The elemental stiffness, damping, and mass matrices are given by

$$\mathbf{K}_e = \int_e \nabla\mathbf{N}^T \cdot \nabla\mathbf{N}\,\mathrm{d}V\,, \tag{5}$$

[1] This mean free path is based on a diffuse field assumption.

$$\mathbf{C}_{e',j} = \int_{e'} \mathbf{N}^T \mathbf{N} \, \mathrm{d}S_j \,, \tag{6}$$

$$\mathbf{M}_e = \frac{1}{c^2} \int_e \mathbf{N}^T \mathbf{N} \, \mathrm{d}V \,, \tag{7}$$

where the subscript e represents an interior element, e' represents a boundary element, S_j is a sound absorbing surface, and $\mathbf{N}$ are interpolating shape functions. The elemental matrices are solved numerically and used to assemble $\mathbf{K}$, $\mathbf{C}$, and $\mathbf{M}$. (For details, see, e.g., Prinn [17]).

3.3 Post-processed quantities

The solution of Eq. (4) provides a set of eigenvalues and eigenvectors. We only concern ourselves with the first (smallest) eigenvalue, λ_0, and the first eigenvector, ϕ_0.

3.3.1 *Reverberation time*

An estimation of the reverberation time of an uncoupled room can be computed from the eigenvalue as follows:

$$\widehat{T}_{60} = \frac{2 \log(1 \times 10^3)}{\lambda_0} \,. \tag{8}$$

3.3.2 *Field index*

A normalized energy flux, or intensity, can be computed from the eigenvector, as follows

$$\widehat{\mathbf{I}} = -D \nabla \cdot \phi_0(\mathbf{x}) \,. \tag{9}$$

The intensity normal to an absorbing surface is of interest in this work. This is given by

$$\widehat{\mathrm{I}}_n = \widehat{\mathbf{I}} \cdot \mathbf{n} \,, \tag{10}$$

where $\mathbf{n} = (n_x, n_y, n_z)$ is an outwardly pointing unit normal vector. Finally, for the absorption coefficient estimation, the (normalized) intensity to energy density ratio, or field index [9], is defined as

$$\widehat{\mathrm{F}} = \frac{\widehat{\mathrm{I}}_n}{\phi_0 \, c} \,. \tag{11}$$

Note that the field index can be related to the absorption coefficient (as observed in Ref. [10]). This can be shown by replacing the energy density in Eq. (2) with the eigenvector. Thus, for the boundary condition considered,

$$\widehat{\mathrm{F}} \sim \frac{\alpha}{2(2-\alpha)} \,. \tag{12}$$

4. ESTIMATION METHOD

The impedance estimation method consists of two main steps. First, measurements of sound pressure and normal velocity are taken at one or more positions within a room, but close to the surfaces at which we would like to estimate the absorption coefficient. This is because the intensity and energy density normal to the surface are required. The reverberation time, intensity, and energy density at these measured positions are calculated. A field index is then computed from the intensity and energy density.

Second, a diffusion equation model of the room is generated, which incorporates unknown absorption characteristics. For example, if the surfaces of a room have one impedance but an absorbing material with a different impedance is placed on the floor, two frequency-dependent absorption coefficients need to be estimated. An eigenvalue analysis of the diffusion equation model is performed, from which the reverberation time and field index at the measurement positions can be computed. An optimization algorithm is subsequently employed to minimize the differences between the observed and simulated values, allowing for the estimation of the absorption coefficients.

4.1 Reference data

An impulse response measurement is made at each surface of interest. The impulse response's energy decay curve is computed using backward Schroeder integration, and the reverberation time is estimated from a straight-line fit to the energy decay curve between -5 dB and -35 dB.

Using two microphones placed normal to the surface of interest, the normal velocity can be estimated from the conservation of momentum, approximated by a finite difference scheme. The frequency domain pressure, P, and normal velocity, V_n, are computed via Fourier transformation. For a given frequency, ω, the time-averaged active normal intensity is given by

$$\mathrm{I}_n(\omega) = \mathrm{Re}\left(\frac{P(\omega) V_n^*(\omega)}{2}\right) \,, \tag{13}$$

and the normal energy density is given by (see, e.g., [18])

$$\mathrm{E}_n(\omega) = \frac{|P(\omega)|^2}{2\rho c^2} + \frac{\rho |V_n(\omega)|^2}{2} \,. \tag{14}$$

A normal field index can be computed from these two quantities, as follows:

$$\mathrm{F}(\omega) = \frac{\mathrm{I}_n(\omega)}{\mathrm{E}_n(\omega) \, c} \,. \tag{15}$$

4.2 Absorption coefficient estimation

A diffusion equation model of the room under test is generated, and an optimization algorithm is used to refine initial guesses of the unknown absorption coefficients to arrive at final estimates. For the optimization, the reverberation times and field index are compared.

If the absorption coefficient sought is spatially uniform, the field index is not computed. The reason is that the field index will be the same on all surfaces. If two absorption coefficients are needed, an average of the reverberation times measured close to each of the surfaces of interest is compared to the diffusion equation prediction. Noting that the diffusion equation predicts a spatially uniform reverberation time, the averaging can lead to improved estimates. For two unknown impedances, two field indices are measured (one for each surface of interest). The ratio of the field indices is used as a measure of the amount of absorption at each surface. The ratio of field indices for two unknown absorption coefficients is given by

$$\mathrm{G}_{2,1} = \frac{\mathrm{F}_{n,2}}{\mathrm{F}_{n,1}}\,, \tag{16}$$

where the subscripts indicate the first and second surfaces. A similar expression is used for the ratio of the diffusion equation field indices. If three or more surfaces are considered, G will be a vector containing the ratios of the field indices with respect to one of the surfaces. Note that using the field index ratio results in a fully determined optimization problem.

To demonstrate the optimization routine, we consider a problem with three unknown absorption coefficients. An array that contains the reverberation time and two field index ratios is generated. For each frequency, or frequency band, considered, the optimization problem reads

$$\tilde{\boldsymbol{\alpha}}(\omega) = \underset{\hat{\alpha}_1,\hat{\alpha}_2,\hat{\alpha}_3}{\arg\min} \|\mathbf{C}(\omega) - \widehat{\mathbf{C}}(\hat{\alpha}_j(\omega))\|_2^2. \tag{17}$$

where $\mathbf{C} = [T_{60}\ \mathrm{G}_{2,1}\ \mathrm{G}_{3,1}]$ and $\widehat{\mathbf{C}} = [\widehat{T}_{60}\ \widehat{\mathrm{G}}_{2,1}\ \widehat{\mathrm{G}}_{3,1}]$. Using Sabine's formula [19], we obtain an initial estimate of the absorption coefficient from a measurement of the reverberation time, as follows:

$$\hat{\alpha}_j(\omega) = 0.161\frac{V}{S\,T_{60}(\omega)}\,, \tag{18}$$

where V is the room volume, S is the total surface area, and T_{60} is the measured reverberation for a given frequency band. This value serves as an initial guess for all unknown absorption coefficients. A nonlinear least-squares solver is used to obtain the estimates.

5. SIMULATION TEST CASE

The test data is generated using a finite element model of a reverberant room. Details of the room, the simulation setup, and the post-processing applied to the simulation solutions are presented here.

5.1 Room and parameters

The room is a reverberant space with non-parallel surfaces. It may either be empty or contain a sample of material placed on the floor. A diagram of the room is provided in Fig. 1a, and the coordinates of the corners can be found in Tab. 1.

The air in the room has a speed of sound of 343 m/s, and a density of 1.2 kg/m³. A point source is located at the second corner of the room. Impedance boundary conditions are specified for the room's surfaces and the material sample. Four impedance conditions are considered; these are described below.

5.1.1 Case 0

A spatially uniform, frequency-independent, real-valued impedance is considered to demonstrate the inherent error of the diffusion equation-based estimation method. The room is empty, and all surfaces have a normalized impedance of 19.77, which results in a random incidence absorption coefficient of 0.3.

5.1.2 Case 1

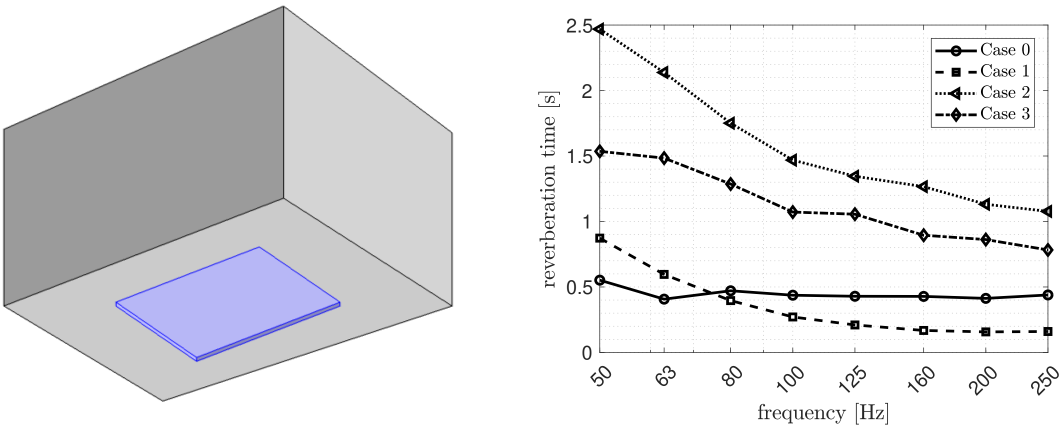
In this case, a complex-valued, frequency-dependent impedance is specified on all surfaces of the empty room. The impedance is described using a Delany-Bazley-Miki model [20]. The impedance is given by:

$$\zeta = (R + \mathrm{i}X)\coth\left[(\gamma + \mathrm{i}\beta)l\right] \tag{19}$$

where

$$\begin{aligned} R &= 1 + 0.070\left(\frac{f}{r}\right)^{-0.632}, \\ X &= -0.107\left(\frac{f}{r}\right)^{-0.632}, \\ \gamma &= \frac{2\pi f}{c}\left[0.160\left(\frac{f}{r}\right)^{-0.618}\right], \\ \beta &= \frac{2\pi f}{c}\left[1 + 0.109\left(\frac{f}{r}\right)^{-0.618}\right], \end{aligned}$$

f is frequency, $l = 0.1$ m, and $r = 41 \times 10^3$ rayl/m.



(a) (b)

Figure 1: Test case: (a) reverberant room with non-parallel absorbing walls and a material sample on the floor. (b) average reverberation time as a function of third-octave bands for four different boundary conditions.

Table 1: Coordinates of the corners of the simulated reverberation chamber. The corner at which the sound source is located is shown in bold.

corner	1	**2**	3	4	5	6	7	8
x [m]	0.36	**8**	7.8	0	0.36	8	7.8	0
y [m]	0.57	**0**	6	6	0.57	0	6	6
z [m]	0	**0**	0	0	4.17	4.51	5	4.6

Table 2: Coordinates of sample corners.

corner	1	2	3	4
x [m]	0.50	2.50	2.50	0.50
y [m]	0.50	0.50	2.00	2.00
z [m]	0.10	0.10	0.10	0.10

5.1.3 Case 2

The normalized impedance of the surfaces of the room is 100, which gives a random incidence absorption coefficient of 0.0734. A material sample with a height of 0.1 m is placed on the floor. The coordinates of four corners of the sample are given in Tab. 2. The impedance model described in Eq. (19) is specified on the sample's surface.

5.1.4 Case 3

The impedance of the walls is 100, while the impedance of the ceiling is 19.77. These values yield random incidence absorption coefficients of 0.0734 for the walls and 0.3 for the ceiling. The sample placed on the floor utilizes the Miki impedance model, as introduced in Eq. (19).

5.2 Simulation and post-processing

Finite element method simulations of the reverberation chamber are performed in the frequency domain to incorporate complex-valued, frequency-dependent boundary conditions in the impulse responses. Impulse responses are obtained via inverse Fourier transformation and filtered into third-octave frequency bands using a second-order Butterworth filter. Decay curves are then calculated from these responses using Schroeder integration [21]. The reverberation times, shown in Fig. 1b, are estimated by fitting straight lines to the energy decay curves between -5 dB and -35 dB, adjusting for offset, and determining the time at -60 dB. Field indices are computed from the pressure and normal velocity at the center of each surface of interest.

6. RESULTS

In this section, we compare the specified and estimated absorption coefficients for four test cases. It is important to note that the specified random incidence absorption coefficients are calculated from the specified impedances.

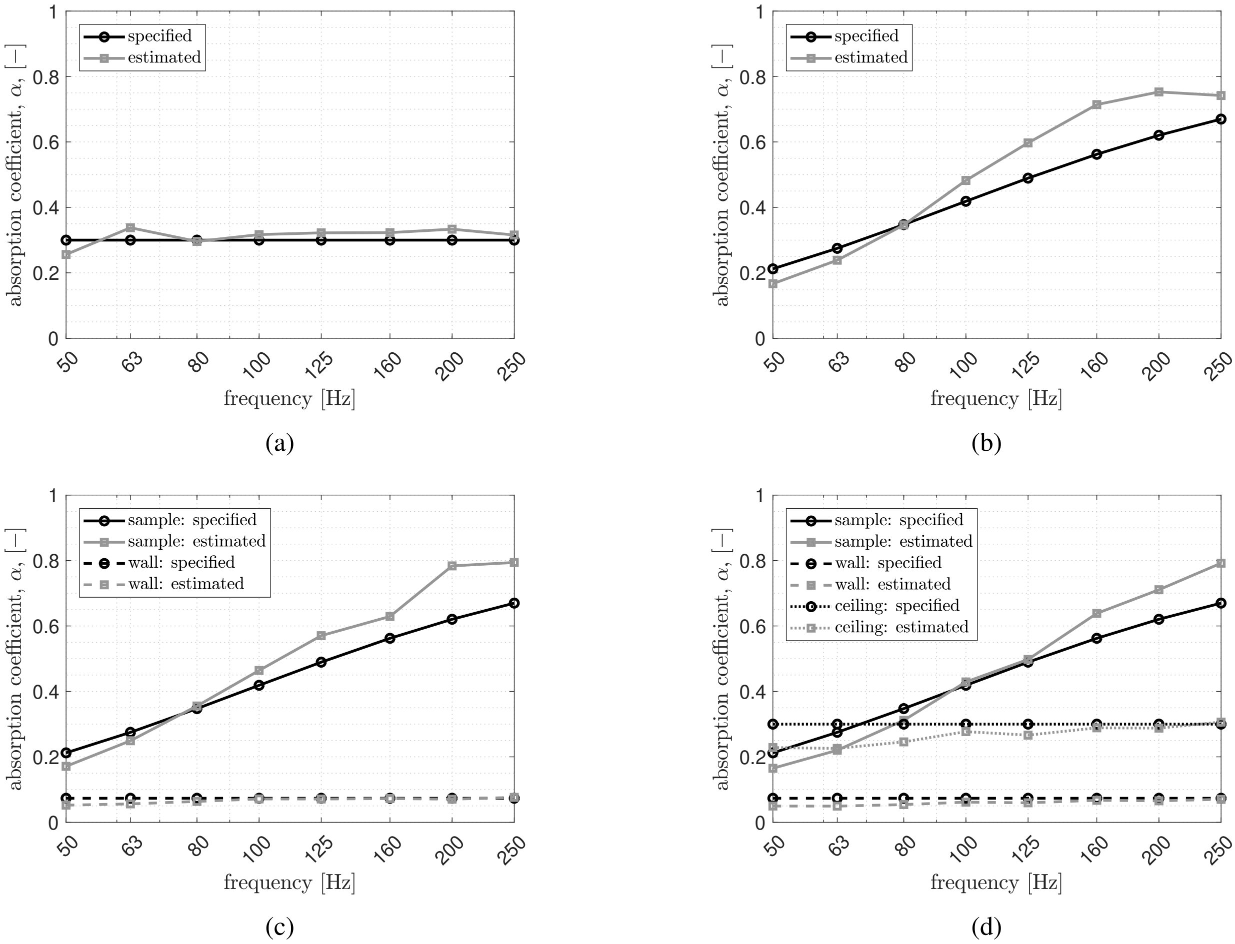


Figure 2: Comparison of specified and estimated absorption coefficients. (a) Case 0 (1 real-valued impedance), (b) Case 1 (1 complex-valued impedance), (c) Case 2 (2 impedances), and (d) Case 3 (3 impedances).

The absorption coefficients for Case 0 are shown in Fig. 2a. A reasonable level of agreement is found. Slight differences are caused by the inability of the diffusion equation to model the spatial non-uniformity of the reverberation time [22] and the assumption of a random incidence absorption coefficient in the diffusion equation, which is an idealization of sound absorption based on a perfectly diffuse field. In Fig. 2b, the spatially uniform absorption coefficients given by the Miki model are compared with the estimated values. The error in this comparison is larger than Case 0. Although the exact cause of this increased error is not fully known, it is believed that the reverberation time and the assumption of random incidence contribute to it. In Case 2, two unknown absorption coefficients are estimated: one for the room's walls and another for the sample itself. As can be seen in Fig. 2c, the absorption coefficient estimation method effectively captures both the frequency trends and the general levels of the unknown coefficients. For Case 3, three unknown absorption coefficients are estimated. This data is compared to the specified absorption coefficients in Fig. 2d. Once again, a reasonable level of agreement is found. We observe that, while there is some qualitative agreement, there are also differences between the specified and estimated absorption coefficients. We expect that these differences will diminish as the frequency increases; however, high-frequency models were not created for this study due to the high computational costs involved.

7. DISCUSSION

The present study has demonstrated that the proposed method can be used to estimate the unknown absorption coefficients of multiple surfaces. In this section, we address limitations and discuss possible future studies.

7.1 Reverberation time

The diffusion equation assumes a nearly diffuse field. Thus, the reverberation time predicted by the diffusion equation in an uncoupled room is spatially uniform. However, the reverberation time in a room tends to be spatially non-uniform. [2] Thus, more accurate measures of the average reverberation time may improve the estimates. While this might be reasonable in a dedicated measurement setting, a high number of measurements might be impractical for some audio applications, e.g., estimating the absorption coefficients of a listening environment using only a few microphones.

7.2 Normal velocity measurement

Accurately measuring the velocity normal to an absorbing surface can be challenging. The spacing of the microphones and their distance from the surface may affect the estimates. Errors in the measured velocity will lead to inaccuracies in the intensity and energy density, which will subsequently impact the estimates of the absorption coefficient.

7.3 Impedance estimation

In-situ absorption coefficients can be useful for certain simulation tools, but wave methods usually require boundary conditions based on impedance or admittance. Additionally, research has shown that using angle-dependent reflection coefficients at room boundaries enhances the accuracy of the image source method, which requires knowledge of the surface impedance. The absorption coefficients estimated in this study can be converted into real-valued impedances using theories like those from Kuttruff [16]. However, these values were calculated during the preparation of this paper but were deemed too inaccurate to be included. While literature exists on estimating complex-valued impedance from absorption coefficients, such as the work by Mondet *et al.* [23], this topic is outside the scope of the current study.

7.4 Modal field

This study has considered a range of frequencies that would generally be associated with the modal region of a sound field. While the diffusion equation is frequency-independent, by construction, it does not model modal fields. Some of the observed errors may be related to missing modal effects. While it is expected that the estimation method will perform better at frequencies above the Schroeder frequency, i.e., for frequencies at which the diffuse field assumption becomes reasonable, further testing is required to determine whether or not improved estimates might be found at higher frequencies.

7.5 Further studies

While the proposed method holds potential for real-world applications, several aspects still need to be investigated. These include the effects of the room geometry, source and receiver positions, measurement noise, and the diffusion coefficient, D, on the absorption coefficient estimates. Also, the impact of multiple complex-valued impedance surfaces and the method's performance for a broader range of frequencies should be investigated. Finally, extending the method to wave methods might also be interesting.

8. CONCLUSION

This paper presents an inverse method to estimate spatially non-uniform absorption coefficients in a room. The proposed method is based on the idea that field index ratios can identify areas of varying sound absorption in the room. An eigenvalue analysis of a diffusion equation model of the room is performed, from which the reverberation time and field index ratio(s) at the measurement positions are computed. An optimization algorithm is employed to minimize the differences between the observed and simulated values, allowing for the estimation of the unknown coefficients. The method is demonstrated using low-frequency finite element simulations. This proof-of-concept work has shown that, at least in principle, the diffusion equation can be used to estimate spatially non-uniform, frequency-dependent absorption coefficients. Future work should consider experimental validations of the proposed method.

[2] This is well known in the community, evidenced by the need to average multiple reverberation time measurements in the reverberation chamber method [2], and recently demonstrated for low frequencies [22].

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