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DYNAMIC WAVE MANIPULATION IN SPATIOTEMPORAL INTERFACES AND SLABS FOR ACOUSTIC CONTROL

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ABSTRACT

Time-varying systems have emerged as a pivotal approach in wave physics, enabling unprecedented control over wave dynamics through the modulation of spatial and temporal properties. In this work, we analyze wave interactions at spatiotemporal interfaces, exploring the resulting scattering phenomena and their implications for acoustic wave manipulation. Building upon foundational principles, we classify interaction regimes—subsonic, supersonic, and intersonic—through a novel $\alpha - \gamma$ diagram, offering a comprehensive framework for understanding wave-interface dynamics. We extend this analysis to spatiotemporal slabs, which consist of two parallel interfaces moving with the same velocity. These structures combine spatial and temporal modulations, enabling intricate wave behaviors such as asymmetric interference patterns and dynamic frequency shifts. We demonstrate the scattering coefficients and frequency conversion characteristics within these slabs. The results highlight the transformative potential of spatiotemporal slabs for broadband acoustic wave control, paving the way for innovative applications in sound insulation, communications, and wave manipulation technologies.

Keywords: *Spatiotemporal, interface, slab, time-varying media, scattering*

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1. INTRODUCTION

Spatiotemporal modulation of acoustic media has opened new avenues for controlling wave propagation beyond the limitations of stationary systems. By dynamically varying properties such as sound speed or impedance, these systems allow for functionalities including frequency conversion, wave amplification, and nonreciprocal transmission [1–3]. Among the different strategies explored, the use of moving interfaces has proven particularly effective in generating new wave behaviors. When such interfaces propagate through the medium, they induce shifts in frequency and wavelength of the scattered waves, governed by the relative velocities of the interface and the acoustic media [4, 5].

Building upon these ideas, *spatiotemporal slabs*, structures defined by two parallel moving interfaces, represent a more complex configuration capable of producing rich scattering phenomena. Unlike a single interface, slabs introduce internal reflections and interference effects that depend on both the acoustic contrast and the motion of the slab. These configurations enable fine control over transmitted and reflected wave components, with potential applications in advanced sound control and acoustic metamaterials.

In this work, we present a study of wave scattering in spatiotemporal slabs. We peopoaw analytical expressions for the scattering coefficients in the subsonic and supersonic regimes, incorporating internal phase accumulation and frequency shifts due to the motion of the interfaces.

2. THE SPATIOTEMPORAL SLAB

A spatiotemporal slab is a dynamic structure defined by two parallel interfaces moving through an acoustic medium. It introduces a finite thickness region in which the wave speed changes from a background value c_1 to a new value $c_2 = \gamma c_1$, and then returns to c_1 after the slab has passed. Unlike single-interface systems, slabs support multiple internal reflections and wave interactions that significantly affect the overall scattering behavior.

These structures are the dynamic counterpart of classical spatial slabs used in layered media acoustics with additional complexity due to their motion. Conceptually, spatiotemporal slabs also generalize the idea of temporal slabs—media with time-varying properties studied in prior works [6, 7]—by combining both spatial and temporal discontinuities [8].

The interaction between an incident wave and the slab depends on the relative velocity of the interfaces ($v = \alpha c_1$) and the acoustic contrast γ. The first interface generates scattered components whose behavior is governed by the coefficient $\Gamma_f(\gamma, \alpha)$. Once the wave enters the slab, it travels in a medium with speed c_2, and the second interface must be analyzed with transformed parameters $1/\gamma$ and α/γ, effectively reversing the roles of the media.

In the subsonic regime, the slab moves slower than the wave, allowing multiple internal reflections that accumulate over time. In contrast, the supersonic regime occurs when the slab moves faster than the wave, leading to only a few distinct wave components without recursive reflections.

2.1 Subsonic regime

In the subsonic regime ($|\alpha| < \min(1, \gamma)$), the interface moves slower than both media. As shown in Fig. 1, an incident wave hits the first interface, producing a *reflected* wave and a *forward* wave that enters the slab. After traveling a distance L_A, the forward wave reaches the second interface, generating a new forward wave that exits the slab, and a reflected wave that remains within it. This process repeats, forming recursive internal reflections and building up multiple transmitted and reflected components. The final transmitted and reflected waves are superpositions of these components, each phase-shifted and frequency-modulated due to the slab motion.

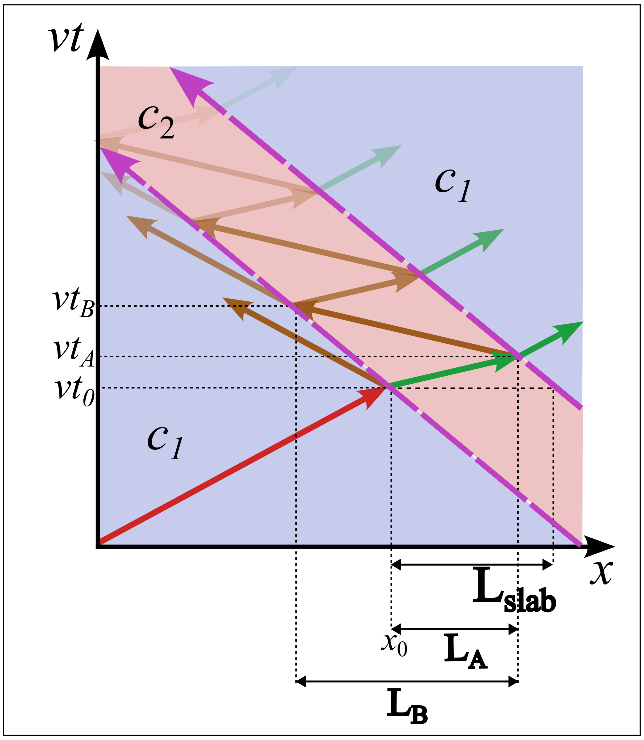

Figure 1. Subsonic slab: multiple internal reflections generate cumulative transmitted and reflected components.

2.2 Supersonic regime

In the supersonic regime ($|\alpha| > \max(1, \gamma)$), the interface moves faster than both media. Upon interaction with the first interface, the incident wave generates a *forward wave* and a *backward wave*, both propagating inside the slab in opposite directions as illustrated in Fig. 2. These components interact with the second interface, producing new forward and backward waves. Unlike in the subsonic case, no recursive reflections are generated.

3. SCATTERING COEFFICIENTS

The scattering response of a spatiotemporal slab is determined by the interference of the wave components that are generated through internal reflections. To compute the global scattering coefficients, three key aspects must be considered:

1. **Acoustic path lengths:** Each wave component travels a different distance inside the slab depend-

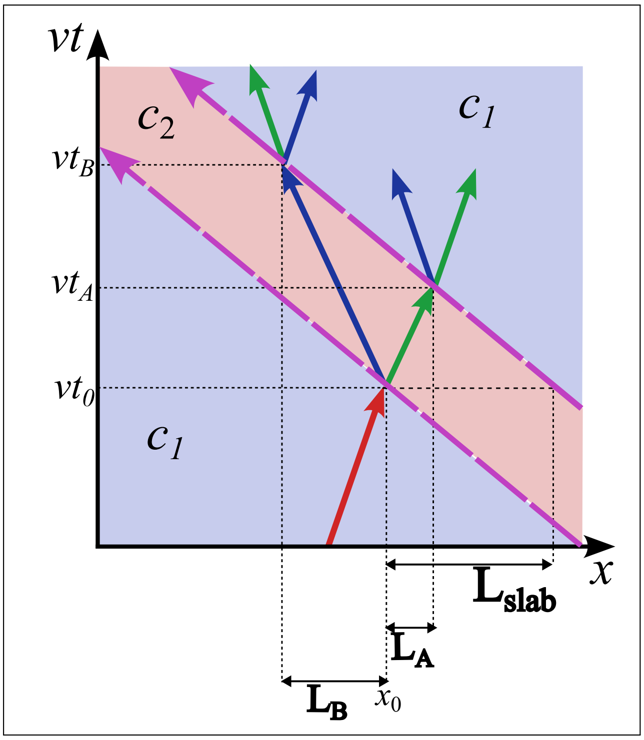

Figure 2. Supersonic slab: wave components interact only once at each interface, producing distinct forward and backward waves.

ing on its propagation direction. These distances are denoted as L_A for waves traveling in the same direction as the moving slab (copropagating), and L_B for waves traveling in the opposite direction (counterpropagating). In the subsonic regime, they are given by:

$$L_A = \frac{\gamma}{\gamma - \alpha} L_{\text{slab}}, \qquad L_B = \frac{\gamma}{\gamma + \alpha} L_{\text{slab}}. \quad (1)$$

2. **Frequency conversion:** Each interaction with a moving interface induces a frequency shift, modifying the phase evolution of the wave inside the slab. These shifts are described by frequency conversion factors.

3. **Interface scattering:** At each interface crossing, the wave amplitude is modified according to a scattering coefficient that depends on the propagation regime.

3.1 Forward Scattering Coefficient

The total forward wave transmitted through the slab is the sum of all forward components. In the subsonic regime, the closed-form expression is:

$$\mathbb{G}_F(\gamma, \alpha, L_{\text{slab}}) = \frac{4\gamma e^{\frac{j\alpha\gamma k_1 L_{\text{slab}} A_{1,F}}{\Phi_F}}}{(\gamma+1)^2 e^{\frac{j\gamma k_1 L_{\text{slab}} A_{2,F}}{\Phi_F}} - (\gamma-1)^2 e^{\frac{j\gamma^2 k_1 L_{\text{slab}} A_{1,F}}{\Phi_F}}}, \quad (2)$$

where:

$$\Phi_F = (\alpha^2 - \gamma^2)^2,$$
$$A_{1,F} = \alpha^2 + 3\alpha\gamma + \alpha - \gamma,$$
$$A_{2,F} = \alpha^2(\gamma + 2) - \alpha\gamma(\gamma - 1) + \gamma^2.$$

3.2 Reflection Scattering Coefficient

The total reflected wave is composed of the direct reflection at the first interface and all components returning after internal reflections. Its final expression is:

$$\mathbb{G}_R(\gamma, \alpha, L_{\text{slab}}) = \frac{(\gamma^2 - 1)\left(1 - e^{\frac{2jk_1 L_{\text{slab}} A_{1,R}}{\Phi_R}}\right)}{(\gamma+1)^2 - (\gamma-1)^2 e^{\frac{2jk_1 L_{\text{slab}} A_{1,R}}{\Phi_R}}}, \quad (3)$$

where:

$$A_{1,R} = \gamma(\alpha-1)(2\alpha^3+\alpha^2+\gamma^2), \quad \Phi_R = (\alpha+1)(\alpha^2-\gamma^2)^2.$$

These analytical expressions capture the essential features of wave propagation through a moving slab, including internal phase accumulation, frequency shift, and multiple reflections.

The analytical results are illustrated in Fig. 3 and 4, which show the forward and reflected (or backward) scattering coefficients for both subsonic and supersonic regimes. The calculations are performed for a slab of thickness $L_{\text{slab}} = 0.1\,\text{m}$ and contrast $\gamma = 2$. The frequency at which the scattering coefficients reach their maxima and minima depends on the slab velocity. The motion of the slab changes the effective thickness that the waves experience as they propagate through it. The relative motion between the slab and the wave causes the wave to travel a longer or shorter distance inside the slab, depending on the direction of propagation. This change in effective thickness modifies the phase accumulation and, consequently, shifts the interference pattern that determines the scattering behavior.

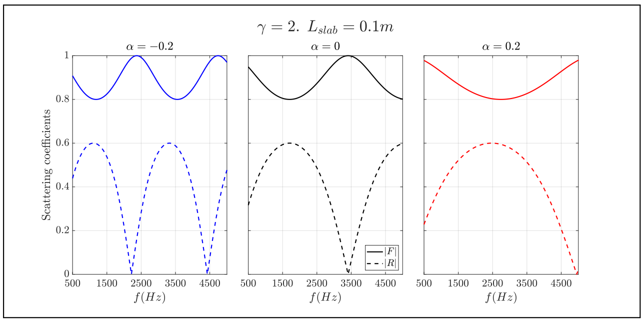

Figure 3. Scattering coefficients in the subsonic regime. Solid and dashed lines correspond to forward and reflected waves. Colors represent counterpropagating (blue), static (black), and copropagating (red) cases.

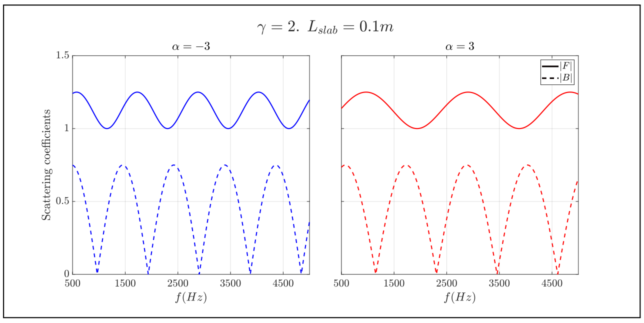

Figure 4. Scattering coefficients in the supersonic regime. Solid and dashed lines correspond to forward and backward waves. Colors represent counterpropagating (blue), static (black), and copropagating (red) cases.

4. CONCLUSIONS

We have presented a study of wave propagation through spatiotemporal acoustic slabs. By considering the combined effects of internal reflections, frequency conversion, and moving interface scattering, we derived closed-form expressions for the global scattering coefficients in subsonic regimes. The results show that slab motion introduces frequency-selective behavior and asymmetric transmission characteristics that are not present in static layered media. These findings provide a useful analytical framework for the design of spatiotemporally modulated acoustic devices for sound control, isolation, and filtering.

5. AKNOWLEDGEMENTS

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