DOI: 10.61782/fa.2025.0350

ESTIMATING SOUND RADIATION CHARACTERISTICS OF COMPLEX-SHAPED PLATES USING REDUCED SETS OF ELEMENTARY SOURCES

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ABSTRACT

When calculating the acoustic radiation from vibrating planar structures, engineers are most often obliged to use computationally heavy approaches, like BEM or FEM. These are, however, largely unfit for parametric studies, optimizations, or real-time sound synthesis. The approach proposed here aims to approximate the acoustic fields produced by a vibrating plate via minimal sets of elementary sources. Based around a modal description of the plate vibration, each lobe region in a mode shape is to be represented by an equivalent punctual source. Aside from its evident computationally efficiency, the proposed approach is versatile since baffled and unbaffled cases can be treated using monopole and dipole arrays, respectively, and complex-shaped plates can be dealt with via automatic image segmentation techniques. Additionally, it enables the calculation of both modal radiation curves as well as forced multi-modal scenarios and transient responses. Because it is set in a modal framework, it can easily be combined with typical structural vibration models. The benefits of this reduced multipole approach are demonstrated through a series of illustrative examples, where approximate solutions are compared to reference results from either analytical or large-scale FEM models.

Keywords: *plate radiation, minimal models, equivalent sources, multi-modal radiation, monopoles and dipoles.*

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1. INTRODUCTION

Whether driven by the need of understanding or of predicting sound radiation for control requirements, the radiation from plates has long been a topic of considerable relevance in engineering acoustics. The problem consists in predicting the sound field radiated by a planar structure of arbitrary shape, surrounded by a fluid and vibrating with a given surface velocity distribution. To provide useful insights, the analysis commonly proceeds on two approaches, addressing (1) the sound radiation from a specific mode, the so-called modal radiation, and/or (2) the overall radiation caused by unevenly forced excitation (e.g. point excitations), where many modes contribute to the vibrational response, the so-called multi-mode radiation.

Early works were mainly concerned modal radiation from simple analytical configurations and well-known results include asymptotic formulae and closed forms solutions for simply supported rectangular plates [1] [2] [3] [4]. One fact to emphasize is the difficulty of producing predictions for the unbaffled configuration, which considerably complicates the mathematical modelling and might lead to instabilities in the numerical implementations [5] [6]. In general, the radiation from more realistic complex-shaped plates typically requires the use of numerical approaches to solve the integral equations, for which finite element and/or boundary element methods are probably the most use techniques. Although precise solutions can usually be achieved by these techniques it is well known that the computation costs still represent a significant limitation particularly when dealing with optimization problems, parametric studies or real-time sound-synthesis.

Recently, a form of lumped model approach has been presented by Garcia et al. [7] by using combination of circular pistons that replicate modal radiation. In this model, each lobe of a mode shape is viewed as a baffle piston that radiates the same amount of energy and with characteristics including size, amplitude, phase, and position, determined by the mode shape geometry. This concept, essentially a reduced form of the equivalent source method [8], is surely a powerful one, leading to very low computational cost and easily combined with modal-based structural vibration models. The authors have however dealt only with limited scenarios, namely, modal radiation in baffled configurations. It remains unclear how this could be extended to the unbaffled cases as well as the estimation of radiation patterns from unevenly excited plates (multi-modal).

In generic terms, the present formulation is a variant of the method proposed by Garcia et al. [7] as we use a spatial distribution of punctual sources to represent each lobe of a mode shape function. One major difference resides in considering monopoles and dipoles as the elementary radiators, which presents practical and conceptual advantages to deal with the problem. Monopole and dipole are the simplest acoustic sources and hence, are easy to model. Also, physically, it is well known that at low frequency, radiation from baffled plates can be represented by sets of monopoles and, similarly, unbaffled plates can be represented by sets of dipoles. In practice, the geometry of the mode shape directly determines the number of equivalent sources and the velocity distribution within each modal region defines the source strength and location of the equivalent point-source. Global measures of the pressure far-field, including total power and radiation efficiency, are then estimated using general analytical expressions derived for planar arrays of monopoles or dipoles. It is obvious that our formulation directly produces predictions for modal radiation but one practical advantage of the formalism is to deal easily with the forced multi-modal problem when one adopts a modal description for the plate dynamics. By doing so, the multi-modal problem is a natural extension of the modal case and hence can be addressed using the same expressions. This type of approach is obviously well adapted to programming and offers the possibility of efficient computations through solely one simple generic code for both baffled and unbaffled configurations. Other important feature of our work is the ability to deal with plates of arbitrary shape addressed by optimal and automatic positioning of the point sources achieved through techniques of image segmentation. Finally, note that by deriving the radiation model from the mode shape geometry, our formulation is readily well-suitable for integrating typical structural vibration models and could provide intuitive physical understanding and interpretation of the solutions.

2. MODAL RADIATION DESCRIBED BY A SET OF POINT SOURCES

The case of modal radiation is concerned with the vibration of a flexible plate animated by a simple harmonic motion in one of its natural modes. The velocity distribution of its surface is then written as

$$v(x,y) = v_m \phi_m(x,y) \tag{1}$$

where ϕ_m and v_m are the mode shape and modal velocity of mode m, respectively. The mode shape contains several local maxima/minima and its overall distribution can be then divided into a set of J convex regions containing one single minima or maxima (lobe). As illustrated in Figure 1, within the area of each region S_j, the location of an equivalent point source (x_j, y_j) will be defined by the weighted centroid of the mode shape, given by

$$x_j = \frac{\int_{S_j} x\phi(x,y)dxdy}{\int_{S_j} \phi(x,y)dxdy} \quad ; \quad y_j = \frac{\int_{S_j} y\phi(x,y)dxdy}{\int_{S_j} \phi(x,y)dxdy} \tag{2}$$

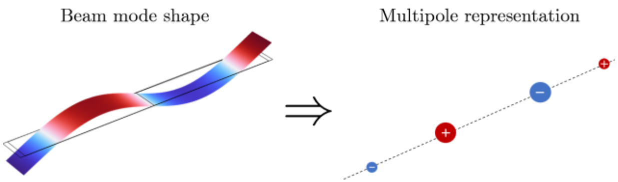


Figure 1. Illustration of the multipole method applied to a free-free beam mode.

2.1 Baffled case and monopoles

Assuming simple harmonic motion with frequency ω, the pressure field from a monopole may be expressed as

$$\hat{p}(r) = i\omega\rho\hat{U}\frac{e^{-ikr}}{4\pi r} = \hat{Q}\frac{e^{-ikr}}{4\pi r} \tag{3}$$

where $k = \omega / c$ is the wavenumber, i is the imaginary unit and quantities with a hat are complex amplitude containing phase information. The source strength $\hat{Q} = i\omega\rho\hat{U}$ represents the injection/suction of fluid into the region (a volume velocity). A generic representation of the source strength for plane radiators with continuous convex velocity distribution $v(x,y,t) = \hat{V}(x,y)e^{i\omega t}$ is given by

$$\hat{Q} = i\omega\rho\int \hat{V}(x,y)dxdy \tag{4}$$

The extension to account for N simple harmonic monopoles of complex source strength $Q_n = i\omega\rho U_n$ results from a direct application of the principle of linear superposition in acoustics. For such arrangement, the complex pressure at point $\mathbf{r}$ in the far-field is given by

$$p(\mathbf{r}) = \sum_{n=1}^{N} Q_n \frac{e^{-ikR_n}}{4\pi r} \tag{5}$$

where $R_n = |\mathbf{r} - \mathbf{r}_n|$ is the distance from the n-th source. Using Eq. (5), it can be shown that the acoustic power radiated by the set of monopoles is given by the sound intensity generated by

$$\Pi = \frac{\rho c k^2}{8\pi}\left(\sum_{n=1}^{N} |U_n|^2 + 2\sum_{n=1}^{N}\sum_{m>n}^{N} |U_n||U_n| \frac{\sin kl_{mn}}{kl_{mn}}\right) \tag{6}$$

where l_{mn} are the distance between source n and m. Eq.(6) states that the total acoustic power is determined by the power radiated by all individual sources when acting in isolation, and their mutual acoustic interaction, which is one of the most important mechanisms governing the effectiveness of sound radiation by vibrating surfaces [9] . It can also be written in the following compact matrix form

$$\Pi = \mathbf{u}^{\mathrm{H}}\mathbf{W}\mathbf{u} \tag{7}$$

where the superscript H is the complex conjugate transpose, $\mathbf{u}$ is the vector of the complex source volume velocities U_n and $\mathbf{W}$ is a matrix whose element (m,n) is

$$W_{mn} = \frac{\rho c k^2}{8\pi}\frac{\sin kl_{mn}}{kl_{mn}} \tag{8}$$

In this equivalent representation of modal radiation, the diagonal terms in $\mathbf{W}$ produce the "self" power radiated by the individual lobes whereas the off-diagonal terms, that clearly are symmetric, are associated to the "mutual" radiated power, i.e. the interference of the sound field radiated by the different lobes.

A useful measure of the effectiveness of sound radiation by vibrating surface, originally defined by Wallace [2], is the radiation efficiency, which is a normalized form of the radiated power. It is defined as

$$\sigma = \frac{\Pi}{\rho c S \langle |v(x,y)| \rangle} \tag{9}$$

where $\langle |v(x,y)| \rangle$ is the spatial and temporal average of the squared velocity distribution, which is defined by

$$\langle |v(x,y)| \rangle = \frac{v_n}{2S}\int_S |\phi(x,y)|^2\, dx dy \tag{10}$$

Finally, note that all the previous derivations have been presented assuming monopole sources, hence radiating energy in free space. Adaptation to the baffle configuration implies to account for the fluid loading effects on a single side of the plate, so that expressions for the acoustic power must be multiplied by a factor of two.

2.2 Unbaffled case and dipoles

A dipole is typically an oscillating point source that can be conceptually modeled by two close monopoles of identical strength Q but opposite phase, and separated by a distance d. The pressure radiated by a dipole is given by

$$p(r,\theta) = -k^2\rho c\left(1 + \frac{1}{ikr}\right)Ud\left(\frac{e^{-ikr}}{4\pi r}\right)\cos\theta \tag{11}$$

where the dependence on the elevation angle θ evidences the directional character of the pressure field. Similarly with monopole source, a dipole source strength can be defined as

$$F = i\omega\rho U d \tag{12}$$

where it is important to underline that here the source strength F has the units of a force. Contrary to generating a (unsteady) net volume outflow as in monopoles, a dipole source then exerts a force on the surrounding fluid and this renders their behavior physically and qualitatively different. If the establishment of a low-frequency equivalence between baffled plane radiators and monopoles is relatively straightforward by considering equivalent volume velocities, it becomes difficult to apply an equivalence between unbaffled plane radiators and dipoles. A simple example is that of a piston of area S and arbitrary shape, vibrating harmonically. If the piston is baffled, its volume velocity is independent of the piston shape. However, if the piston is unbaffled, difficulties arise because the same independence on the piston shape does not hold. That is, pistons with the same surface area but different shapes will not radiate the same amount of acoustic power, even at low frequencies. This is a consequence of localized inertial flows along the piston edges, which present large tangential velocities and significantly alter the force applied on the fluid by the vibrating piston. Analytical solutions for the low-frequency radiation of unbaffled circular [4] or elliptical [10] pistons are known, but deriving a generic expression for an arbitrary shaped piston is a more difficult task. However, in a recent work by the authors [11], a relatively generic approximation was proposed based on the piston compactness (ratio of area to perimeter). When looking at Eq.(12), the physical role of the equivalent distance d can be viewed as a correction factor $(d<1)$ on the volume velocity U, encapsulating the "edge effects", such that the equivalent force can be written as in (12). In the case of a circular disk for example, the equivalent distance is $d = (8/3\pi)a$. From a broad set of numerical results from a finite element model, the authors have proposed the following approximation for the equivalent distance, based on the piston compactness ratio

$$d = 2\left(\frac{8}{3\pi}\right)\frac{S}{P} \approx 1.7\frac{S}{P} \tag{13}$$

where P and S are the perimeter and area of the piston respectively. This expression is anchored on the known result for a circular disk, but also provides an excellent approximation for pistons of arbitrary shape. In our representation by means of equivalent modal radiators, Eq.(13) is used to estimate the equivalent dipole source strength F_j for each modal region j, using $d_j = 1.7\ S_j / P_j$ while the local volume velocity U_j is calculated as before (see Eq.(4)). Then, the pressure radiated in the far-field ($(kr \gg 1)$ by a set of N dipoles located arbitrary within a plane surface at position defined by vector $\mathbf{r}_n$ and pointed along the z-axis, is given by

$$p(\mathbf{r}) = -k^2\rho c\sum_{n=1}^{N} F_n \frac{e^{-ikR_n}}{4\pi r}\frac{\mathbf{r}_n \cdot \mathbf{d}_n}{r} \tag{14}$$

where $\mathbf{d}_n$ is the vector distance between the two dipoles. The total acoustic power radiated by the dipoles array is given by

$$\Pi = \mathbf{f}^{\mathrm{H}}\mathbf{D}\mathbf{f} \tag{15}$$

where $\mathbf{f}$ is a vector containing the dipole source strengths and the elements in the radiation matrix $\mathbf{D}$ are

$$D_{mn} = \frac{\rho c k^4}{8\pi}\left(\frac{\sin kl_{mn}}{(kl_{mn})^3} - \frac{\cos kl_{mn}}{(kl_{mn})^2}\right) \tag{16}$$

ILLUSTRATIVE RESULTS

2.3 Modal radiation of a baffled and unbaffled plate

The surface velocity of a rectangular baffled plate with dimensions L and w, with simply supported boundaries, vibrating at one of its natural modes is given by

$$v(x,y,t) = v_{mn}\phi_{mn}(x,y)e^{i\omega t}$$
$$\text{with}\quad \phi_{mn}(x,y) = \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{w}\right) \tag{17}$$

In this simple case, each mode (m,n) is divided into $J = m \times n$ regions with the same area

$$S_j = \frac{Lw}{mn}\ \forall j \tag{18}$$

The source strength of each equivalent monopole is the same for all sources

$$Q_j = \pm 2ik\rho c\left(\frac{Lw}{mn}\right)\frac{4}{\pi^2}v_{mn}\ \forall j \tag{19}$$

where a factor of two is present to account for the baffle. Note however that the sign of Q_j alternates between adjacent lobes. The weighted centroid of each modal region (monopole location) is located at the center of the lobe, define in a regular grid as

$$\mathbf{x}_j = \frac{L(p-1)}{2m}\ ;\ \mathbf{y}_j = \frac{w(q-1)}{2n}\ \text{ for } \begin{cases} p = 1,2\ldots m \\ q = 1,2\ldots n \end{cases} \tag{20}$$

from which the inter-monopoles distance matrix l_{mn} can easily be obtained (as in Eq.(6)).

To illustrate the capability of the proposed method, Figure 2 shows the modal radiation efficiency of various modes of a simply supported square baffled plate calculated with the multipole approximation compared to the analytical solutions presented by Wallace [2]. The three plots shown pertain to the different modal families indexed: odd-odd, odd-even and even-even. The radiative behavior of these three modal families is characterized by their low-frequency behavior, which equates to a monopolar, dipolar or quadrupolar radiation efficiency, i.e. with σ_m proportional to $(kL)^2$, $(kL)^4$, $(kL)^6$, respectively. This simple example serves to underline the importance of symmetry in the radiation efficiency of vibrating structures. Here, much of the radiation from lobes of opposite phases, separated by a symmetry line, will cancel each other and severely reduce the amount of radiated power at low-frequencies. We also note, as expected, that the multipole formulation does not provide accurate results at high-frequencies. Since point sources have no characteristic dimension, a finite set of monopoles will not be able to reproduce the radiation of a physical structure when wavelengths λ exceed a critical length λ_c, related to the distance between the point sources (internodal distances).

Similarly, we illustrate the capacity of the dipole approximation considering modal radiation efficiency of a simply supported square unbaffled plate. Here, the dipolar equivalent distance can be easily calculated from Eq.(13)

$$d_j = 1.7\left(\frac{S}{P}\right) = 1.7\frac{2Lw}{Ln + wm}\ \forall j \tag{21}$$

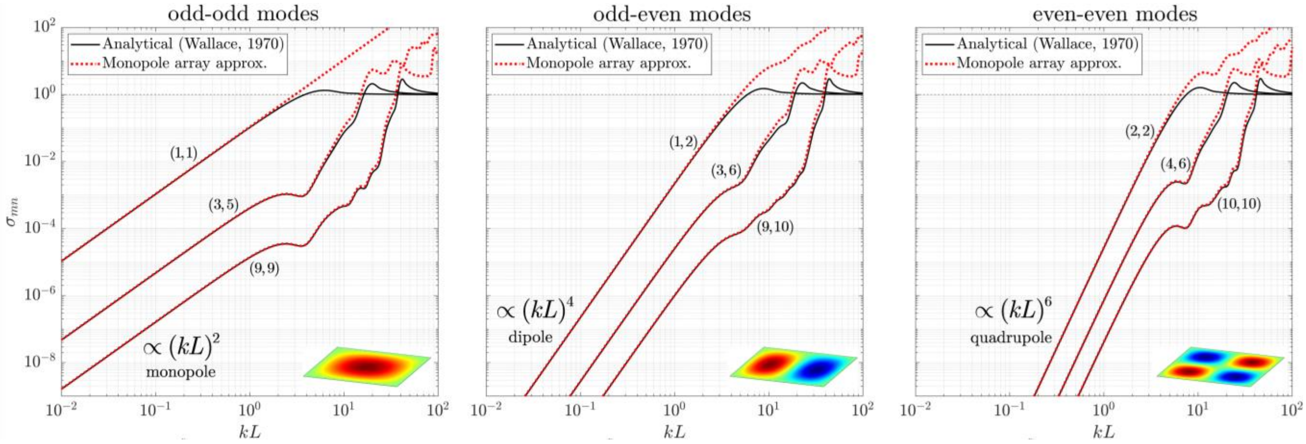


Figure 2. Modal radiation efficiency of square baffled plate with simply supported boundary conditions for odd-odd, odd-even and even-even modes as a function of the dimensionless frequency. The monopole approximations (red dotted lines) are compared to the analytical results (black lines) given by Wallace [2].

In this case, reference results were calculated using the method proposed by Laulagnet [5]. Figure 3 shows the modal radiation efficiency of some modes of an unbaffled plate from the three modal families. It shows that, at least for the case of a simply supported plate, the proposed formulation provides very reasonable approximations, almost as good as for the baffled case. We note however some minor quantitative differences, likely related to the validity of the approximate expression for the equivalent distance.

2.4 Watershed transform and automatic mode shape segmentation

The illustrative results shown in the previous section are simple to obtain since this case is simplified by the fact that all mode shapes have well-defined lobe regions, separated by the nodal lines of the mode shape. Hence, there is no ambiguity on the number and location of point sources to consider. However, this is not the general case and, particularly in complex-shaped plates, mode shapes often contain multiple local minima (representing multiple lobes) whose basins are not separated by nodal lines. Consider for example the well-known Matlab's "peaks" function (Figure 4). We note that this function contains three local minima and three local maxima. Hence, the proposed method would, in principle, require a minimal set of six-point sources to provide an approximation of its modal radiation efficiency. The three local minima (in blue) are indeed contained in regions bounded by nodal lines as opposed to the three local maxima (red/orange), which are in a single

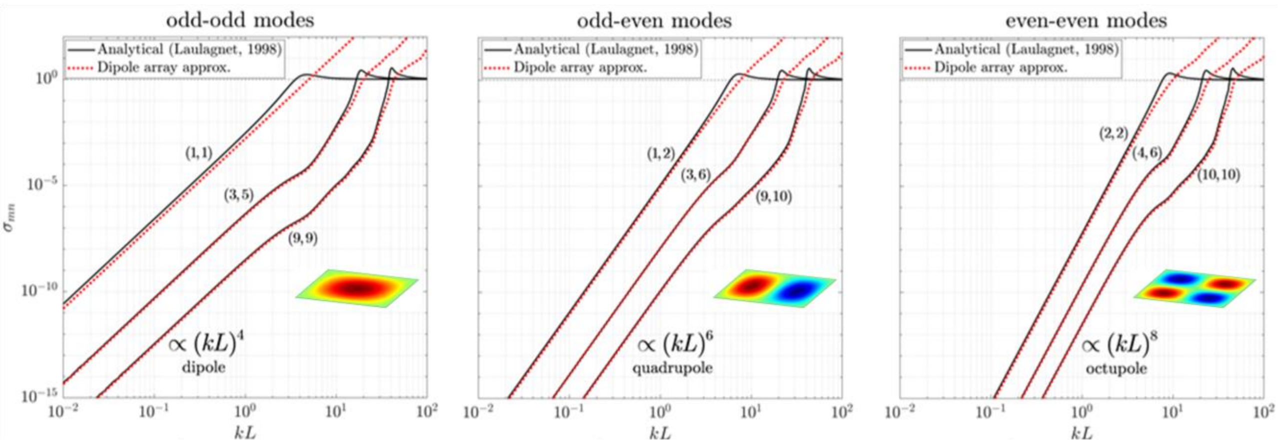
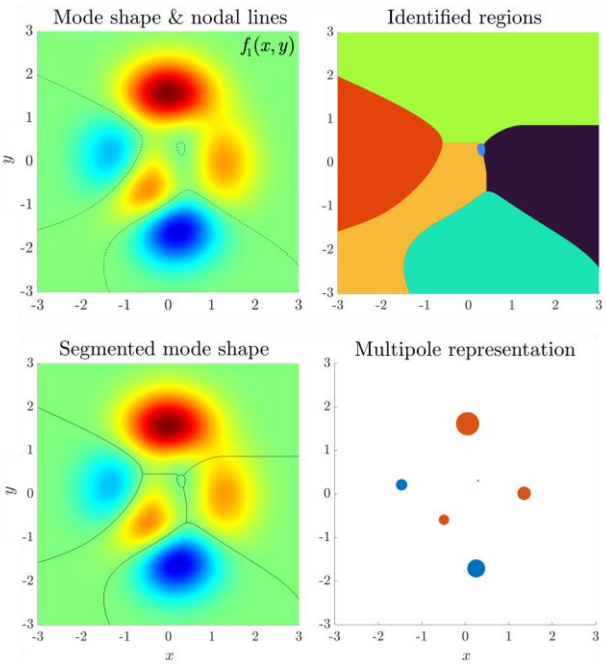


Figure 3. Modal radiation efficiency of square unbaffled plate with simply supported boundary conditions for odd-odd, odd-even and even-even modes as a function of the dimensionless frequency. The dipole approximations (red dotted lines) are compared to the analytical results (black lines) given by Laulagnet [5].



“connected” region. We are then tasked to find lines that enable the segmentation of the three local maxima into different regions. This can be achieved in an automatic fashion using a variety of tailored algorithms. Here we

Figure 4. Illustration of the process of mode shape segmentation using the watershed transform on the “peaks” function, as well as the resulting point source representation.

suggest the use of the watershed transform [12] a widely used algorithm for image segmentation, which is simple, computationally efficient and perfectly fits the intended purpose. For brevity, we restrain from showing the details of the procedure here, but it essentially consists in performing the watershed transform on the modulus of the mode shape function $|f_1(x,y)|$. Figure 4 illustrates watershed transform on the peaks function and as well as the resulting multipole representation.

3. RADIATION DUE TO FORCED RESPONSE

3.1 Formulation for multi-modal scenarios

The natural extension of the previous formulation dealing with the sound power radiated by individual modes is to address the acoustic radiation from unevenly excited plates (e.g. from point force excitation), where many modes contribute to the vibratory and radiative responses. The difficulty in the multi-modal case stems from the fact that, although mode shapes are orthogonal from a vibratory point of view, the associated modal radiation fields are not. Hence, the total sound power is not equal to the sum of the sound powers generated by each mode separately but must also include the sound power produced by the radiative interaction between all modes [13]. Following our multipole representation of sound radiation, the total power radiated by a forced plate described by a set of N structural modes can be formulated by considering a large vector $\mathbf{Q}$ built by stacking a set of vectors $\mathbf{q}_n$, containing the source strengths of the point sources associated to each mode. Essentially, we aim to consider the interaction between all point sources in all modal sets. In view of such a structure, the radiation matrix $\mathbf{W}$ for a multimodal baffled plate is now a block matrix, in which the diagonal elements are square submatrices of modal radiation (as in Eq.(8)) and the off-diagonal blocks are submatrices of radiation stemming from inter-modal coupling, thus describing the production of acoustic power due to the interaction between all elementary sources pertaining to the different modes involved in the vibration. Knowing the modal response of the structure in terms of modal velocities u_n, the total acoustic power radiated for multimodal plates can thus be calculated from

$$\Pi = \begin{Bmatrix} q_1^1 \\ \vdots \\ q_1^{M_n} \\ \vdots \\ q_N^1 \\ \vdots \\ q_N^{M_n} \end{Bmatrix}^{\mathrm{H}} \begin{bmatrix} \mathbf{W}_{11} & \cdots & \mathbf{W}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{W}_{11} & \cdots & \mathbf{W}_{NN} \end{bmatrix} \begin{Bmatrix} q_1^1 \\ \vdots \\ q_1^{M_n} \\ \vdots \\ q_N^1 \\ \vdots \\ q_N^{M_n} \end{Bmatrix} \quad (22)$$

where $\mathbf{W}_{pq}$ are full matrices whose elements $\mathrm{w}_{pq,rs}$ are given by

$$\mathrm{w}_{pq,rs} = \frac{\rho c k^2}{8\pi} \frac{\sin(k l_{pq,rs})}{k l_{pq,rs}} \quad (23)$$

in which we denote $p,q = 1,2 \ldots N$ the modal index and r,s the index of elementary sources in each set, whose upper limit depends on the mode shape. For brevity, we present only the baffled case but the unbaffled case can similarly be formulated using an analogous procedure based on Eqs.(15)-(16) instead.

The local volume velocities q_n^m in Eq.(22) are calculated from the complex modal velocities v_n that can be calculated, when considering a point excitation at $\mathbf{r}_0 = (x_0, y_0)$ as

$$v_n = \frac{i\omega F_0 \phi_n(\mathbf{r}_0)}{m_n(\omega_n^2 - \omega + 2i\omega\omega_n\zeta_n)} \tag{24}$$

where F_0 is the amplitude of the point force and m_n, ω_n and ζ_n are the modal mass, angular frequency and damping ratio of mode n, respectively.

3.2 Illustrative example on a rectangular plate

For illustrative purposes, we now consider a baffled simply supported rectangular plate with length $L = 0.6\,\mathrm{m}$, width $w = 0.5\,\mathrm{m}$ and thickness $h = 2\,\mathrm{mm}$, centered at $(0,0)$, such that it is defined in $x = [-0.3,\ 0.3]$ and $y = [-0.25,\ 0.25]$. Its natural frequencies and modal masses are given by

$$\omega_{mn} = \pi^2 \sqrt{\frac{B}{\rho_s h}} \left[\left(\frac{m}{L}\right)^2 + \left(\frac{n}{w}\right)^2 \right];\ m_{mn} = \frac{\rho_s L w h}{4} \tag{25}$$

where B is the bending stiffness and ρ_s is density. Modal damping was fixed uniformly at $\zeta_n = 0.01\ \forall n$.

For simplicity, we consider a unit force amplitude $F_0 = 1$ and two different locations for the excitation: $\mathbf{r}_1 = (0.19,\ 0.11)$ and $\mathbf{r}_2 = (0.24,\ 0)$. Reference results were calculated using an FEM model composed of a plate with prescribed velocity profile, surrounded by a hemisphere with non-reflecting boundaries (PMLs).

Figure 5 shows the vibratory response (collocated mobility), radiated power and radiation efficiency for excitation points $\mathbf{r}_1$, where results from the multipole method were calculated using $N = 100$ modes.

The results in Figures 5 show how the reduced set of point sources is clearly able to reproduce radiative properties of the plate in forced, multi-modal scenarios. At relatively low frequencies $(kL < \pi)$, the estimated radiated power and radiation efficiency are indistinguishable from the reference FEM results. Above $kL < \pi$, we note a drift of the estimated values, with the multipole method slightly overestimating radiated energy. Nevertheless, approximations are still reasonable even at these high frequencies. This is explained by the fact that the frequency limit of each modal approximation is related to the average distance between monopoles in each modal set, as seen in Figures 2 and 3. Hence, this frequency limit increases with

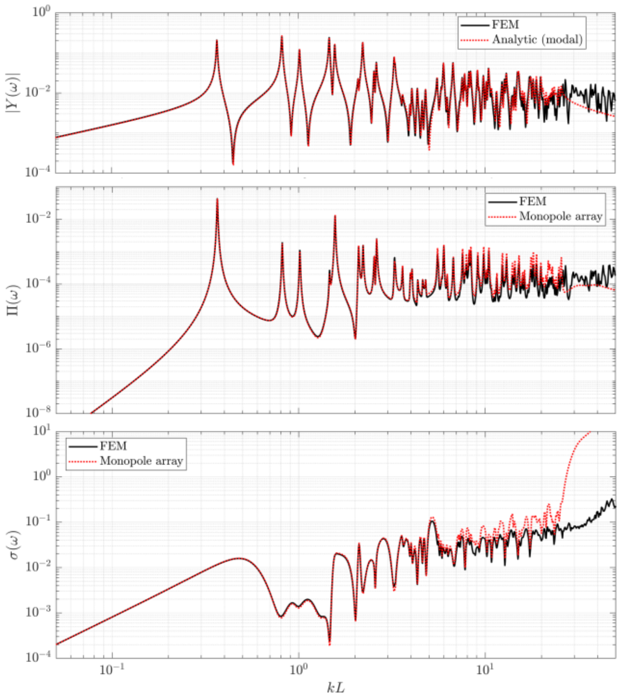


Figure 5. Magnitude of co-located mobility (top), total radiated power (center) and radiation efficiency (bottom) of the baffled rectangular plate excited at $\mathbf{r}_1 = (0.19,\ 0.11)$ as a function of the dimensionless frequency kL. Here, 100 modes were considered.

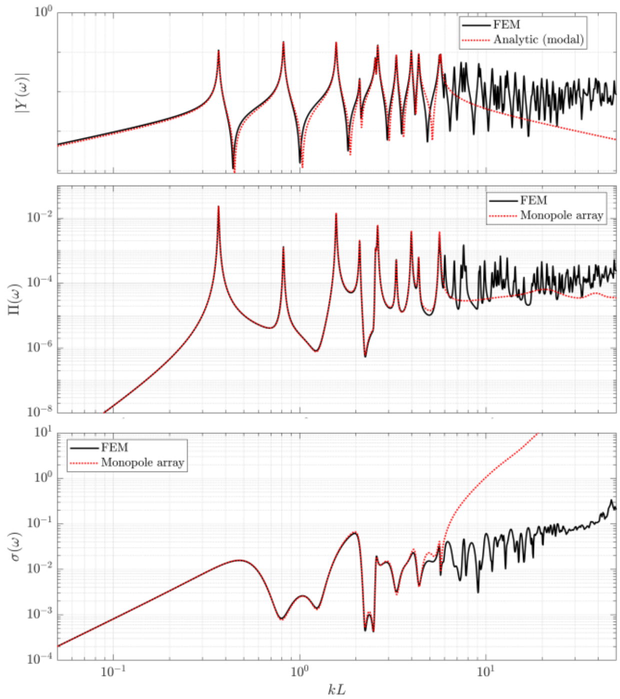


Figure 6. Magnitude of co-located mobility (top), total radiated power (center) and radiation efficiency (bottom) of the baffled rectangular plate excited at $\mathbf{r}_2 = (0.24,\ 0)$ as a function of the dimensionless frequency kL. Here 20 modes were considered.

modal order, as do the natural frequencies of the modes (!).

Figure 6 shows the analogous results for excitation point $\mathbf{r}_2$, and here, only $N = 20$ modes were used for the multipole method estimations. The responses in Figure 6 present less peaks since the excitation point is located on one of the symmetry lines and hence, some modes of the plate are not excited (those of even order in y-direction). Again, we clearly see a good approximation only limited in this case by the number of modes considered.

Finally, and most importantly, is it worth underlining that while the FEM model required approximately 1 to 5 seconds of computation time, per frequency bin, on a desktop computer, the reduced multipole model, with $N = 100$ (i.e. ~2500 sources) took approximately 30-40ms per frequency bin. Furthermore, when $N = 20$ (i.e. ~125 sources) it took about 0.3-0.4ms. Still, this computational time could be further reduced if one considers a threshold on the modal vibratory response. That is, for each frequency bin, considering only a limited set of modes whose vibratory response is dominant.

4. CONCLUSIONS

This paper shows how a method based on the distribution of point sources is able to recreate the radiation properties from vibrating plates, both from single mode excitation as well as multi-modal (unevenly forced) cases. Despite its simplicity, results show a relatively wide range of validity (not necessarily limited to low frequencies). Aside from the commonly treated baffled problem, we demonstrate how this approach can be generalized to unbaffled scenarios by using dipoles instead of monopoles. Also, to deal with complex-shaped plates, we propose an automatic mode-shape segmentation procedure that identifies the spatial distribution of the sources, based on the watershed transform. Illustrative results for the multi-modal case highlight the computational efficiency of the approach reaching acceptable approximations to reference results at computational costs that are several orders of magnitude below the analogous FE model.

An important feature of the formulation is the fact that it is built on a modal description of the plate vibration, which makes it readily integrable in standard structural models. This allows parametric studies and optimization strategies to be carried out for improving sound radiation of structural components, and gives the possibility to simulate very realistic sounds of bars and plates, even in real time.

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