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ESTIMATING THE IMPEDANCE OF A MATERIAL SAMPLE AT LOW FREQUENCIES

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ABSTRACT

Reliable descriptions of a room's physical properties are essential for accurately simulating room acoustical fields. An important model input parameter is the impedance of the materials that comprise the boundaries of the room. However, the impedance information of commonly used sound-absorbing materials is generally unavailable. While databases of absorption coefficients are available for commonly used materials, these are of limited use for simulation tools that require impedance (or angle-dependent reflection coefficients) as input data, and it is uncommon to find absorption coefficients below 100 Hz. This study estimates the normal impedance of a sample of sound-absorbing material at frequencies below 100 Hz. An eigenvalue-based impedance estimation method is used to determine a sample's complex-valued, frequency-dependent impedance from measurements in a reverberation chamber. While the results are encouraging, the method is sensitive to model input parameters, and further refinement is required to make the method practical.

Keywords: *Reverberation chamber, Eigenvalue estimation, Impedance estimation.*

1. INTRODUCTION

Understanding the impedance characteristics of surfaces is crucial for accurately modeling room acoustical fields. However, obtaining this information can be challenging, particularly at low frequencies and for complex-valued impedance. Eigenvalue analysis has been suggested in the literature as a solution [1, 2]. This work employs eigenvalue analysis to estimate the normal impedance of an absorbing surface at low frequencies. In this report, the proposed method is used to estimate the normal impedance of the walls of an empty reverberation chamber and the impedance of a material sample placed in the same chamber. The approach is checked by comparing the measured and simulated reverberation times, absorption coefficients, and room transfer functions.

This paper is organized as follows. In Sec. 2, we provide background information by reviewing relevant literature and offer motivation for the work. Section 3 outlines the impedance estimation method. The measurements conducted in the reverberation chamber are described in Sec. 4, and the results of the impedance estimation are presented in Sec. 5. Finally, we discuss the findings of this study in Sec. 6 and conclude the paper in Sec. 7.

2. BACKGROUND

In general, the impedance of an absorbing material is best described by an extended reaction model. However, a local reaction model can be used to approximate the behavior of some commonly used materials. The local reaction model assumes that the surface impedance is independent of the angle of incidence, meaning it only models the impedance at normal incidence, yielding a normal impedance. Descriptions of absorbing materials based on normal impedance can be easily integrated into wave modeling tools. The normal impedance model is also advantageous for certain geometrical modeling tools, such

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as the image source method, because it allows practitioners to specify angle-dependent reflection and absorption coefficients (see, e.g., Ref. [3]).

In practice, absorption coefficient tables are often utilized in lieu of normal impedance values, a practice that has several disadvantages. The standard method for quantifying a material's sound-absorbing properties, known as the reverberation chamber method [4], provides absorption coefficients. In theory, these coefficients represent the sound absorption at all angles of incidence, but in practice, only a subset of all possible angles is represented in the data. This limitation leads to measured absorption coefficients that depend on the specific environment in which they were collected. The lack of angle-dependent information and the dependence of the data on the measurement room restrict the usefulness of these absorption coefficients. Therefore, having access to the normal impedances of commonly used materials would be highly beneficial for the room acoustics community.

The impedance tube method [5, 6] yields normal impedances of absorbing materials. The method is limited to small samples, which excludes edge effects and potentially includes unwanted installation effects. Depending on the physical dimensions of the tube being used, there is also a limited range of frequencies for which such a measurement is valid. Measurements at low frequencies typically require tubes of potentially impractical length. Due to these limitations, there are ongoing efforts within the community to develop in-situ impedance measurement methods. These usually depend on multi-microphone measurements and advanced signal processing techniques. The interested reader is directed towards Ref. [7] for a recent review of the work being done in this field. Long wavelengths and modal effects in the rooms under test complicate the in-situ measurement of surface impedance at low frequencies. An interesting approach is to use the room's modes to measure the surface impedance.

Hull and Radcliffe propose an eigenvalue-based impedance method [8]. They successfully demonstrate the method using one-dimensional analytical descriptions of the modes in an impedance tube. Nowakowski *et al.* [9] use the eigenmodes of a two-dimensional problem to estimate surface impedance. Prinn *et al.* [1, 2, 10] present an estimation method based on eigenvalue analysis of impulse responses simulated and measured in rooms containing a test sample. The proposed method is extended here to make the approach more robust and is used to estimate a sample's impedance in a reverberation chamber.

3. ESTIMATION METHOD

The eigenvalue-based estimator has two main steps: first, the reverberation chamber's eigenvalues are computed, and second, the measured eigenvalues are compared to the predicted eigenvalues obtained from a model, and an initial guess of the impedance is refined to obtain an impedance estimate. The following sections will describe these steps. For brevity, derivations are omitted, and the interested reader is directed towards Refs. [1, 2].

3.1 Chamber eigenvalues

The first step is to estimate the empty chamber's eigenvalues. We do this for a set of measured room impulse responses using the matrix pencil method [11], which provides a set of potential eigenvalues. The matrix pencil method can be written as follows: Solve the eigenvalue system

$$\det\left(\boldsymbol{H}_1^{\dagger}\boldsymbol{H}_2 - \gamma\boldsymbol{I}\right) = 0\,, \tag{1}$$

where superscript $\dagger$ indicates the Moore-Penrose pseudoinverse,

$$\boldsymbol{H}_1 = \begin{bmatrix} h_1 & h_2 & \dots & h_p \\ h_2 & h_3 & & h_{p+1} \\ \vdots & & \ddots & \vdots \\ h_{N-p} & h_{N-p+1} & \dots & h_{N-1} \end{bmatrix}_{(N-p)\times p}, \tag{2}$$

and

$$\boldsymbol{H}_2 = \begin{bmatrix} h_2 & \dots & h_p & h_{p+1} \\ h_3 & & h_{p+1} & h_{p+2} \\ \vdots & \ddots & & \vdots \\ h_{N-p+1} & \dots & h_{N-1} & h_N \end{bmatrix}_{(N-p)\times p}. \tag{3}$$

The entries of Hankel matrices (2) and (3) are samples from a measured impulse response, h_n, with $n = 1, 2, ..., N$, and pencil parameter $p = \text{round}(N/2)$. Solving (1) yields eigenvalues γ_m, which are used to compute the mth resonance frequency

$$f_m = f_\text{s}\arg(\gamma_m)/(2\pi)\,, \tag{4}$$

and the mth damping constant

$$\sigma_m = f_\text{s}\log(|\gamma_m|)\,, \tag{5}$$

where f_s is the sampling frequency. The mth measured chamber eigenvalue is then

$$\lambda_m = f_m + \mathrm{i}\sigma_m/(2\pi)\,. \tag{6}$$

The matrix pencil method generates both valid and spurious eigenvalues (see, e.g., Ref. [12]). Given data from multiple measurement positions, the valid eigenvalues tend to form local clusters. From this clustering, the distance of each eigenvalue to its neighbors is calculated, and the eigenvalues with the smallest differences from one another are identified. Thus, in this work, k-means clustering (see, e.g., Friedman *et al.* [13]) is used to identify the valid eigenvalues. This provides us with a set of chamber eigenvalues that contain information on the resistance and reactance, i.e., the complex-valued impedance, of the surfaces in the chamber.

3.2 Impedance estimates

With a set of chamber eigenvalues, a numerical wave model of the chamber is used to find the unknown impedance. The finite element method is chosen to model the chamber since the method robustly models problems with arbitrary geometries. The Helmholtz equation-based eigenvalue problem is written as

$$\left[\boldsymbol{K} + (2\pi \mathrm{i})\frac{\hat{\lambda}}{c}\sum_{j=0}^{J}\hat{\zeta}_j^{-1}\boldsymbol{C}_j + (2\pi \mathrm{i})^2\frac{\hat{\lambda}^2}{c^2}\boldsymbol{M}\right]\boldsymbol{v} = \boldsymbol{0}\,, \quad (7)$$

where matrices $\boldsymbol{K}$, $\boldsymbol{C}$, and $\boldsymbol{M}$, are the usual finite element matrices (see, e.g., Ref. [14]). Here, $\hat{\zeta}_j$ is the impedance of the jth surface of the chamber, $\boldsymbol{v}$ is an eigenvector, and the mth simulated eigenvalue is given by

$$\hat{\lambda}_m = \hat{f}_m + \mathrm{i}\hat{\sigma}_m/(2\pi)\,. \quad (8)$$

For a given eigenvalue, the present method provides one impedance estimate. Therefore, to estimate the impedance of the jth surface for the mth resonance frequency, we assume that, either all impedances of the other surfaces in the chamber are known, or the impedance is spatially uniform ($J = 0$). To obtain an estimate, we refine an initial guess for $\hat{\zeta}_j$. This guess is refined using an optimization algorithm to minimize the differences between the chamber and simulated eigenvalues. Thus, we solve the problem

$$\hat{\zeta}_m = \min_{\hat{\zeta}\in\mathbb{C}}\left[\frac{\|\lambda_m - \hat{\lambda}_m\|_2}{\|\lambda_m\|_2}100\right]\,. \quad (9)$$

The cost function is a relative error. Estimates that fail to reach an upper bound on the error (in this case, 1%) are assumed to be erroneous and are removed from the frequency-dependent estimates set. This is necessary due to the sensitivity of the impedance estimation method.

A significant issue that can arise during optimization is when an eigenvalue is associated with an eigenvector that does not correspond to that eigenvalue. This situation occurs when eigenvalues are closely spaced in the frequency domain. As the impedance is varied, the eigenvalues can exchange their chronological order; for instance, an eigenvalue that was previously higher in frequency than its lower-frequency neighbor can become lower than that same neighbor after a change in impedance.

The consequence is that the impedance estimator may attempt to fit an impedance to the incorrect eigenvalue. To improve the method's robustness, we make use of measurements at multiple positions. We use the magnitude of the normalized transfer functions at the measurement positions to approximate the mth normalized eigenvector. This approach allows us to compare the error of the chosen eigenvector with the errors of its immediate neighbors (in terms of resonance frequency). If an eigenvalue with a lower eigenvector error is identified, it is assumed that the eigenvalues have exchanged their chronological order. The eigenvector with the lowest error is used for the estimation.

This work uses the commercially available finite element tool COMSOL [15] to solve the eigenvalue problem.

4. MEASUREMENT SETUP AND DATA

The reverberation chamber under test is shown in Fig. 1. The chamber is formed by two rooms with non-parallel walls joined by a large open window. The sound field produced by two sound sources is recorded at multiple positions within the room. One sound source is a full-range loudspeaker placed in a corner of the reverberation chamber, while the other is a low-frequency woofer placed in the same position. The measurements are taken in two chamber conditions: the empty chamber and the occupied chamber, in which two copies of a sound-absorbing material sample are placed on the chamber floor. This section outlines the experimental setup and presents the relevant data.

4.1 Empty chamber

The chamber is excited with a swept sine signal. An omnidirectional microphone receives the signal, emitted by either the loudspeaker or the woofer, at 12 evenly spaced receiver positions. These positions are reported in Tab. 1. The received signals are deconvolved to obtain a set of measured impulse responses. The speed of sound is estimated to be 343 m/s, and the medium density is 1.2 kg/m^3.

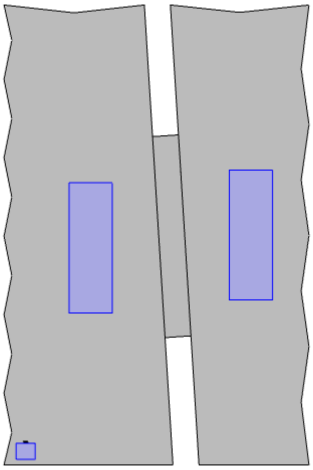


Figure 1: Top view of the reverberation chamber under test. The origin of the coordinate system is the bottom left-hand corner of the chamber. The highlighted rectangle close to the origin is the loudspeaker, facing in the y-dimension, and with the driver's center located at (0.5, 0.5, 0.2) [m]. The highlighted rectangles in the middle of each chamber section are two identical material samples.

Table 1: Measurement receiver positions.

id.	x	y	z
1	1.00	1.00	1.25
2	1.00	5.00	1.25
3	1.00	9.00	1.25
4	3.00	1.00	1.25
5	3.00	5.00	1.25
6	3.00	9.00	1.25
7	5.00	1.00	1.25
8	5.00	5.00	1.25
9	5.00	9.00	1.25
10	6.00	1.00	1.25
11	6.00	5.00	1.25
12	6.00	9.00	1.25

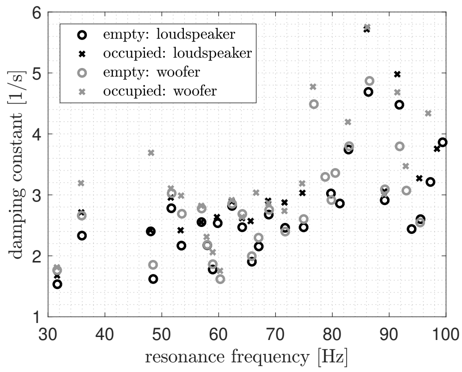


Figure 2: Comparison of empty and occupied chamber eigenvalues, for two sources of sound.

4.2 Chamber with sample

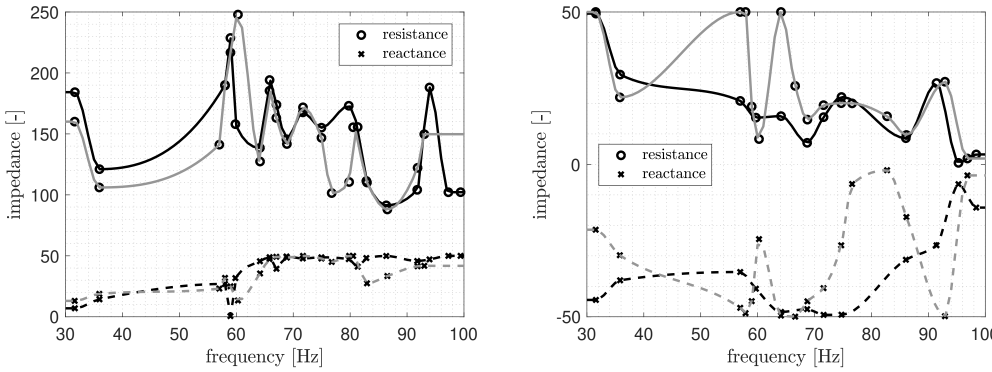
The absorbing material is a polyurethane foam with a porous foam top layer. It is divided into two parts (this is necessary due to the chamber construction; see Fig. 1). The two samples, each with dimensions $1.0 \times 3.0 \times 0.14$ m, are placed on the floor. The measurement procedure described above is repeated for the occupied chamber.

5. RESULTS

First, the unknown impedance of the chamber walls is estimated using the eigenvalues of the empty chamber. This impedance data is then used to estimate the sample's impedance using the eigenvalues of the occupied chamber. The empty and occupied chamber eigenvalues are presented in Fig. 2. Note the increase in damping with the introduction of the sample, and the unexpected damping offset between the sources. This section presents these data and compares the measured and simulated reverberation times, absorption coefficients, and transfer functions.

5.1 Impedance estimates

The estimated impedance of the chamber walls is shown in Fig. 3a for both the loudspeaker and the woofer. Although there are some differences between the two data sets, the general shape of the graphs is consistent. At low frequencies, the resistance of the chamber walls lies in the region between 100 and 250. It is important to note that the lower bound for the resistance optimization is 50, while the upper bound is 250. Thus, for the woofer data, the impedance estimate in the region of 60 Hz might not be correct. For the reactance, the lower and upper bound



(a) (b)

Figure 3: Impedance estimates for the loudspeaker (black lines) and the woofer (gray lines). (a) spatially uniform impedance of all surfaces of the empty chamber. (b) impedance of the material sample.

of the optimization are -50 and 50, respectively. The estimated reactance above approximately 65 Hz is close to the upper bound. It is important to note that this data likely does not represent the actual impedance of the chamber walls, as not all chamber surfaces may exhibit the same impedance (this is a necessary assumption in this work). Instead, this impedance estimate represents the spatially uniform complex-valued impedance.

The estimated impedance of the material sample is presented in Fig. 3b. As expected, the resistance of the sample is lower than that of the chamber walls. Upon comparison of the loudspeaker and woofer data, we see that while the data have similar trends, there are noticeable differences. It is expected that these differences appear due to inaccuracies at the eigenvalue estimation stage.

5.2 Reverberation time estimates

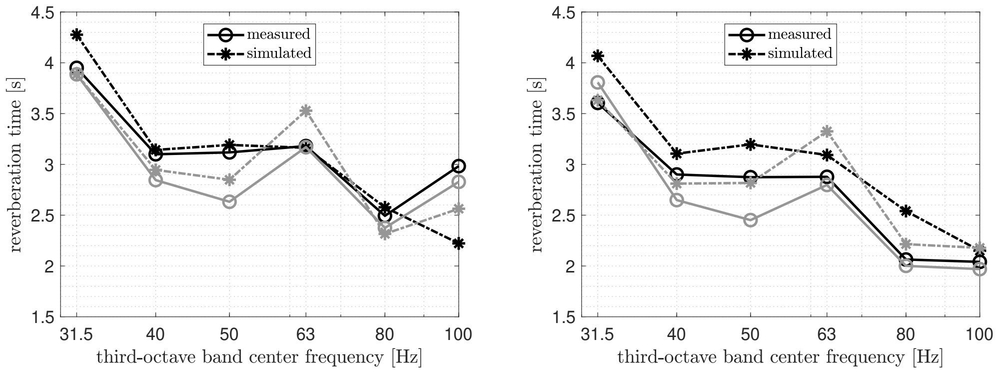
A set of spatially averaged reverberation times can be computed from the measured data. To simulate this quantity, the estimated impedance values are used to describe the boundary conditions of finite element models for both the empty and occupied chambers. A two-microphone method measures the velocity of the loudspeaker and woofer drivers during the measurements. The measured velocity is then post-processed to remove noise and is used to describe the source excitation in the models.

The measured and simulated reverberation times for third-octave bands are compared in Fig. 4. While there is some qualitative agreement between the two, noticeable differences exist. Notice also that the measured reverberation time depends on the source type. Since the same position was used for both sources, it is expected that these differences appear due to the changes in excitation levels as a function of frequency. The exact source of the differences between the measured and simulated data is unknown, but it is expected that improving the estimates of the sample impedance could help reduce them.

5.3 Absorption coefficients

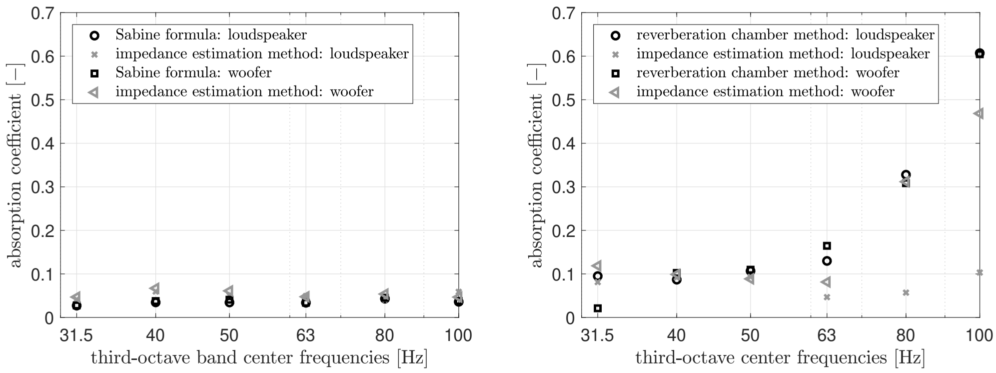
The measured and simulated absorption coefficients are compared in Fig. 5. The measured absorption coefficients for the chamber walls were calculated from the reverberation times obtained in the empty chamber using Sabine's formula. The absorption coefficients for the sample were determined using the reverberation chamber method [4]. The random incidence absorption coefficients (see, e.g., Kuttruff [16]) were derived from the estimated impedances to obtain simulated absorption coefficients. We do not expect the measured and simulated absorption coefficients to be in perfect agreement since the random incidence assumption might not be fulfilled in this reverberation chamber at the low frequencies of interest.

Data for both the loudspeaker and the woofer are included. The results show that the absorption coefficients for the chamber walls are similar, which is reassuring. However, there are noticeable differences between the measured and estimated sample impedances. While improved impedance estimates are expected to yield better agreement for the woofer data, the loudspeaker absorption coefficients at 80 and 100 Hz are concerning.



(a) (b)

Figure 4: Comparison of measured and simulated spatially averaged reverberation times for the loudspeaker (black lines) and the woofer (gray lines). (a) empty chamber. (b) occupied chamber.



(a) (b)

Figure 5: Comparison of the measured and simulated absorption coefficients. (a) chamber walls, (b) material sample.

5.4 Transfer functions

The empty chamber's measured and simulated transfer functions are compared in Fig. 6. In general, there is some agreement between the transfer functions; however, noticeable differences appear at certain frequencies. The cause of these discrepancies is not immediately apparent. It is possible that the positioning of the sources, or their acoustic centers, is not identical to those in the simulation.

For instance, at approximately 48 Hz, we observe a significant difference. An eigenvalue analysis of the chamber reveals two eigenvalues in this frequency range, with resonance frequencies around 48 Hz and 49 Hz. For the 48 Hz mode, there is a pressure minimum (a node) at the speaker location, whereas for the 49 Hz mode, there is a pressure maximum (an antinode). (Please refer to Fig. 7 for mode shapes.) One would expect that the 48 Hz mode would not be excited while the 49 Hz mode would be, but in the measured data the opposite seems to be true. Upon examining Fig. 6a and Fig. 6b, it appears that the 48 Hz mode is present in the measured data, while the 49 Hz mode is reflected in the simulations. Clearly, this study could be improved by increasing the precision of both the experimental and simulation setups.

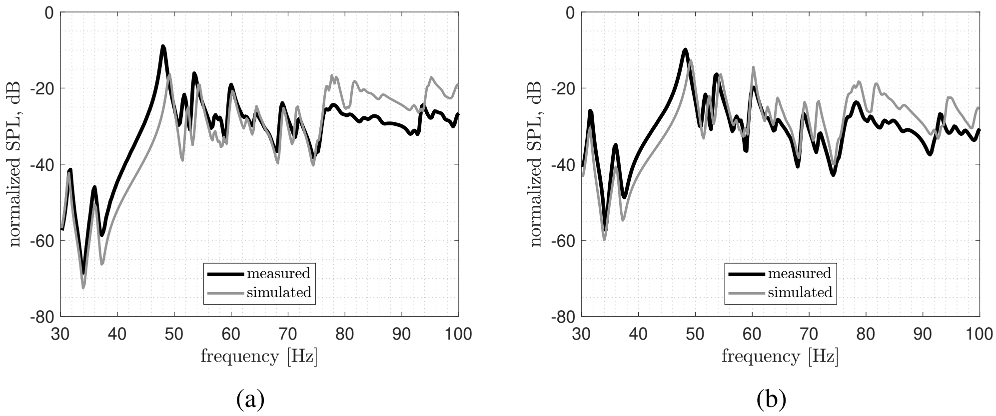


Figure 6: Comparison of empty chamber transfer functions at measurement position 1. (a) loudspeaker, (b) woofer.

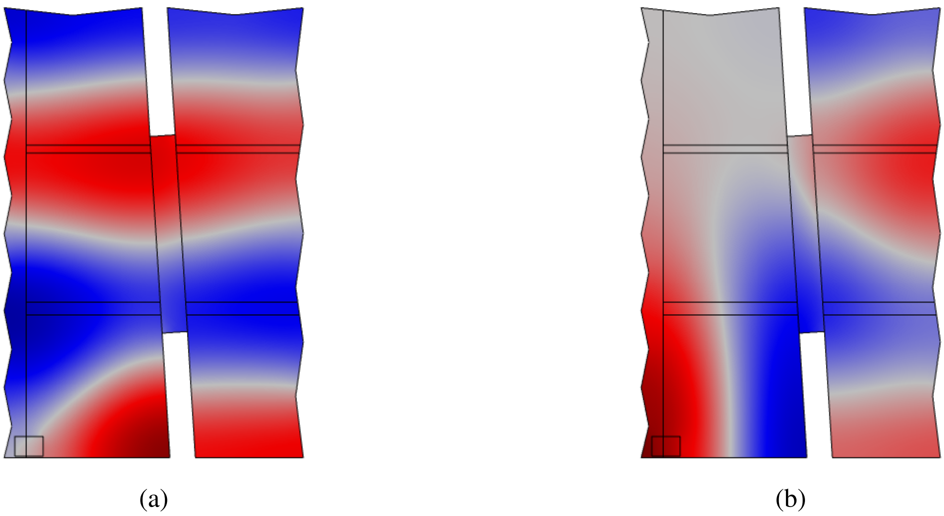


Figure 7: Modes of the empty chamber (a) 48 Hz, (b) 49 Hz.

6. DISCUSSION

While the results presented in this paper are promising, the comparisons between the measured and simulated data shown in Figs. 4, 5, and 6 indicate that further work is necessary for this method to be effectively used in impedance measurements. Some of the assumptions made to derive the impedance estimates may need to be modified. For example, we have assumed that the surface impedances of the floor, walls, and ceiling of the reverberation chamber are spatially uniform, which may be an oversight. Efforts should be made to quantify the impedances of all surfaces in the reverberation chamber, possibly including ventilation systems. The differences between the chamber eigenvalues computed for the two different sound sources, shown in Fig. 2, are concerning. In theory, since the chamber eigenvalues are global quantities, these should not depend on the sound source. The observed differences result in similar differences between the impedance estimates, shown in Fig. 3. The cause of the observed differences between the chamber eigenvalues is unknown. A detailed analysis of the measurement setup is needed to refine the input data for the impedance estimation method.

7. CONCLUSION

Measuring the normal impedance of a material sample at low frequencies can be challenging. In this study, we employed an eigenvalue-based method to estimate both the impedance of the walls of an empty reverberation chamber and that of a material sample at frequencies below 100 Hz. The impedance estimation method yielded two different results for two different sound sources, due to unexpected variations in the chamber's eigenvalues. The cause of the discrepancy remains unclear. Therefore, while the method is theoretically sound, it may produce unreliable impedance estimates due to measurement noise and uncertainties in the input parameters for the inverse model.

Future research should aim to minimize the uncertainties associated with this impedance estimation method. For example, more detailed descriptions of the chamber's geometry, the sound source, and the properties of the medium could improve the accuracy of the estimates. Additionally, reevaluating the assumption that a spatially uniform impedance adequately represents the impedance of the chamber walls may also lead to better estimations.

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