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GENERATION OF SOUND WAVES BY NONLINEARLY EVOLVING TWO-DIMENSIONAL COHERENT STRUCTURES ON A TURBULENT SUBSONIC MIXING LAYER

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ABSTRACT

Coherent structures (CS) are present on a subsonic turbulent mixing layer or a wing wake and are known to constitute an important source of aircraft noise. With these structures being treated as wavepackets of instability modes supported by the mean flow, two acoustic radiation mechanisms have been identified. The first, referred to as generalised Mach-wave radiation (GMWR), is associated with the fact that a CS undergoing amplification and attenuation consists of supersonic components in its spectral tail, which radiate to the far field as sound waves. On the other hand, the nonlinear interaction of the CS generates a temporally and spatially modulated mean-flow distortion, which emits low-frequency sound waves. This second mechanism is referred to as envelope radiation (ER). We investigate, in a common mathematical setting, these two radiation processes for nonlinearly evolving CS of planar modes, which are described by strongly nonlinear critical-layer theory. The emitted noise for each mechanism is predicted on the basis of first principles. Nonlinear effects are found to induce jittering, which enhances the GMWR significantly but suppresses ER slightly. The two mechanisms are both viable for CS of moderate amplitude, with GMWR and ER being dominant in the near downstream and sideline regions respectively.

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1. AEROACOUSTICS THEORY ON THE BASIS OF FIRST PRINCIPLES

Generation of sound waves by turbulent aerodynamic motions remains a topic not only of fundamental scientific interest, but also of technological relevance for effective reduction of noise. Earlier experiments and theoretical studies indicated that coherent structures (CS) in a free shear flow such as a mixing layer can be represented by instability modes on the turbulent mean flow ([1–3]), and recent work has led to general acceptance of this notion ([4–6]). This understanding opened a promising prospect of elucidating the mechanisms of sound generation as well as modelling noise, in terms of instability waves or wavepackets ([7, 8]). There has been resurgence of research activities in this area ([9–11]). Experiments showed that the generation of noise is primarily associated with interactions and breakdown of CS ([12–14]) and at lower Reynolds numbers with vortex pairing ([13, 15]). The intermittent nature of such events was found to be responsible for strong acoustic emission ([16–19]). Since these events and the intermittency are caused by nonlinearity, nonlinear dynamics of CS must be taken into account if the acoustic radiation is to be predicted correctly.

Unsteady fluctuations within a shear flow (termed as the ‘near field’) almost always radiate sound waves to large distances (referred to as the ‘acoustic field’). Although in the shear region the fluctuations are usually complex and energetic, they acquire a simple character of sound waves of much reduced intensity in the far field, where the shear almost vanishes. The main aim of aeroacoustics is to predict far-field sound waves thereby helping find effective means to reduce noise or mitigate its impact. For that purpose, it is necessary to understand adequately sound generation mechanisms. There are mainly three types of methods to obtain the sound field, including (a) solving the full compressible N–S equations in a sufficiently large domain, (b) acoustic analogy and (c) aeroacoustic theory on the basis of asymptotic analysis ([20]); relatively detailed surveys on these approaches were given by [21–23]. In the present investigation, we adopt the approach (c) that views the sound waves radiated to the far field as 'ripples' of the near-field hydrodynamic/aerodynamic fluctuations. Mathematically, the acoustic field corresponds to the far-field asymptote of the latter. By analysing the large-distance asymptotic behaviours of the unsteady fluctuations, the true physical sources may be determined without the arbitrariness associated with pre-designation of 'source' and 'sound' in acoustic analogy. This amounts to probing into the precise process of acoustic radiation, through which the fundamental questions of why and how CS emit sound waves can be addressed, and the sound waves are predicted on the basis of first principles. The present paper is mainly a theoretic investigation so that we will illustrate in detail the mathematical derivations and the physical insights; the numerical results and parametric studies will be presented at the Conference.

Dynamically, the triple decomposition is adopted to decompose an instantaneous flow field into the time-averaged mean field, coherent motion and fine-scale turbulence. Of these, CS are separated from the instantaneous flow field by the (Favre) time and phase averages. The effect of small-scale turbulent fluctuations on CS is modelled by a gradient model with possible time delays. A CS, represented by a wavepacket of instability modes, undergoes amplification in the upstream region, which is well described by linear stability theory. Our focus is on the streamwise region where the CS is nearly neutral and thus prone to nonlinear effects ([24]). Near the neutral position, a CS in the main part of shear layer can be represented as a modulated travelling wave, namely,

$$\tilde{q}(\tau,\bar{x};t,x,y) = \epsilon A^{\dagger}(\tau,\bar{x})\hat{q}_0(y)\,\mathrm{e}^{\mathrm{i}(\alpha_N x-\omega_N t)} + \text{c.c.} + O(\epsilon^{3/2}), \tag{1}$$

where q represents any of velocity components, temperature, density and pressure; $\alpha_N, \omega_N \in \mathbb{R}$ are the wavenumber and frequency of the locally neutral mode respectively; $\epsilon \ll 1$ is a measure of its magnitude, and $\hat{q}_0(y)$ characterises its transverse distribution; $A^{\dagger}$ is an amplitude function of the slow temporal and spatial variables,

$$\tau = \epsilon^{1/2}t, \qquad \bar{x} = \epsilon^{1/2}x/c_N, \tag{2}$$

with $c_N = \omega_N/\alpha_N$ being the phase speed of the neutral mode. Note that the origins of coordinates x and $\bar{x}$ are both chosen to be the neutral position of CS. The amplitude of CS, $A^{\dagger}$, is governed by the dynamical system derived by the critical layer theory based on the high-Reynolds-number assumption. Furthermore, a modulated wavetrain may consist of discrete or a continuum of sideband modes to describe which $A^{\dagger}$ is Fourier expanded with respect to τ. This dynamical theory successfully describes the roll-up and break-up of CS, the spectral broadening, the wavepacket modulation and the amplitude jittering. The reader is referred to [21] for the detail.

2. HIGH-FREQUENCY SOUND WAVES: GENERALISED MACH-WAVE RADIATION (A LINEAR MECHANISM)

The question of great importance is the physical mechanisms and processes by which instability modes or CS emit sound waves. As will be discussed in detail in the following, there are two kinds of mechanisms of mixing noise and both can be described on the basis of first principles. In supersonic mixing layers, a supersonic-mode CS can emit sound directly in the form of Mach-wave radiation (e.g. [7, 25–28]), which stands as an efficient radiation mechanism.

In subsonic flows, the radiation mechanism is complex because instability modes propagate subsonically relative to the ambient flow, and their eigenfunctions decay exponentially in the transverse direction. Consequently these modes do not radiate directly if their amplification–attenuation is neglected ([21–23, 29, 30]). However, when the amplification–attenuation of a subsonic mode is taken into account, the tail of its envelope spectrum contains supersonic components, which radiate to the far field a sound wave with the same frequency as that of the instability mode, as was first shown by [31]. Strictly speaking, the amplitude of the emitted sound wave is exponentially small, but in practice may be significant in small angles to the downstream direction. This radiation mechanism may be viewed as a generalisation of the Mach-wave radiation, and hence is termed as 'generalised Mach-wave radiation (GMWR)' hereafter. It was studied in a simplified mathematical model by [32], who showed that for a wavepacket with Gaussian envelope, the directivity of the acoustic field exhibits an exponential dependence on the cosine of the polar angle, a feature referred to as 'superdirectivity'. This distinctive feature was observed in [33]. Employing Lighthill acoustic analogy and modelling the linear source term by a wavepacket of linearly evolving instability modes, [33] calculated the sound, and the prediction was found to be in agreement with the measurement for a range of radiation angles ([33, 34]). Recently, linearised Euler equations were solved to predict the sound radiated by a modulated mode ([35]). A simplified approach of one-way linear Euler or N–S equations, which retain non-parallel-flow effect at leading order, was proposed. This approach predicts simultaneously the linear development of an instability mode and the acoustic field radiated due to the non-parallelism induced modulation ([36]). While the GMWR itself is linear, nonlinearity influences amplitude evolution, inducing jittering and enhancing the acoustic emission. Indeed studies accounting for jittering in an *ad hoc* manner suggest that this radiation efficiency is increased ([34]) since the 'supersonic-mode spectral tail' is significantly amplified. With jittering being appropriately taken into account, which requires an investigation of nonlinear dynamics of CS, the GMWR is likely to be a viable noise generating mechanism in subsonic jets ([37, 38]). The mechanism is supported by the experimental finding that when a jet is subject to a harmonic excitation, a rather sharp peak at the excitation frequency appears in the far-field acoustic spectrum ([39]).

When the amplification–attenuation of an instability mode or CS is accounted for, supersonic components are present in the spectral tail of the amplitude function and so would emit sound waves with the same frequency as that of the fundamental. This is somewhat similar to the Mach-wave radiation of supersonic modes and is thus referred to as GMWR. Strictly speaking, however, the sound intensity is exponentially small with respect to the ratio of the wavelength of the carrier wave to the envelope length. Nevertheless, noise generated by vortical structures of a subsonic mixing layer was largely attributed to emission of this kind, to which much attention has been paid ([33, 40, 41]).

The leading-order pressure of the CS, (1), is dual-Fourier transformed with respect to (t, x) and denoting the transformed quantity in spectral space (ω, α) by a wide-hat '$\widehat{\bullet}$',

$$\widehat{\tilde{p}}(\omega,\alpha,y) \sim c_N \hat{p}_0(y)\left[\widehat{A^\dagger}\left(\frac{\omega-\omega_N}{\epsilon^{1/2}}, c_N\frac{\alpha-\alpha_N}{\epsilon^{1/2}}\right) + \widehat{A^{\dagger *}}\left(\frac{\omega_N-\omega}{\epsilon^{1/2}}, c_N\frac{\alpha_N-\alpha}{\epsilon^{1/2}}\right)\right]. \tag{3}$$

In the transverse region outside the main shear layer ($|y| \gg 1$, but we do not specify the exact asymptotic scaling), the mean flow is uniform. The eigenfunction $\hat{p}_0(y)$ decays exponentially and has the asymptotes

$$\hat{p}_0\Big|_{y\to\pm\infty} \to \mathscr{P}_{\pm\infty}\, \mathrm{e}^{\mp\alpha y\sqrt{1-Ma^2(\bar{U}_\pm - c_N)^2/\bar{T}_\pm}}, \tag{4}$$

where $\mathscr{P}_{\pm\infty}$ are two constants to be determined globally. The perturbation in this region can be expressed as $\left(\tilde{U}_M^\pm, \tilde{V}_M^\pm, \tilde{T}_M^\pm, \tilde{P}_M^\pm\right)$, and it follows that the governing equations can be reduced to a two-dimensional convected wave equation for $\tilde{P}_M^\pm$,

$$Ma^2\left(\frac{\partial}{\partial t} + \bar{U}_\pm\frac{\partial}{\partial x}\right)^2 \tilde{P}_M^\pm = \bar{T}_\pm\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\tilde{P}_M^\pm, \tag{5}$$

which implies that the perturbation is possible to acquire the character of sound waves.

Equation (5) is solved in the frequency–wavenumber (ω–α) space, where the system is converted to

$$\left[\frac{\partial^2}{\partial y^2} + K_\pm^2(\omega,\alpha)\right]\widehat{\tilde{P}_M^\pm}(\omega,\alpha,y) = 0 \tag{6}$$

with

$$K_\pm(\omega,\alpha) = \sqrt{Ma^2\left(\bar{U}_\pm\alpha-\omega\right)^2/\bar{T}_\pm - \alpha^2}\,. \tag{7}$$

The solution is found to be

$$\widehat{\tilde{P}_M^\pm}(\omega,\alpha,y) = \widehat{\mathcal{C}}_\pm(\omega,\alpha)\,\mathrm{e}^{\pm\mathrm{i}\,K_\pm(\omega,\alpha)y}, \tag{8}$$

where $\widehat{\mathcal{C}}_\pm(\omega,\alpha)$ are to be determined by matching with (3) as $|y| \gg 1$, the far-field asymptote of the wavepacket. Taking into account the fact that the 'eigenfunction', $\hat{p}_0(y)$, is obtained for a near-neutral mode, we substitute

$$\omega = \omega_N + \epsilon^{1/2}\bar{\omega}, \qquad \alpha = \alpha_N + \epsilon^{1/2}\bar{\kappa}/c_N, \tag{9}$$

with $\bar{\omega}, \bar{\kappa} = O(1)$, into (8) to match with (3). The function $K_\pm$ is purely imaginary, which is the same as the far-field asymptote of $\hat{p}_0(y)$. The matching for α close to α_N gives

$$\widehat{\mathcal{C}}_{\pm}(\omega,\alpha) = \mathscr{P}_{\pm\infty} c_N \left[\widehat{A^\dagger}\left(\frac{\omega-\omega_N}{\epsilon^{1/2}}, c_N \frac{\alpha-\alpha_N}{\epsilon^{1/2}}\right) + \widehat{A^{\dagger *}}\left(\frac{\omega_N-\omega}{\epsilon^{1/2}}, c_N \frac{\alpha_N-\alpha}{\epsilon^{1/2}}\right)\right]. \quad (10)$$

The function $\widehat{\mathcal{C}}_{\pm}(\omega,\alpha)$ can be viewed as the analytic continuation of the known function $\widehat{A^\dagger}$ onto the entire α plane. Equations (7)–(8) and (10) indicate that in the spectral tail of the amplitude function, a spectral component with a certain wavenumber α satisfying $|c(\alpha)| = |\omega/\alpha| > a_{\pm}$ ($a_{\pm}$ being the sound speeds of the two ambient flows) propagates supersonically in the ambient flow and radiates Mach waves. Equation (6) and solution (8) have been used to extrapolate the acoustic field in previous studies, where $\widehat{\mathcal{C}}_{\pm}(\omega,\alpha)$ were determined either by imposing a boundary condition ([31]) or by numerically 'patching' with the near-field solution ([42]) at a suitably chosen (but still *ad hoc*) transverse location. The present approach of asymptotic matching and analytic continuation avoids introducing such an artificial transverse location, and may be considered more satisfying.

Furthermore, the higher-order pressure in the expansion (1) contains all nonlinearly excited harmonics. These components have the similar far-field behaviours as that of the fundamental, and will also radiate sound waves via GMWR as the fundamental does.

3. LOW-FREQUENCY SOUND WAVES: ENVELOPE RADIATION (A NONLINEAR MECHANISM)

A spatially and temporally modulated wavepacket, which consists of frequency sideband components, is a more realistic representation of a CS. The nonlinear interaction of the wavepacket generates a mean-flow distortion, which is also modulated slowly with respect to time and space, and acts as an emitter to radiate low-frequency sound waves on the scale of the wavepacket envelope. This mechanism, referred to as 'envelope radiation (ER)' hereafter, was described by [21–23, 29]. Using asymptotic techniques, they described the physical process of radiation, whereby the physical sources are identified and the relation to the equivalent source is established. In the special and simplest case of a wavepacket consisting of two modes with frequencies that differ by a small amount, the emitter is the nonlinearly forced difference mode. The ER mechanism is nonlinear in the sense that the interactions leading to emission take place in the phase of nonlinear amplification and attenuation of the CS, and that the intensity of the radiated sound waves is proportional to the wavepacket amplitude squared. Such an experiment was performed, and strong sound was found to radiate at the difference frequency ([43]), supporting the present ER mechanism. It is also suggested by the noticeable feature that the dominant far-field noise concentrates in a spectral band with its peak frequency being just about $\frac{1}{10}$ of the frequency of the most unstable modes in the near nozzle region ([44, 45]) and the spectral peak of hydrodynamic fluctuations within the entire jet flow.

The mean-flow distortion caused by the nonlinear interactions is a part of the CS, corresponding to the modulated components without the fast-varying carrier wave factor. In the main layer, the expansion of the leading order takes the form $\epsilon^2(u_M, \epsilon^{1/2}v_M, T_M, p_M + \epsilon^{1/2}p_{M_2})$.

The transverse velocity at this order is governed by the long-wavelength Rayleigh equation,

$$\left(\frac{\partial}{\partial\tau} + \frac{\bar{U}}{c_N}\frac{\partial}{\partial\bar{x}}\right)\frac{\partial v_M}{\partial y} - \frac{\bar{U}'}{c_N}\frac{\partial v_M}{\partial\bar{x}} = -\mathcal{S}(\tau,\bar{x},y), \quad (11)$$

where the forcing $\mathcal{S}$ on the right-hand side is produced by the nonlinear interactions (Reynolds stresses).

Noting $\mathcal{S} \to 0$ as $y \to \pm\infty$, we have the complementary solution to (11) in physical space,

$$v_M(y \to \pm\infty) \to v_{M,c} = \left(\frac{\partial}{\partial\tau} + \frac{\bar{U}}{c_N}\frac{\partial}{\partial\bar{x}}\right) a_M^{\pm}(\tau,\bar{x}), \quad (12)$$

where $a_M^{\pm}(\tau,\bar{x})$ are arbitrary functions and may take different values for $y \gtrless y_c$.

The determination of $a_M^{\pm}(\tau,\bar{x})$, or its transformed $\widehat{a_M^{\pm}}(\bar{\omega},\bar{\kappa})$, needed two conditions that are (a) the continuous condition across the generalised inflectional point and (b) the matching condition with the far-field sound fields. The detailed derivations can be found in [21].

A two-dimensional CS with initial amplitude of $O(\epsilon)$ induces the sound fluctuations $\epsilon^{5/2}(\tilde{U}_E^{\pm}, \tilde{V}_E^{\pm}, \tilde{T}_E^{\pm}, \tilde{P}_E^{\pm})$ on the scale of $(\tau, \bar{x}, \bar{y})$ with

$$\bar{y} = \epsilon^{1/2}(y - y_c)/c_N = O(1) \quad (13)$$

being the slow transverse variable to describe the sound field. The low-frequency sound fields, in spectral space, are governed by

$$\left[\frac{\partial^2}{\partial\bar{y}^2} + \mathscr{K}_{\pm}^2(\omega,\kappa)\right]\widehat{\tilde{P}_E^{\pm}}(\omega,\kappa,\bar{y}) = 0, \quad (14)$$

subject to the Neumann boundary condition

$$\left.\frac{\partial \widehat{\tilde{P}_E^{\pm}}}{\partial \bar{y}}\right|_{\bar{y}=0^{\pm}} = \frac{1}{c_N \bar{T}_{\pm}} \left(\bar{U}_{\pm}\kappa - c_N\,\omega\right)^2 \widehat{a_M^{\pm}}(\omega,\kappa), \quad (15)$$

where

$$\mathscr{K}_{\pm}(\omega,\kappa) = \sqrt{Ma^2 \left(\bar{U}_{\pm}\kappa - c_N\,\omega\right)^2 / \bar{T}_{\pm} - \kappa^2}. \quad (16)$$

The solution can be written as

$$\widehat{\tilde{P}_E^{\pm}}(\omega,\kappa,\bar{y}) = \mp \widehat{\mathcal{E}}_{\pm}(\omega,\kappa) \widehat{a_M^{\pm}}(\omega,\kappa)\, \mathrm{e}^{\pm\,\mathrm{i}\,\mathscr{K}_{\pm}(\omega,\kappa)\bar{y}}, \quad (17)$$

where $\widehat{\mathcal{E}}_{\pm}(\omega,\kappa)$ are determined by the boundary conditions (15) as

$$\widehat{\mathcal{E}}_{\pm}(\omega,\kappa) = \mathrm{i}\left(\bar{U}_{\pm}\kappa - c_N\,\omega\right)^2 / \left[c_N \bar{T}_{\pm} \mathscr{K}_{\pm}(\omega,\kappa)\right]. \quad (18)$$

4. RELEVANT IMPORTANCE OF THE TWO MECHANICS

For a wavepacket of subsonic modes, which undergo nonlinear amplification and decay, the GMWR and ER mechanisms operate simultaneously. However, they were investigated separately in previous studies using different mathematical approaches with the presumed sources being modelled differently. As a result, the relative importance of the two mechanisms cannot be elucidated properly. Moreover, with a few exceptions the majority of the work employed linear wavepackets while it is now becoming increasingly clear that nonlinear development influences radiation significantly. In this presentation, we will carry out a comparative investigation of the two mechanisms by which nonlinearly evolving CS on a turbulent mixing layer radiate sound waves, with the aim to assess (a) the effects of nonlinearity on the two mechanisms respectively, and (b) the relative importance of the two. Similar to [34] that investigated this topic on a subsonic circular jet, the main differences are present on (a) the flow structures, (b) the far-field behaviours of the eigenfunction and low-frequency components, and (c) the directivity of ER. In detail, on a mixing layer, on the one hand, there are upper and lower sound fields for both GMWR and ER rather than just an outer one on a jet. On the other hand, the eigenfunction is lack of a $r^{-1/2}$ factor (with r being the radial variable) and the low-frequency transverse velocity keeps constant in the far-field rather than decaying algebraically on a jet . These two aspects give the ER directivity to behave like a 'quadrupole' rather than a 'monopole' or a 'dipole' on a jet (see [21] and [22] respectively).

Taking the dual-Fourier inversions of (8) and (17), we have the sound pressures in physical space. Of interest are the acoustic waves in the far field ($|y| \gg 1$ and $\bar{y} = O(1)$), for which the polar coordinates (R,φ) are introduced, where

$$R = \sqrt{x^2 + y^2}, \qquad \tan\varphi = y/x \quad (-\pi \leqslant \varphi \leqslant \pi). \quad (19)$$

Noting the definitions of $\bar{x}$ and $\bar{y}$, (2) and (13), the polar coordinates (R,φ) are also proper to describe ER by rescaling R and keeping φ. The integrals of α and $\bar{\kappa}$ can be approximated by the stationary-point method.

The overall intensity of the acoustic pressure is measured by the root–mean–square values of $\tilde{P}_M^{\pm}$ and $\tilde{P}_E^{\pm}$ according to Parseval's theorem, from which the overall directivity functions $\mathscr{D}_M(\varphi)$ and $\mathscr{D}_E(\varphi)$ are defined as the superposition of the corresponding spectrum functions $\mathscr{S}_M(\omega,\varphi)$, $\mathscr{S}_E(\bar{\omega},\varphi)$, namely,

$$\left\{\mathscr{D}_M(\varphi),\ \mathscr{D}_E(\varphi)\right\} = \sqrt{\int_{-\infty}^{\infty} \left\{ \left|\mathscr{S}_M(\omega_N + \epsilon^{1/2}\bar{\omega},\varphi)\right|^2,\ \left|\mathscr{S}_E(\bar{\omega},\varphi)\right|^2 \right\} \mathrm{d}\bar{\omega}}. \quad (20)$$

Note that we have substituted ω by $\omega_N + \epsilon^{1/2}\bar{\omega}$ for GMWR, where $\bar{\omega} = O(1)$ is the scaled frequency corresponding to the slow temporal variable τ, and the different asymptotic magnitudes are also included.

The physical dimensionless pressure generated by the GMWR and ER are thus found as,

$$\left[\tilde{P}_M^{\pm}(R,\varphi),\ \tilde{P}_E^{\pm}(R,\varphi)\right] = \frac{1}{\sqrt{R}} \left[\mathscr{D}_M(\varphi),\ \mathscr{D}_E(\varphi)\right]. \quad (21)$$

Finally, the sound pressure level (SPL) functions of the two mechanisms, SPL_M and SPL_E, are defined as

$$(\mathrm{SPL}_M,\ \mathrm{SPL}_E) = 20\log_{10}\left[\rho_0^* U_0^{*2}\left(\tilde{P}_{M,\mathrm{rms}}^{\pm},\ \tilde{P}_{E,\mathrm{rms}}^{\pm}\right)/p_{ref}^*\right], \quad (22)$$

where $p_{ref}^* = 2.0 \times 10^{-5}$ Pa is the reference pressure, ρ_0^* is the reference density chosen to be the dimensional fast-stream density and U_0^* is the reference velocity chosen to be the half of velocity difference of the two ambient flows.

5. NUMERICAL RESULTS

These theories are applied to a streamwise-slowly-varying compressible mixing layer, which is formed by two streams with velocities U_1^* and $U_2^* < U_1^*$. The mean streamwise-velocity profile is chosen to be an empirical one, e.g. a hyperbolic tangent profiled modified by a hyperbolic secant function. The mean temperature profile is related to the velocity profile via Crocco's relation.

Supported by the non-parallel mean-flow profile, we firstly calculated the eigenvalue and eigenfunction of the neutral mode to describe the linear feature of CS. Following a linear upstream condition, the CS enters the nonlinear regime, for which the coupled amplitude equation and critical-layer velocity–temperature equations are solved to obtain the nonlinear development of the amplitude of CS. Using the obtained amplitude function and some other near-flow hydrodynamical behaviours of CS, the equivalent sound sources of the two radiation mechanisms are determined. In the same mathematical and numerical frameworks, the sound spectrum, directivity and sound pressure level of the two mechanisms are calculated.

Due to the length limitations of the paper, the numerical results will be presented at the Conference.

6. CONCLUSION

In this presentation, we have carried out a theoretical investigation of two fundamental mechanisms by which nonlinearly evolving CS on a subsonic mixing layer generate sound waves. With a CS being represented as a wavepacket of instability modes, which undergo amplification–attenuation in the streamwise direction, the first mechanism is GMWR: each mode in the wavepacket consists of supersonic components in the high-frequency tail of its amplitude spectrum, which radiate sound waves with the same frequencies as those of the mode and its harmonics. The second mechanism is ER: the nonlinear interactions of the modes generate a slowly breathing mean-flow distortion, which emits low-frequency long-wavelength sound waves on the scales of the wavepacket envelope. The two mechanisms operate simultaneously for a CS (wavepacket) consisting of sideband components. In the present study, the two mechanisms are considered together in a common mathematical framework so that their relative importance can be clarified. Nonlinearity was found to reduce the amplitude but cause oscillatory attenuation, which is a form of jittering; these have important implications for the effects on the ER and GMWR respectively. By analysing the large-transverse-distance asymptotic behaviours of relevant hydrodynamic fluctuations, the emitted sound waves as well as their physical process and sources are all determined on the basis of first principles using the high-Reynolds-number asymptotic framework. For a CS of wavepacket form with a moderate amplitude, the sound waves emitted through the two mechanisms are comparable but exhibit different features. Nonlinearity enhances the GMWR dramatically, but suppresses the ER moderately. The opposite effects were attributed respectively to the jittering and attenuation mentioned earlier: the former amplifies the components in high-wavenumber tail of the amplitude spectrum whereby strengthening the GMWR, while the latter weakens the mean-flow distortion and hence the ER. The present study suggests strongly that nonlinear evolution of CS plays a crucial role and must be included in the prediction of noise. For a wavepacket with a continuum of sidebands, the GMWR is suppressed but the ER is enhanced with the increase of bandwidth. The GMWR takes place primarily in the region making small angles to the axis, beyond which the ER dominates.

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