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HIGH BULK MODULUS PENTAMODES: THE THREE-DIMENSIONAL METAL WATER

Sebastiano Cominelli[1*] **Giacomo Brambilla**[1] **Marco Verbicaro**[1]
Matteo Pozzi[1] **Gabriele Cazzulani**[1] **Francesco Braghin**[1]
[1] Department of Mechanical Engineering, Politecnico di Milano, Italy

ABSTRACT

Pentamode metamaterials (PMs) mimic the acoustic properties of liquids and offer a powerful means of controlling sound propagation. However, their design and realisation remains an open challenge, especially in three dimensions. Existing PMs rely on thin junctions to simultaneously achieve high bulk modulus, low shear modulus and adequate density. This is successful in 2D but not in 3D, where an upper bound on the bulk modulus prevents efficient interaction with underwater sound waves. Hence, a fundamentally different geometry is needed.

We first apply topology optimisation to find different geometries and test it against the design of a two-dimensional Lüneburg lens. We show that it offers greater flexibility than conventional parametric optimisation approaches and the usual geometry based on effective pins is recovered. However, extending this method to three dimensions leads to geometries that are extremely difficult to manufacture. We show that simple pin-like kinematics are no longer sufficient in 3D, but that slider-like joints are required. Applying this paradigm, we propose the first 3D periodic lattice that successfully mimics the acoustic behaviour of water at low frequencies.

Keywords: *pentamode, underwater acoustics, cloaking, phononic crystal.*

1. INTRODUCTION

Pentamode metamaterials (PMs) have attracted considerable attention for their potential in underwater acoustics, enabling applications such as acoustic lenses, waveguides and invisibility cloaks [1]. PMs are solids in which the propagation of shear waves is prevented by a low shear-to-bulk modulus ratio, allowing them to behave like artificial fluids. It is common in the literature to present the concept in a two-dimensional approximation, where the so-called bimode metamaterials (BMs) are typically designed by parametric optimisation. All current designs are based on thin junctions that act as kinematic pins, making it possible to achieve a material with the same quasi-static properties as water. The same concept has been applied to the design of three-dimensional PMs [2, 3], and materials with the same sound speed as water and tunable anisotropy have been proposed to build acoustic devices [4]. However, the acoustic impedance of such materials is two orders of magnitude lower than that of water, preventing effective implementation. In this work, we first rely on topology optimisation to look for a new geometry of BMs. A unit cell shape similar to those existing in the literature is recovered, with the advantage of being a more efficient and versatile approach than the usual parametric optimisation. When aiming for a 3D PM, topology optimisation provides a geometry that is extremely difficult to realise [5], so we take a step back and focus on the kinematics of a PM. We show that the pin-like junctions are no longer sufficient to achieve the high bulk modulus of water and that a more complex topology is required [6]. We therefore propose slide-like junctions and design the first material whose mechanical properties are truly indistinguishable from water in the quasi-static limit.

**Corresponding author: sebastiano.cominelli@polimi.it.*

2. TOPOLOGY OPTIMIZATION FOR BIMODE METAMATERIALS

To design a BM with a desired acoustic behaviour, we apply a density-based topology optimisation strategy to a square unit cell. For simplicity, we focus on the design of an isotropic material, targeting only the effective bulk modulus and mass density, along with a low shear modulus. We define the inverse problem for a unit cell as:

$$\begin{aligned} &\min_{\rho} \mathbb{C}^{\mathrm{H}}_{33} + \alpha_T C_T \\ &\text{subject to :} \\ &\quad \mathbb{C}^{\mathrm{H}}_{11} < \kappa_L \\ &\quad \mathbb{C}^{\mathrm{H}}_{12} > 0.99\, \kappa_L \\ &\quad 0.99\, V_L < V < V_L \\ &\quad 0 \le \rho_k \le 1 \quad \forall k = 1, \dots, N \\ &\quad \text{Cell problem} \\ &\quad \text{Thermal problem} \end{aligned} \tag{1}$$

where the tensor $\mathbb{C}^{\mathrm{H}}$ is the homogenised elasticity tensor [7]. The objective function minimises a combination of the shear stiffness $\mathbb{C}^{\mathrm{H}}_{33}$ and the thermal compliance C_T, scaled by a weight factor α_T. The thermal compliance acts as a proxy for joint thickness, introduced following the virtual temperature method so that overly thin or disconnected regions are penalised [8]. The constraints enforce that the effective moduli $\mathbb{C}^{\mathrm{H}}_{11}$ and $\mathbb{C}^{\mathrm{H}}_{12}$ match the target bulk modulus κ_L, while a volume constraint ensures that the amount of solid material remains within bounds (V close to a prescribed value V_L). The design field $\rho \in [0, 1]$ describes the solid and void regions inside the cell. We apply this topology optimisation framework to design the unit cells of a 2D Lüneburg lens [9] for underwater acoustics. A circular Lüneburg lens is discretised into square unit cells that are optimised independently. The target properties for each cell are given by the local bulk modulus κ_L and density ρ_L, derived from the analytical profile of an impedance matched lens. The results demonstrate that the optimised structures provide accurate refractive index control, preserve constant impedance, and enable fluid-like wave propagation, making them ideal candidates for underwater metamaterial applications. Figure 1 shows that the effective properties of the cells are very close to the desired ones.

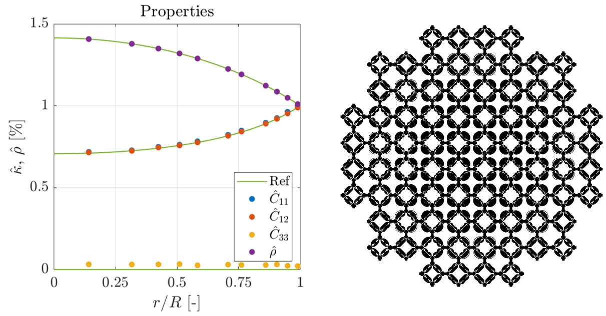

Figure 1: Left, properties of the optimised cells compared to the target; right, microstructure of the lens.

3. TRANSITION TO 3D: A NEW KINEMATIC APPROACH

The inverse design of a 3D PM can be solved by topology optimisation as before, but the solution is generally hard to realise, see for example [5]. In fact, while thin joints are sufficient for 2D metaliquids, applying this approach to three dimensions limits the bulk modulus to two orders of magnitude less than a liquid [6]. The effective stress tensor $\mathbb{C}^{\mathrm{H}}$ of an elastic network is computed as the summation over the boundary nodes of a cell as [10]

$$\mathbb{C}^{\mathrm{H}} = V^{-1} \sum_i \mathbf{r}_i \otimes \mathbf{f}_i, \tag{2}$$

where $\otimes$ is the outer product, $\mathbf{r}_i$ the position of the i^{th} node, and $\mathbf{f}_i$ the force applied to it. Note that the moments acting on the nodes have no effect. Therefore, we relax the hinge-hinge kinematics and instead design a link that acts as a slider, transferring not only axial forces but also momenta from one node to another. To address this, we propose a novel 3D pentamode metamaterial (PM) geometry inspired by the kinematics of ropes. Instead of relying on traditional pin-like junctions, our structure uses thin fibre links that behave like slider joints. This design enables the PM to replicate the acoustic properties of water in the low-frequency regime, achieving a near-perfect pentamode response. The proposed design is illustrated in Figure 2, where each titanium unit cell consists of massive nodes connected by multiple thin fibres acting as sliders. Repeating this unit cell results in a face-centred cubic lattice whose effective elastic tensor is:

$$\frac{\mathbb{C}^{\mathrm{H}}}{\kappa_{\mathrm{H_2O}}} = \begin{bmatrix} 1.04 & 1.01 & 1.01 & 1.4\times10^{-5} & -2.9\times10^{-5} & -2.1\times10^{-5} \\ & 1.04 & 1.01 & 1.9\times10^{-5} & -2.2\times10^{-5} & -1.9\times10^{-5} \\ & & 1.04 & 1.2\times10^{-5} & -2.3\times10^{-5} & -1.8\times10^{-5} \\ & & & 0.039 & -5.2\times10^{-6} & -5.8\times10^{-6} \\ & \text{sym} & & & 0.039 & 5.7\times10^{-6} \\ & & & & & 0.039 \end{bmatrix}, \tag{3}$$

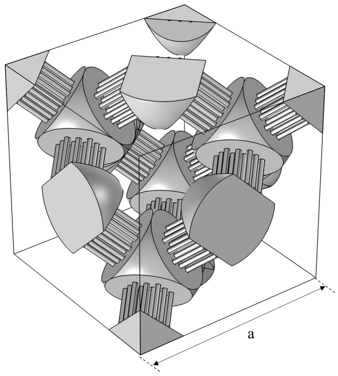

Figure 2: The PM lattice of the metal water.

where $\kappa_{\mathrm{H_20}} = 2.2\,\mathrm{MPa}$ is the bulk modulus of water. The tensor $\mathbb{C}^{\mathrm{H}}$ is very close to that of water, with the terms in the upper left 3×3 matrix close to 1 and the remaining terms very small.

4. CONCLUSION AND FUTURE PERSPECTIVES

In this work, we propose two different approaches to the design of pentamode metamaterials aimed at replicating the acoustic properties of liquids. Topology optimisation has proven to be a flexible strategy for designing bimode materials for underwater applications, ensuring both manufacturability and precise control over the acoustic parameters.
When extending the design to three dimensions, a more complex topology is required. To address this, we introduce fibre-based slider-like connections that overcome the limitations of traditional pin-junction designs, achieving a nearly ideal pentamode material with a bulk modulus comparable to that of water. The proposed 3D structure exhibits a low shear modulus and impedance matching with water, making it a promising candidate for underwater acoustic applications such as invisibility cloaking. As a next step, we are preparing for experimental validation, which will include the fabrication of the proposed structures and their acoustic characterisation.

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