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INVESTIGATING THE ACOUSTIC RADIATION OF COHERENT STRUCTURES IN A SUBSONIC JET

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ABSTRACT

In the present paper, a non-Lighthill approach based on first principles is developed. The physical sound sources were clearly defined, and two acoustic radiation mechanisms were investigated. The sound field spectrum obtained by calculation was found to be in relatively good agreement with previous study.

Keywords: *jet noise; hybrid methods; equivalent sound source; acoustic radiation mechanism.*

1. INTRODUCTION

Turbulent jets at high subsonic Mach numbers account for a significant proportion of commercial aircraft noise. Despite considerable advances in the characterisation and reduction of jet noise, this subject persists as a salient issue, primarily due to a lack of understanding of the physical mechanisms underlying sound generation.
A turbulent jet is composed of two distinct structures: large orderly coherent structures (CS) and fine-scale random fluctuations [1]. A substantial corpus of research has indicated that the generation of noise is predominantly associated with the evolution and interaction of coherent structures [2-4].
Computational aeroacoustics, defined as solving the full compressible Navier-Stokes equations numerically within a sufficiently large domain encompassing both the near and far fields, is an extremely challenging task, even for the most powerful computers at present.
Theoretical approaches based on reduced equations are indispensable, and two theoretical methods have been developed by taking different perspectives to generation of noise by unsteady flows. The primary approach to aeroacoustics is acoustic analogy, a concept initially proposed by Lighthill [5].
A methodology that differs from the acoustic analogy is the asymptotic approach, which examines the sound waves radiated to the far field as the asymptotic behaviour of the near-field hydrodynamic fluctuations. The mechanism of acoustic radiation was initially investigated using matched asymptotic expansion by Tam & Burton [6]. Building upon this, Zhang and Wu [7-10] have extended the analysis to study the nonlinear evolution and acoustic radiation of coherent structures in subsonic turbulent jets. The physical sound source can be derived directly.
The objective of the present study is to formulate a hybrid method based on first principles. The near/hydrodynamic field is simulated numerically, while the far acoustic field integrates with asymptotic theoretical analysis. The physical sound sources were calculated, and two acoustic radiation mechanisms were identified.

2. HYBRID METHOD

The two-dimensional compressible Navier-Stokes equations in cylindrical coordinates are solved by a finite-difference DNS code. The axial and radial derivatives are computed with sixth-order center difference scheme, while the second-order total-variation-diminishing (TVD) Runge-Kutta scheme is used for time integration.
According to latest research, we know that the nonlinear evolution of large-scale coherent structure produces the

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equivalent sound source $\widehat{a_M^+}$ consist of three parts, namely $\mathcal{U}$, $\mathcal{S}$ and $\mathcal{Y}$. The formula is as follows, and only the upper half of the solution of the free shear flow is considered:

$$\widehat{a_M^+}(\omega,k)=\frac{\hat{\mathcal{E}}_-(\omega,k)}{\hat{\mathcal{E}}_++\hat{\mathcal{E}}_-}\left[\frac{ik\hat{\mathcal{U}}(\omega,k)}{c(k-\omega)^2}+\int_{-\infty}^{\infty}\frac{\hat{\mathcal{S}}(\omega,k,y)}{(\bar{U}k/c-\omega)^2}dy\right]-\frac{\hat{\mathcal{Y}}(\omega,k)}{\hat{\mathcal{E}}_++\hat{\mathcal{E}}_-} \quad (1)$$

where $\mathcal{U}$ is contributed by the interactions in the critical layer, corresponding to the streamwise velocity jump across the critical layer, $\mathcal{S}$ is reflected in the effect of the small-scale structure on the coherent structure, $\mathcal{Y}$ represents the back action from the sound field acted on the source, corresponding to the pressure jump, k is the axial wavenumber, c is the phase speed, y represents the transverse direction, $\mathcal{E}_+$ and $\mathcal{E}_-$ are functions related to the boundary conditions, $\hat{\blacksquare}$ means the Fourier transform of the corresponding quantity. More details can be found in [9].
Then the acoustic pressure $\tilde{P}^+$ can be obtained by integrating the sound source. Here, we directly give the final far-field pressure at any equivalent distance $\bar{R}$ and angle θ to jet axis as follows,

$$\tilde{P}^+\sim\frac{1}{2\pi^2\bar{R}}\int_{-\infty}^{\infty}e^{sgn[\phi''(\omega,k_s,\theta)]i\pi/4-i\pi/4}D_P(\omega,k_s)\,\widehat{a_{Ml}^+}(\omega,k_s)e^{i\phi(\omega,k_s,\theta)\bar{R}-i\omega\tau}d\omega \quad (2)$$

3. NUMERICAL RESULTS

In present study, the parameters of disturbance waves were listed in table 1, and ω_1, ω_2 are the two inflow primary waves. It should be noted that this simplified "double-wave" model was proposed by Sandham [11-12], and they pointed out there would be a strong nonlinear interaction creating the difference mode wave. Therefore, we calculated these classic cases to verify the feasibility of our hybrid method on the one hand, and on the other hand to analysis the mechanisms of sound radiation of this simplified model in detail.
The dimensionless parameters, Mach number and Reynolds number, are chosen the same as the experimental parameters of Stromberg *et al.* [13], which are *M=0.9* and *Re=3600* respectively.

Table 1. Parameters of inflow.

ω_1	ω_2
2.2	3.4

Figure 1 shows the instantaneous dilation rate field calculated by the present method. The sound radiation is highly directional, and noise is emitted from the vortex roll-up position, implying that the sound sources are just hidden here. The wavelength of the emitted sound corresponds to the difference mode frequency *Δω=1.2*, which means that the difference mode generated by the nonlinear interaction between two inflow disturbances dominates the acoustic radiation.

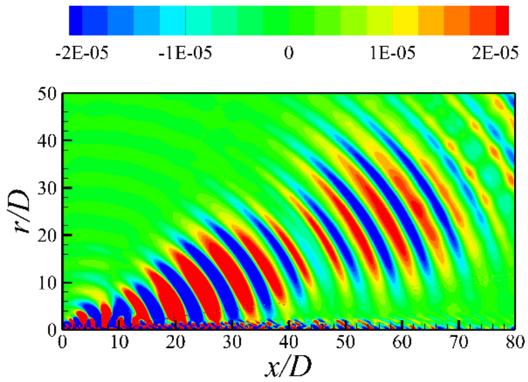

Figure 1. The instantaneous dilation rate field.

By integrating the equivalent sound source, the far-field pressure can be easily obtained. For an intuitive description, the sound pressure level was calculated and compared with the results of DNS, see in Figure 2. And the results of DNS are calculated by the empirical formula

$$SPL=20log_{10}\left(\frac{p'_{DNS}p_s}{p_c p_{ref}}\right), \quad (3)$$

where p'_{DNS} is the calculated pressure of DNS, the relative pressure $p_{ref}=2\times10^{-5}N/m^2$, p_s is the standard atmospheric pressure and $p_c=0.0072p_s$.

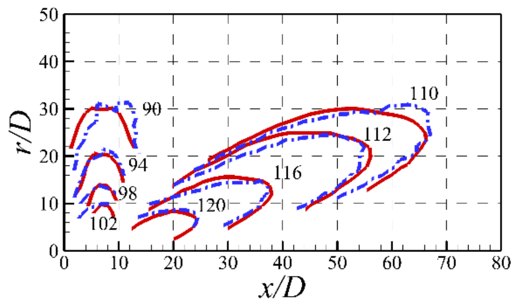

Figure 2. The sound pressure level contours. (red line for present result; blue dashed line for DNS)

It is evident that the results of the present method are exactly congruent with those of direct simulation. For the low angle region, the sound radiation exhibits super-directive characteristics at an angle of nearly *30* °relative to the jet axis. This radiation mechanism can be considered as an extension of the Mach-wave radiation.
In addition, there is another direction of radiation at an angle approximately *90* °to the jet axis. This finding is an indication of the presence of an additional high angle radiation mechanism. The nonlinear interaction of the instability waves gives rise to a mean-flow distortion. This distortion is slowly modulated in both time and space and acts as an emitter, radiating low-frequency sound waves on the scale of the wavepacket envelope at a high angle. This phenomenon is so called envelope radiation.

4. CONCLUSION

This paper presents a novel non-Lighthill hybrid method for the study of the acoustic radiation of axisymmetric modes in subsonic jets. The results, of our hierarchical hybrid methodology, exhibit a remarkable agreement with experimental observations and those obtained from DNS. It is therefore concluded that the non-Lighthill hybrid method is a feasible approach for predicting acoustic noise in subsonic jets.
The findings also indicate the presence of two distinct sound sources and two sound radiation mechanisms: general Mach-wave radiation and envelope radiation.

5. ACKNOWLEDGMENTS

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