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MATCHING PURSUIT DECOMPOSITION OF JET NEAR-FIELD PRESSURE DATA

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ABSTRACT

The Matching Pursuit (MP) decomposition is applied to an extensive set of pressure data obtained in experiments carried out in two distinct laboratories, at the University Roma Tre and at the École Centrale de Lyon. Both data sets contain pressure signals measured simultaneously in the near and in the far-field of free stream compressible circular jets of different diameters and different subsonic Mach numbers. The MP method provides a decomposition of the pressure time series into a linear combination of waveforms selected from a dictionary composed of Gabor atoms. A subset of these waveforms is chosen to provide a Reduced Order Model of the original pressure signals that matches at best the energy content at the frequencies of interest. It is demonstrated that the reconstructed signals preserve significant features related to the near-field hydrodynamic pressure field and the far-field noise generation mechanisms. Additionally, it is shown that relevant statistical properties are universal and can serve as a valuable foundation for future modeling.

Keywords: *Reduced Order Modeling, Jet Noise, Matching Pursuit*

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1. INTRODUCTION

The far-field noise in compressible jets arises from the high-speed flow exiting the jet, which is inherently unsteady and spatially random. In fact, acoustic analogies that model pressure wave radiation in subsonic jets identify the turbulent flow—represented by the Reynolds stress tensor—as the primary source of noise [1]. According to this view, experiments targeted to characterize the jet noise sources, necessities the measurement and processing of non-deterministic signals characterized by time-localized events and time-varying frequency content. To this extent, the application of Time-Frequency decomposition techniques is crucial to overcome the limitations of standard Fourier analysis that is not well-suited for extracting time-localized features.

In the present approach, to the purpose of analyzing efficiently complex signals with time varying characteristics, we propose the use of the Matching Pursuit (MP) method, a technique that has not been used extensively in aeracoustics so far, except for a few applications for beamforming analyses [2] and [3]. The MP procedure provides the decomposition of a given signal (time series) into a linear combination of waveforms taken from a predefined set of functions called Dictionary. The algorithm selects the best basis, not necessarily orthogonal, that efficiently approximates the signal of interest overcoming the limitations of other more traditional approaches, such as the Short Time Fourier Transform, or the Wavelet Transform.

The objective of this work is to demonstrate the method's capability to provide a reliable reduced order modeling of jet noise data while preserving relevant physical properties related to both the evolution of near-field pressure and its correlation with the far field. Additionally, potential advancements in jet noise modeling are outlined, along with examples of statistical outcomes that highlight universal features.

2. THE PROCESSING METHOD AND THE EXPERIMENTAL DATABASE

The main features of the MP technique are briefly worked out in the following, while details can be found in the literature. This method has been introduced by Mallat & Zhang [4] and has been successively applied for the analysis of temporal signals or images in several fields out of aeroacoustics or aerodynamics (e.g. [5-7]). The MP decomposition provides an optimal non-linear approximation of a signal by adaptively choosing the proper basis depending on the signal nature. The basic idea is to construct an optimal basis, not necessarily orthogonal, that approximates efficiently the signal of interest. The MP algorithm iteratively constructs an approximation by selecting a set of elementary waveforms, called *atoms*, chosen from an over-complete set of functions, called Dictionary, such that they represent at the best the inherent signal features. To the purpose of the present application, the dictionary is composed of Gabor atoms that provide optimal joint time–frequency localization. The Gabor dictionary indeed contains functions obtained by the combination of a Gaussian envelope of variable time location and extension, with a cosine function of variable frequency. The extraction of the atoms is based on an iterative process that maximizes the correlation between the signal reconstructed through the combination of the atoms and the original signal. We remind to the literature for the details on the procedure and the implementation of the algorithm.
In the present analysis, the MP processing procedure has been applied to two distinct data sets obtained in experiments carried out in two different facilities, hereinafter denoted as ECL and UR3.
The experimental database ECL consists of near and far-field pressure signals synchronously measured in the anechoic wind tunnel of the Laboratoire de Mecanique des Fluides et d'Acoustique (LMFA) of the Ecole Centrale de Lyon. The setup provided near field pressure measurements using a linear array of microphones and far-field pressure measurements through a microphone array displaced on a circular arc. Details about the experimental setup can be found in [8].
The experiment denoted as UR3, has been carried out in the semi-anechoic chamber available at the Laboratory of Fluid Dynamic "G. Guj" of the University Roma Tre of Rome. Also in this case near field measurements have been carried out through a linear array of microphones whereas only one microphone was located in the far field. Details about this second setup are given in [9].
We underline that in both experiments the Mach numbers were subsonic and varied between M=0.6 and M=0.9. On the other hand, the jet diameters in the two experiments were significantly different therefore corresponding to different Reynolds numbers.

3. RESULTS

As pointed out above, the Dictionary used in the present analyses is composed of Gabor atoms obtained by a combination of a trigonometric function with a Gaussian envelope. Figure 1 reports examples of those atoms corresponding to different scales. It is pointed out that the minimum and maximum time-scales are selected in order to be compatible with the frequency range of interest for the present applications.
An example of a signal reconstructed through the combinations of atoms selected from the Dictionary is given in Figure 2. It is shown that the reconstructed signal consists of a few selected atoms randomly distributed in time (thus with a random phase), having different frequencies and amplitudes. Indeed, to the purpose of providing a reduced order modeling of the measured pressure signals, in the present analysis the reconstruction is accomplished using only a few thousand atoms. It is also important to point out that the frequency, amplitude, and timing of the selected atoms are not predetermined, but rather derived as outputs of the MP algorithm. In this sense the MP method is considered an *adaptive* approach.

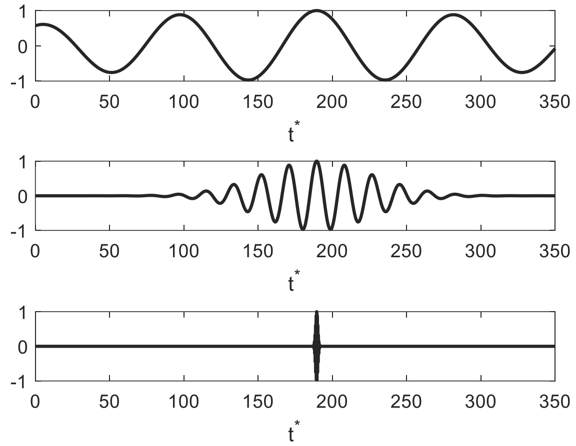

Figure 1. Examples of Gabor functions composing the Dictionary. Amplitudes are normalized in order to vary between -1 and 1.

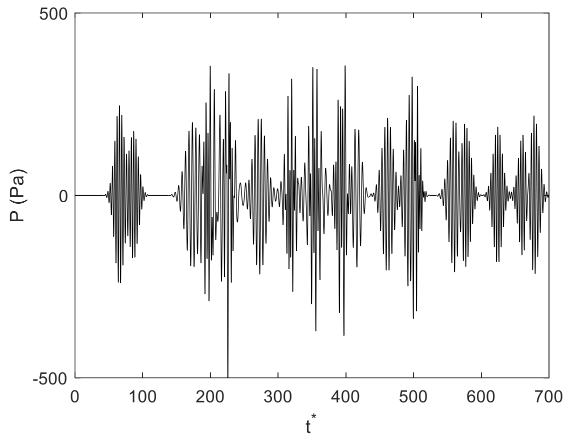

Figure 2. Examples of signal reconstruction. The case reported refers to the UR3 experiment at M=0.7 and x/D=2.

The adaptive nature of the algorithm is demonstrated by the spectral content of the reconstructed signals. An example is given in Figure 3 along with the spectrum of the original signal. The two spectra look very similar in the region where an energy bump related to the Kelvin-Helmholtz (K-H) instability mechanisms, is present. According to the nature of the algorithm, a greater number of atoms results in a more accurate reconstruction of the original signal. However, this analysis demonstrates that a limited number of atoms can still yield a satisfactory reproduction of the signal, particularly within the frequency range that is of primary interest from a physical perspective.

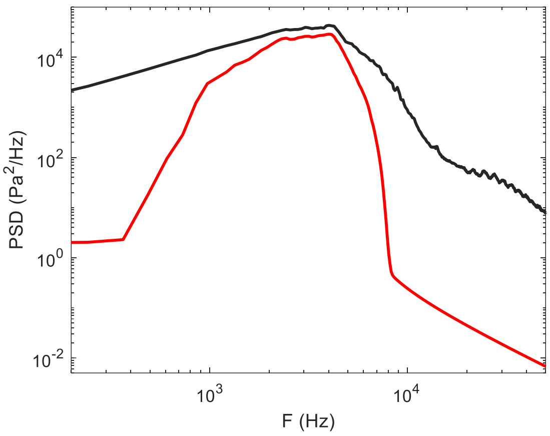

Figure 3. Examples of PSD of the original (black) and reconstructed signal (red). Same case as Figure 2.

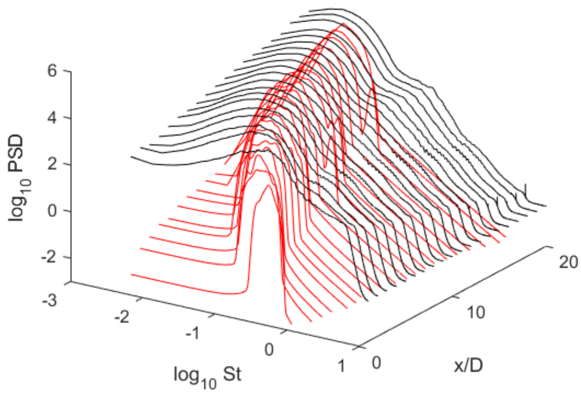

Figure 4. Fourier auto-spectra of the original (black curves) and reconstructed (red curves) signals. Data refers to the UR3 database at M=0.9.

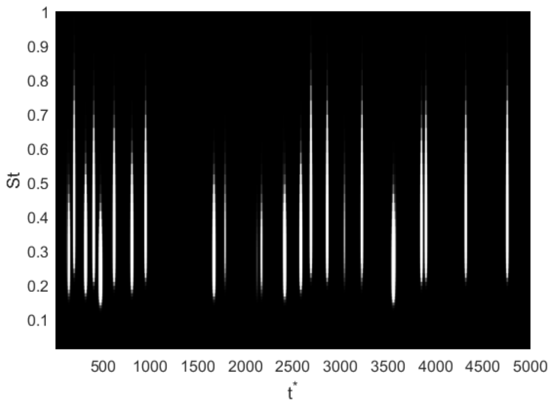

Figure 5. Modulus of the Wavelet Cross-Spectrum between near-field reconstructed signals at x/D=1 and x/D=3. Data refers to the ECL database at M=0.9.

The capability of the method to preserve the physical properties of the original pressure field is examined by analyzing the evolution of atoms in the near field and their correlation with the far field.
The evolution of the auto-spectra in the streamwise direction is examined and examples are reported in Figure 4. It is shown that even though more narrow-banded, the spectra of the reconstructed signals reproduce correctly the energy content of the original signals in the region corresponding to the K-H instability frequency. The reconstruction is satisfactory not only in terms of amplitude but also in terms of its evolutionary behavior along x/D. Similar results are obtained in all the other cases and not reported here for brevity.
Another way to assess the physical significance of the extracted atoms is to verify their *convective* nature, that is to verify whether their trace is presented over different distances from the jet exit. Indeed, atoms that are artifacts generated by the algorithm should not exhibit any coherence in the streamwise direction. Due to the local nature of the atoms in the time frequency domain, the only way to extract a localized coherence is to compute the Wavelet Cross Spectrum (WCS) and an example is reported in Figure 5. The WCS is obtained using a consolidated algorithm [10] and the example reported refers to near-field microphones separated by a distance of 2D. The trace of the atoms is clearly visible (white stripes) showing that most of them are physically correct and are not artifacts generated by the algorithm. It has been checked that the percentage of atoms showing zero correlation length, and thus originated by spurious effects of the algorithm, is less than 10%.

In order to maximize the reliability of the results, all the reconstructed signals are distilled from the spurious atoms and only those showing a non-zero coherence over a distance of at least two diameters are preserved.

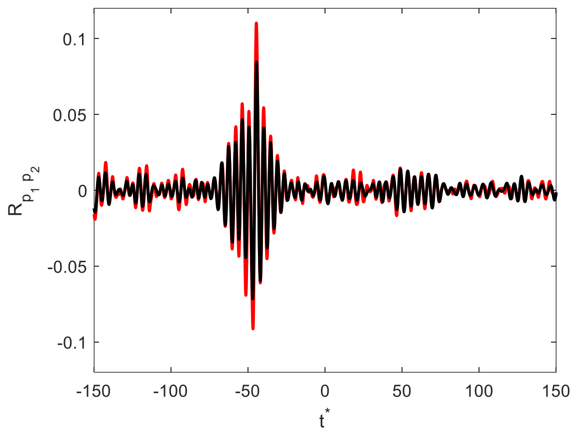

Figure 6. Examples of near- far-field cross-correlations of the original (red) and reconstructed signal (black). Same case as Figure 4.

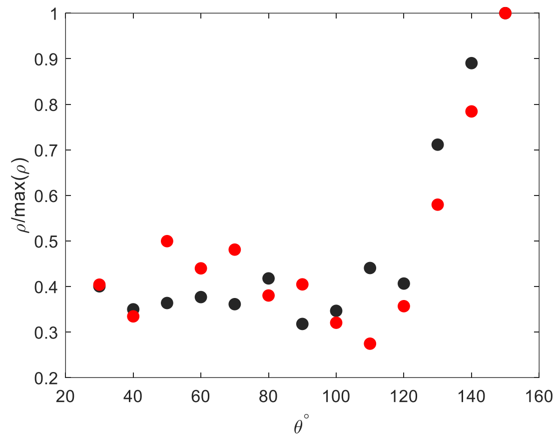

Figure 7. Normalized cross-correlation maxima computed between a near-field microphone located at x/D=1 and all the microphones in the far-field. Black dots refer to the original signals, red dots to the reconstructed ones. The analysis refers to the ECL database at M=0.9.

The set of reconstructed near field pressure time series are correlated with the far field signals in order to verify their connection with the radiated noise. An example of correlations obtained from the UR3 database is given in Figure 6. The original and reconstructed signals exhibit a very similar behavior both in terms of amplitude and in terms of the time location of the correlation maxima. This time delay, when converted to space using the speed of sound, reproduce exactly the distance between the near field and the far field microphone.

The ECL database allowed us to better clarify the correlation between near and far field microphones thanks to the availability of a polar arc in the far field. In order to characterize the directivity patterns, correlations between near field microphones and all the far field sensors have been computed. To this purpose, the normalized amplitudes of the correlation maxima have been extracted. Results obtained at M=0.9 are reported in Figure 7 and 8 considering a reference near-field microphone located within the jet potential core and in the turbulent region, respectively. A normalization has been employed in both cases to emphasize the dependency upon the polar angles rather than the differences in amplitudes, which are inevitably significant due to the limited number of atoms used to reconstruct the signals.

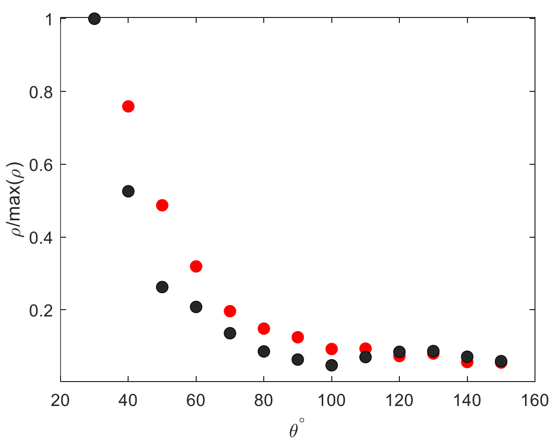

Figure 8. Same as Figure 5 but considering a near-field microphone at x/D=8.

The amplitudes are normalized with respect to the one exhibiting the maximum correlations. As expected, for the microphone located close to the jet exit (Figure 7) the maximum directivity is achieved at around 150° and the trend is very similar for the original and reconstructed cases. When the far field is correlated with a near-field microphone located far from the jet exit, the trend completely changes and the maximum correlation corresponds to very small angles. This is shown in Figure 8 where, again, reconstructed and original cases exhibit very simal behaviors.

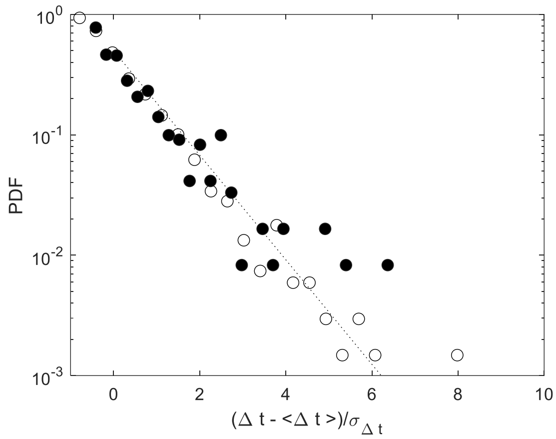

Figure 9. PDFs of the waiting time computed from the UR3 (empty symbols) and the ECL (full symbols) datasets. Both cases are referred to M=0.9 and x/D=3.

The results presented thus far highlight the physical significance of the reconstructed signals, as they accurately reproduce the near-field pressure properties and exhibit proper correlation with the far-field pressure, thereby maintaining the connection with the radiated noise.

The temporal behavior of the extracted waveforms is further explored by extracting the statistics of the time delay between successive atoms. This quantity is often denoted as the waiting time and its statistics is connected with intermittency, an intrinsic property of the turbulent flow. The analysis provides the Probability Distribution Functions (PDF) reported in the so-called reduced form, i.e. by subtracting the mean value from the analyzed random variable and dividing by the standard deviation. Examples are given in Figures 10 where explanatory results obtained in the two experiments are reported. According to the larger number of acquired samples, a better statistical convergence is achieved from the UR3 database but a very good agreement among the normalized PDFs is observed in all cases. For the cases reported, the PDFs exhibit a pure exponential decay, and the exponential fit reported in the figures is very simple, its analytical expression being $F(x) = 0.5\, e^{-x}$.

The same result has been obtained from all the other cases confirming that the time of appearance of the atoms follows a universal statistics.

This universal form of the PDF, together with other statistical properties of the waveforms, can be used to set up a model for reproducing the jet dynamics responsible for the noise emission. This task remains an objective for future studies currently being conducted by the authors, and preliminary results will be presented at the conference.

4. CONCLUSIONS

The MP procedure has been applied to a comprehensive database comprising pressure signals from two distinct laboratory experiments. Both experiments provide pressure signals in the near and far field of circular compressible jets operating at subsonic Mach numbers.
The main outcomes obtained can be summarized as follows:

- The MP procedure offers a reduced-order representation of the original signals by reconstructing them through the superposition of a limited number of Gabor atoms extracted from a specified dictionary.
- The overall properties of the near-field pressure are correctly preserved by the reconstructed field. The modelled signals reproduce correctly different features: the energy contained by the pressure spectra around the Kelvin-Helmholtz frequency; the spectra evolution along the streamwise direction; the convective nature of the selected atoms.
- The correlation between near and far field signals is preserved. According to the expected behavior, the reconstructed signals close to the jet exit exhibit good correlations with far field microphones at aft angles. Also, the measured time delay is in line with the distance between the near and far microphones.
- The statistics of the waiting time between successive atoms are universal, supporting the notion of using the reduced-order representation of the signals as potential components for developing predictive models of jet noise.

We emphasize that the similarity of the results obtained across various configurations demonstrates their independence from flow conditions and, more importantly, from the experimental facility. This outcome represents the basis for setting up a reliable model of the atoms' properties that can be used for jet noise predictions. Further studies on this aspect are currently underway by the authors and, hopefully, will be available by the time of the conference.

5. ACKNOWLEDGMENTS

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6. REFERENCES

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