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DYNAMICAL BEHAVIOR OF A BRASS INSTRUMENT AROUND A DOUBLE HOPF BIFURCATION

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ABSTRACT

Self-sustained musical instruments, such as wind or bowed string instruments, are capable of producing sustained sounds from a continuous supply of energy. These instruments exhibit transient regimes when the sound appears and during note changes. In this study, we focus on certain transient behaviors of a soprano trombone around a double Hopf bifurcation, induced by a slide displacement. To numerically study these transients on a model, the input impedance of the instrument is interpolated between different slide positions. Linear stability analysis and time integration are applied to a brass model for this situation. These simulations show that around a double Hopf bifurcation and at constant blowing pressure, it is possible to select the equilibrium or two different periodic regimes by only controlling the slide position.

Keywords: *Double Hopf bifurcation, transients, multistability, self-sustained musical instruments, trombone impedance*

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1. INTRODUCTION

Brass instrument are self-oscillating systems whose functioning regimes depend on the resonator parameters and on the playing parameters, such as the blowing pressure or the lips resonance frequency. In the scope of dynamical systems, the oscillations of a self-sustained musical instrument usually arise from Hopf bifurcations, where a periodic solution appears and the stability of an equilibrium solution changes. The study of the Hopf bifurcations and the dynamics of a brass instrument around these bifurcations is a common approach [1]. In most works concerning this subject, the considered sets of parameters foster one specific periodic regime at a time [2]. Nevertheless, these instruments exhibit double Hopf bifurcations where two Hopf bifurcations appear for the same values of parameters. Around those double Hopf bifurcations, two different periodic regimes may appear which results in rich transients and multistability issues. In this article, we focus on the transient behavior of a soprano trombone around a double Hopf bifurcation. The model is presented in Sec. 2 along with the input impedance interpolation of a slide instrument. Linear Stability Analysis (LSA) and time integration are then applied to the model in Sec. 3 to study a moving slide scenario. Conclusions are drawn in Sec. 4.

2. BRASS INSTRUMENT MODEL

2.1 Equations of the model

We use the same model as in [3]. The lips are represented by a one degree-of-freedom linear oscillator and the resonator by a modal decomposition of its input impedance. The air flow passing through the lips is modeled by the Bernoulli equation. Overall, the the model writes:

$$\begin{cases} \ddot{x} + \frac{\omega_L}{Q_L}\dot{x} + \omega_L^2(x - x_0) = \frac{p_0 - p}{\mu_L}, \\ \dot{p}_n - s_n p_n = Z_c C_n u \quad \forall n \in [1, N_m], \\ p = 2\sum_{n=1}^{N_m} \Re(p_n), \\ u = W\sqrt{\frac{2|p_0 - p|}{\rho}} \cdot \text{sign}(p_0 - p) \cdot \max(x, 0), \end{cases} \tag{1}$$

where x is the lip position, p_n are the modal pressures, u is the air flow and p_0 is the blowing pressure. ω_L, Q_L and μ_L are the angular frequency, the quality factor and the surface mass of the lips. N_m is the number of modes used to describe the input impedance and C_n and s_n are its residues and poles. Z_c is the characteristic impedance at the input of the resonator, W is the width of the lip channel, x_0 is the lip position at rest and ρ is the air density. In the remainder, we set the following parameters:

$$\begin{cases} f_L = 500 \text{ Hz}, \\ \mu_L = 2 \text{ kg.m}^{-2}, \\ Q_L = 4, \\ x_0 = 1 \cdot 10^{-4} \text{ m}, \\ W = 8 \cdot 10^{-3} \text{ m}, \\ Z_c = 1.83 \cdot 10^6 \text{ kg.s}^{-1}\text{.m}^{-4}. \end{cases} \tag{2}$$

2.2 Studying slide instruments

To explore the behavior of a brass instrument around a double Hopf bifurcation, we choose to fix the lip parameters and to vary continuously the resonator modal parameters using a slide. Indeed, it is difficult to control and estimate the lip parameters on an experimental setup whereas it is much easier to do so with a slide position. That is why we study a soprano trombone.

In order to simulate its behavior at any slide position or with a time evolving slide position, we interpolate the modal coefficients (poles C_n and residues s_n) of the input impedance measured at various slide positions. The input impedance is measured at each slide centimeter (the resonator length modification is twice as long as the slide displacement since the pipe is 180° curved) and its modal decomposition is computed with the peak-picking toolbox [4]. As a result, the resonator is entirely described by only one control parameter d_{slide}, which facilitates numerical implementation.

Using the peak-picking toolbox [4], the input impedance can be decomposed with at least $N_m = 11$ modes at any slide position without having spurious modes. However, in the following, we will only keep two of these modes which is sufficient to highlight a double Hopf bifurcation.

3. NUMERICAL ANALYSIS

3.1 Linear Stability Analysis (LSA)

We consider two control parameters (p_0, d_{slide}) since all other parameters are fixed according to (2). Usually, Linear Stability Analysis (LSA) is used to find threshold pressures and fundamental playing frequencies at the onset of the oscillations. This corresponds to finding Hopf bifurcations in the plane (p_0, f_L). Such a representation is given in Fig. 1 for $d_{slide} = 15$ cm. The intersection of two Hopf curves corresponds to a double Hopf bifurcation (marked with stars on Fig. 1, 2 and 3). In the remainder, we keep only modes 5 and 6 which are associated with the two Hopf bifurcations that cross around $f_L = 500$ Hz. The LSA of the two modes model is plotted in Fig. 2. Reducing the model to only two modes slightly changes the location of the Hopf bifurcations, but they still cross around $f_L = 500$ Hz.

Here, we consider a fixed $f_L = 500$ Hz and a variable d_{slide}. Consequently, the Hopf curves are rather plotted with respect to d_{slide} in Fig. 3. We limit the study to $p_0 \in [0, 5]$ kPa and $d_{slide} \in [10, 16]$ cm. In this region of interest, only two periodic solutions exist along with the equilibrium, and they show a double Hopf bifurcation at $p_{0,DH} = 2.8$ kPa and $d_{slide,DH} = 14.6$ cm.

Additionally, we use Manlab and the Hill-Floquet stability analysis [5] to compute the periodic branches and their stability. In this configuration, all Hopf bifurcations are direct. For a given d_{slide}, the first Hopf bifurcation (i.e. with the lowest p_0) leads to a stable regime whereas the second Hopf bifurcation (i.e. with the highest p_0) leads to an unstable regime. For p_0 higher than the second Hopf bifurcation, this second regime becomes stable through a torus bifurcation represented by dashed lines in Fig. 3.

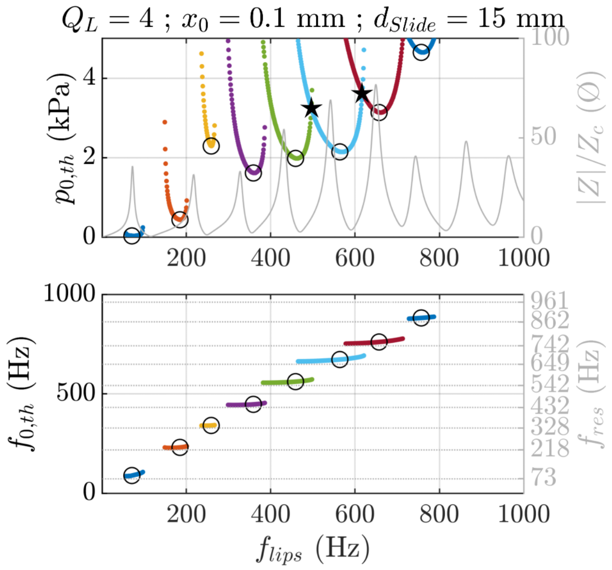

Figure 1: LSA with respect to f_L considering $N_m = 11$ modes. Pressure thresholds (top pane) and playing fundamental frequency at the onset of the oscillations (bottom pane). Double Hopf bifurcations are represented by stars. The dimensionless input impedance modulus is superimposed on the top pane whereas the resonance frequencies are represented as horizontal gray lines on the bottom pane. Each color corresponds to a different playing regime. The local minimum of $p_{0,th}$ are marked with a circle.

The regions where the second periodic regime is unstable are shaded.

3.2 Time simulation

According to Fig. 3, for p_0 slightly below the double Hopf $p_{0,DH}$, one can switch from one periodic regime to another passing through the equilibrium by varying only the slide position. We study this kind of scenario with time integration. The considered parameter trajectory is drawn in black in Fig. 3, with $p_0 = 2.6$ kPa and $d_{slide} \in [14, 16]$ cm. In Fig. 4, the time evolution of the mouthpiece pressure p computed with time integration is plotted along with the control parameters p_0 and d_{slide}, as well as its spectrogram.

Firstly, the brass model indeed passes through both

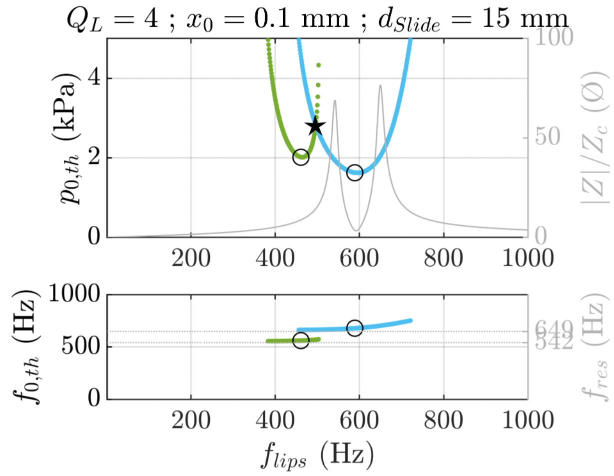

Figure 2: LSA with respect to f_L considering only modes 5 and 6. The double Hopf bifurcation is indicated by a star.

periodic regimes and the equilibrium. Secondly, the system transients last few seconds after both parameters are fixed. This observation is consistent with the fact that the system is close to its Hopf bifurcations. Indeed, in this region, leading LSA eigenvalues (i.e. with the highest real part) are close to zero, which corresponds to slow exponential growth or decrease and therefore, long transients. Finally, the amplitude of the oscillations decreases for both periodic regimes as the slide position moves toward the double Hopf bifurcation, until oscillations disappear when the system reaches the equilibrium. Since d_{slide} varies dynamically, silence appears with some delay after the slide position crossed the static Hopf bifurcation predicted by LSA [6].

This type of behavior has been observed on preliminary experiments using artificial lips and a controlled air supply which suggests that the real system also exhibits double Hopf bifurcations for some sets of parameters.

4. CONCLUSION

In this short paper, we show that brass instruments may exhibit double Hopf bifurcations for some sets of parameters. Around such a bifurcation, two periodic regimes exist and the system has a rich behavior especially regarding transients. Here, we only focus on one specific

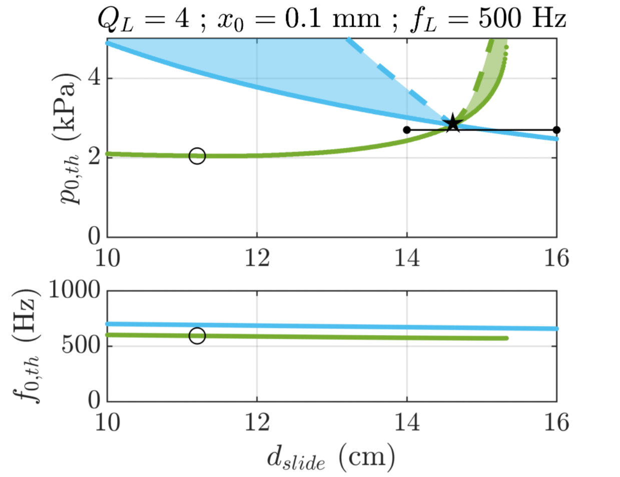

Figure 3: LSA with respect to d_{slide} considering only modes 5 and 6. Continuous lines represent Hopf bifurcations whereas dashed lines represent torus bifurcations. The double Hopf bifurcation is indicated with a star. The solution coming from the second Hopf bifurcation is unstable between its Hopf and its torus bifurcations. This is represented by shaded regions of the corresponding color. The horizontal black line represents the parameter trajectory considered for time integration in Sec. 3.2.

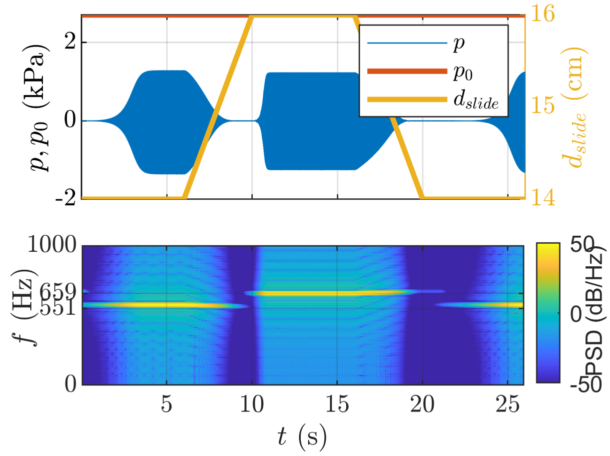

Figure 4: Top pane: time evolution of the control parameters p_0 and d_{slide} and of the resulting mouthpiece pressure p computed with time integration. Bottom pane: Corresponding spectrogram of p in terms of Power Spectral Density (PSD).

scenario where the slide position varies continuously. This scenario highlights the interest of studying the slide position as a bifurcation parameter since it induces interesting behaviors while remaining easy to control and to measure in an experimental context. To compute time integrations with a varying slide position, we propose to describe the input impedance with only one parameter by use of interpolations of the modal parameters. The multistability intrinsic to double Hopf bifurcations, as well as the associated transient behaviors, will be further explored in experimental and numerical studies.

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