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NON RECIPROCITY IN SPATIOTEMPORAL SONIC CRYSTALS

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ABSTRACT

In recent years, there has been considerable interest in spatio-temporal metamaterials and their potential applications in the field of acoustic wave propagation due to their ability to modify the behaviour of sound waves. These active materials are based on the variation in time of their physical characteristics, such as shape, position or intrinsic properties. This results in wave properties that differ significantly from those observed in time-invariant materials. Time-domain simulation methods, are a helpful tool to study the associated phenomenology.A salient property of these materials is the non-reciprocity of wave propagation, a feature that is absent in most static systems. In this study, uncoupled spatio-temporal variations of the properties of a medium are considered, with the aim to show the conditions of non-reciprocal behavior of sound waves.

Keywords: *Metamaterials, Space-Time, FDTD, Reciprocity*

1. INTRODUCTION

The spatiotemporal variation of material properties has been shown a powerful means to control wave propagation, offering new opportunities in comparison to materials that are either spatially homogeneous or temporally invariant [1–3]. Materials engineered to exhibit such dynamic characteristics require tailored design and are classified as spatiotemporal metamaterials. In the field of acoustics, this area has garnered significant research interest in recent years.

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A notable example is the Helmholtz resonator (HR) with a time-dependent volume [4, 5], showing non-reciprocity as a distinctive property. In conventional materials, acoustic wave propagation remains invariant irrespective to the direction of incidence—the well known acoustic reciprocity principle, that states that an acoustic response remains the same when the source and receiver are interchanged. However, in spatiotemporal metamaterials, reciprocity can be disrupted through specific mechanisms. One such mechanism involves selective losses [6], where wave attenuation depends on the direction of incidence. Another approach relies on nonlinear coupling [7], wherein the medium's response varies according to the wave's amplitude and direction.

Temporal modulation of material properties breaks time-reversal symmetry, allowing waves to behave differently depending on their propagation path.

The finite-difference time-domain (FDTD) method is a powerful and versatile numerical tool for the analysis of spatio-temporal systems [8, 9]. This approach resolves the wave evolution in media with variable properties in time and space. The FDTD method is also used for modeling moving electromagnetic structures, where the generalization of the Yee scheme allows the treatment of arbitrary spatio-temporal configurations and addresses the challenges of modeling discontinuities in spacetime. This type of modeling is crucial for capturing effects such as non-reciprocal propagation in media with moving discontinuities. In this work, we propose a non-reciprocal system based on sequential spatial and temporal modulations. We investigate the behavior of acoustic waves in media that exhibit locallt temporal and locally spatial modulations that are applied sequentially and not simultaneously. The term sequential refers to wave-matter interaction, from the point of view of the wave propagation: the wave may experience a temporal change in the medium properties followed by a spatial change or vice versa, but not both at the same time.

The sequential application of temporal and spatial modulations breaks the symmetry of wave propagation, which leads to direction-dependent responses and thus enables non-reciprocal behavior.

2. SPACETIME METAMATERIALS

A spacetime metamaterial consists of a set of elements (or unit cells) distributed in space, whose properties vary in time. In this work, we define systems that consist of two elements: (1) a spatial, one-dimensional sonic crystal and (2) a temporal interface that instantaneously changes the properties of the medium surrounding the crystal at a given time, keeping its spatial uniformity. The transmission of a wave propagating through such a system depends on the temporal arrangement of these two elements. If the wave experiences the temporal interface before interacting with the sonic crystal, the transmission differs from the case in which the temporal change occurs after the wave has passed through the crystal. Next we review the most prominent fetures of the spatial and temporal media.

2.1 The spatial medium: A 1D sonic crystal

Sonic crystals are periodic acoustic structures that affect wave propagation through scattering and interference [10, 11]. They consist of a series of scatterers of a certain size, which are periodically arranged at a certain distance from each other, the lattice constant. This leads to the formation of band gaps in which certain frequencies do not propagate through the structure, while the rest of the bandwidth propagates completely. For a 1-D sonic crystal and an infinite number of periodic scatterers, the dispersion relation is defined by the Rytov formula [12]:

$$\begin{aligned}\cos(kd) &= \cos(k_1 d_1)\cos(k_2 d_2)\\ &\quad -\frac{1}{2}\left(\frac{k_1}{k_2}-\frac{k_2}{k_1}\right)\sin(k_1 d_1)\sin(k_2 d_2),\end{aligned} \tag{1}$$

where k_1 and k_2 are the wavenumbers inside each material, and its corresponding width. The lattice constant of the unit cell is $l_c = d_1 + d_2$.

The first Bragg bandgap occurs when the wavelength of the acoustic wave is nearly twice the lattice constant. The central frequency of the first bandgap results:

$$f_B = \frac{c_{\mathrm{avg}}}{2l_c}, \tag{2}$$

where c_{avg} is the average sound speed in the unit cell. This expression provides a useful approximation of the frequency at which destructive interference suppresses wave propagation due to the periodic arrangement of scatterers.

2.2 The temporal medium: A time interface

A temporal interface is defined as a sudden change in the properties of the medium in which a wave propagates. In acoustic propagation, these changes may concern the speed, c, or the density of the medium ρ. In this work we will focus on changes in propagation speed.

Consider that at a given time t the propagation speed changes from c_1 to $c_2 = \gamma c_1$. The parameter γ characterizes the ratio between the propagation speed after the temporal interface and that of the initial medium ($\gamma = 1$ means no time change).

When a sound wave interacts with a temporal interface, it generates two new components that propagate in different directions. A forward wave that propagates in the same direction as the incident wave, with a scattering coefficient

$$f = \frac{\xi_f}{\xi_i} = \frac{\gamma+1}{2\gamma}, \tag{3}$$

where ξ_f is the amplitude of the generated forward component and ξ_i is the amplitude of the incident wave. Secondly, a backward wave is generated. This component propagates in the opposite direction to the incident wave, with a scattering coefficient of

$$b = \frac{\xi_b}{\xi_i} = \frac{\gamma-1}{2\gamma}. \tag{4}$$

In a time interface, a frequency shift occurs while keeping the wawvelength unaltered. This frequency shift obeys

$$\omega_f = \omega_b = \gamma\omega_i \tag{5}$$

where ω_f and ω_b are the angular frequencies of the forward and backward waves respectively that differ from the frequency of the incident wave ω_i by a factor of γ.

3. RECIPROCITY ON THE TRANSMISSION

Reciprocity of transmission in acoustics is the principle that the transmission properties of an acoustic system remain unchanged when the positions of the source and receiver are reversed. Mathematically, this property is derived from the symmetry of the Green's function [13], which describes the wave propagation between two points in a linear, time-invariant medium. The reciprocity theorem, originally proposed by Helmholtz and later extended by Rayleigh [14], states that in a reciprocal system, the acoustic pressure measured at a given receiver that is due to a source at a different location is equivalent if the roles of the source and receiver are reversed, provided that the medium and boundary conditions remain unchanged.

A system is said to be reciprocal if it conforms to the principles of linearity and time invariance. Linearity is crucial for the validity of the superposition principle, which states that the response of the system to multiple inputs is equal to the sum of the responses to each individual input. Time invariance means that the properties of the system remain constant over time, ensuring that the acoustic waves propagate symmetrically. Typical reciprocal acoustic environments are homogeneous linear fluids and elastic solids, in which the sound waves follow deterministic and reversible paths.

In contrast, non-reciprocal acoustic systems disrupt this symmetry, resulting in directionally dependent wave transmission [15, 16]. The non-reciprocity of such systems can be attributed to various factors, including time-varying media, nonlinearity or external biasing effects such as moving fluids, active metamaterials or magnetoacoustic interactions [17]. For example, the presence of a uniform background flow can lead to convective asymmetry and thus alter the transmission of acoustic waves in a way that depends on the relative direction of propagation. Similarly, phononic structures with active elements can exhibit non-reciprocal behavior by dynamically modulating material properties, resulting in unidirectional wave amplification or isolation.

The study of acoustic reciprocity and non-reciprocity has significant implications for a variety of applications, including noise control, ultrasound imaging, and phononic device engineering. Specifically, the design of materials and systems that exploit non-reciprocity has the potential to enhance sound isolation and improve energy harvesting efficiency in acoustic systems.

3.1 Reciprocity on space-time systems

For the study of the effect of the reciprocity of space-time systems we have designed a model composed by a time interface and the aforementioned one-dimensional sonic crystals. In the initial model, a wave is propagating through an initial medium with propagation speed c_1 untill it interacts with a time interface as shown in Figure 1. This generates a forward and a backward wave to be generated. However, the time interface designed does not change the entirety of the sound propagation speed for the whole system. Some portions of the medium stays with a propagation speed c_1, while the rest changes to $c_2 = \gamma c_1$. This results on the emergence of a one dimensional sonic crystal in the medium. The forward wave generated by the interface soon reaches the sonic crystal and it creates a sequence of transmitted waves due to the multiple reflections within, that add up to an overall transmission of the system.

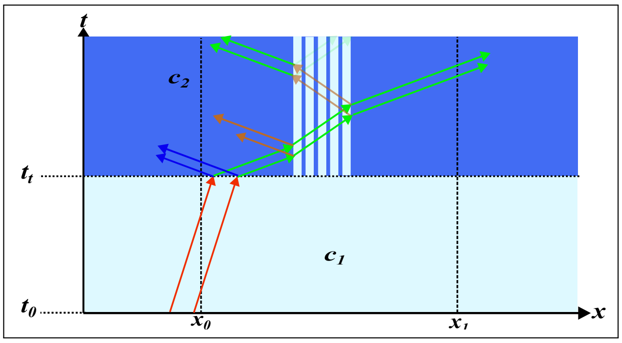

Figure 1. Spacetime schematic of the designed system. The incident wave (red) propagates through the original medium (cyan) with a propagation speed of c_1. When the time interface, at time t_t, changes the sound speed of the medium to c_2 (bright blue) it creates a forward wave (green) and a backward wave (blue). The forward wave then interacts with the sonic crystals originating reflected (brown) and transmitted waves. The overall transmission is evaluated at x_1.

To analyze reciprocity, the system is reversed. Ttemporal and spatial inversions are performed, as shown in Fig. 2. Since the time is reversed, the wave first propagates in the medium at speed c_2 before it reaches the sonic crystal. After interacting with the periodic structure and generating transmitted and reflected waves, the system experiences the temporal interface that sets the propagation speed to c_1. This transition generates a sequence of forward and backward waves that are evaluated ath the measurement point.

Comparing the transmission of each system, we can observe the effects on the reciprocity of propagation through spacetime systems when both the time and space dimensions are inverted.

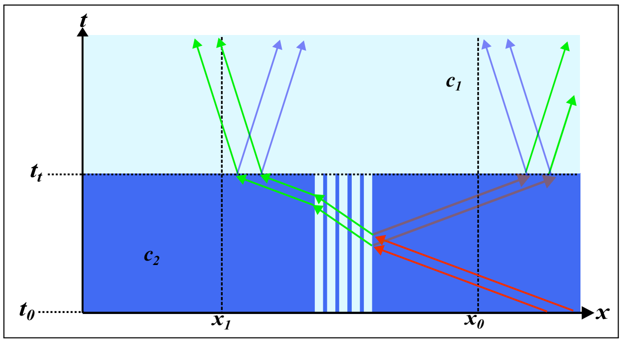

Figure 2. Spacetime diagram of the reversed system, consisting of a temporal interface and a one-dimensional sonic crystal.

4. NUMERICAL METHOD AND SIMULATION MODELS

To simulate the wave behavior in the sequential spatial and temporal configurations analyzed in this work, the numerical method employed is the Finite-Difference Time-Domain scheme centered in time (CIT-FDTD) [18].

Compared to the classic leapfrog approach [8, 9] to this method, the CIT-FDTD allows an instantaneous updating of all the pressure and velocity vectors that are used to model the sound propagation through the model. This method was first introduced to study sound propagation through air with changing properties such as a background wind that modifies the characteristics of the model. In this case, we have used this method to split the one dimensional models into a set of independent simulations, each one with its own propagation speed, that communicate through its boundaries and, since all the updates are synchronous, it guarantees a proper continuity between each sector of the simulation. Its implementation comes from the wave equations of force and momentum, defined as

$$\frac{\partial p}{\partial t} = -\rho c^2 \frac{\partial u_x}{\partial x}, \tag{6}$$

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{7}$$

where p is the acoustic pressure, u_x is the sound particle velocity, c is the sound propagation speed in the medium its propagating within and ρ is the density of the medium. To discretize these equations for its numerical implementation we need to make sure that both pressure and velocity are updated in the same instant. This can be achieved by saving a previous value of each array for its updated values to be simultaneous.

$$p_i^{n+1} = p_i^{n-1} - \frac{2\Delta t \rho c^2}{\Delta h}\left(u^n_{x_{i+1/2}} - u^n_{x_{i-1/2}}\right), \tag{8}$$

$$u^{n+1}_{x_{i+1/2}} = u^{n-1}_{x_{i+1/2}} - \frac{2\Delta t}{\rho \Delta h}\left(p^n_{i+1} - p^n_i\right), \tag{9}$$

where Δt and Δh are the time and spatial increments respectively. It is important to notice that the time increment is multiplied by a factor of two due to the fact that the time derivative takes two time steps to be performed to guarantee the the instantaneous update of the values of pressure and velocity. Each value is calculated in a discrete position i and a discrete time instant n. Following the classic implementation of the FDTD method, velocity and pressure arrays are staggered in space by $\Delta h/2$.

When an instant comes in which a change in the sound propagation speed is set to happen, the spatial increment Δ_h is updated in each sector to maintain the continuity of the Courant number through the simulation. The existing values of sound pressure and particle velocities existing in each position are then interpolated to the new set of positions originated from those changes in the spatial increments.

4.1 Simulated systems

As shown in the section 3.1, the systems investigated in this work are combinations of one-dimensional sonic crystals and instantaneous temporal interfaces. To analyze the reciprocity of this type of systems, a series of numerical simulations have been launched with different values of γ_2. For each value of γ_2, we performed three different simulations: one where the incident wave interacts first with the temporal interface (see Fig. 1), one where the wave starts to propagate in the second medium and interacts first with the sonic crystal (see Fig. 2) and a reference simulation where the system is not affected by a change in its properties.

The sonic crystal designed for these simulations consists of four rows, each with a length of 0.17 m. The bandgap of these crystals depends strongly on the propagation speed of the medium surrounding the structure, as can be seen in equation(1).

The models have a physical length of 130 m to ensure sufficient free-field propagation of the Ricker wavelet used as excitation. The spatial discretization is chosen to allow accurate simulation up to a maximum frequency of $f_{\max} = 4000\,\text{Hz}$, using 8 points per wavelength. The Courant number is set to $C = 0.25$ to ensure the stability of the simulations.

5. RESULTS

As shown in Figure 3, the incident wave propagates towards the center of the model until it interacts with the temporal interface, generating a forward and a backward wave in the second medium. The forward wave then interacts with the sonic crystal, which consists of scatterers with the propagation speed of the original medium c_1. Multiple reflections and transmission phenomena occur there, which lead to the transmitted waves reaching the receiver.

After we have reversed the space and time dimensions,

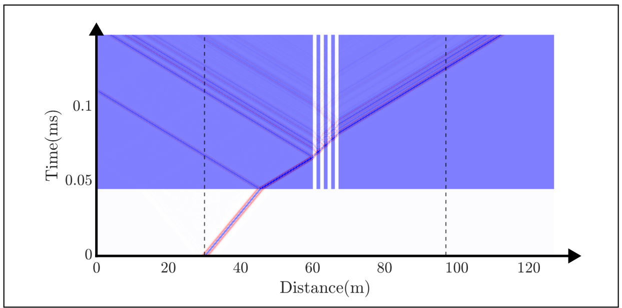

Figure 3. Spatiotemporal simulation of the propagation of a Ricker wavelet traveling through a medium with propagation speed c_1 (white) and encountering a time interface with propagation speed c_2 (blue).

we simulate the system and observe its temporal evolution, as shown in Fig. 4. The figure shows the received wave over time for two different values of γ_2. The potential reciprocity between the reversed and non-inverted system appears to be very low, as the two waveforms show significant differences in their temporal propagation at the receiver.

To evaluate the reciprocity between the original and reversed spacetime systems, we computed the cross-correlation coefficient between the corresponding waveforms, which quantifies their similarity in the time domain. The results are presented in Fig. 5.

6. CONCLUSIONS

In this work we have investigated the effects on reciprocity in the transmission of acoustic waves of a combined system of a temporal interface and a spatial sonic crystals using numerical simulations. The simulations were performed with CIT-FDTD, where the one-dimensional model could be divided into sections with different propagation speeds. The results show a strong relationship between the degree of reciprocity and the difference between the sound propagation speed in the media that make up the system: the greater the difference between γ_1 and γ_2, the lower the observed reciprocity. These results contribute to a better understanding of the behavior of spatiotemporal materials whose properties differ from those

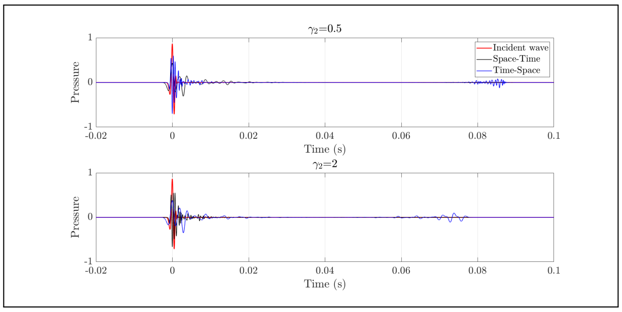

Figure 4. Time evolution of two systems with $\gamma_2 = 0.5$ (top) and $\gamma_2 = 2$ (bottom). The temporal response to an incident wave (red) is shown for the systems in which the time interface interacts with the first wave (black) and in which the initial propagation speed of the medium corresponds to $c_2 = \gamma_2 c_1$ and the sonic crystal interacts with the incident wave in the first place (blue).

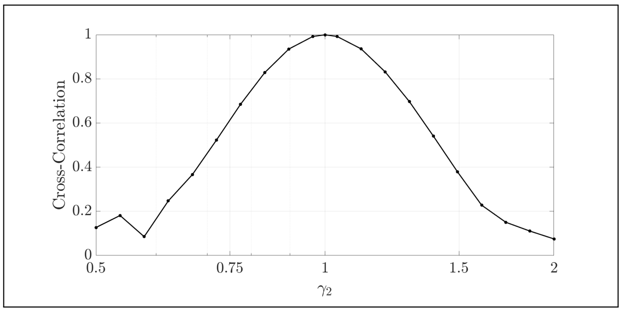

Figure 5. Cross-correlation coefficient of transmitted waves in time-space and space-time systems with the same value of γ_2. If the value of the cross-correlation is 1, the waves are identical and the systems therefore exhibit reciprocity.

of conventional static media and allow advanced control of sound wave propagation. Future work should explore more intrincate spatiotemporal systems, such as moving interfaces, time crystals or spatiotemporal slabs, to further investigate the limits of acoustic reciprocity.

7. ACKNOWLEDGMENTS

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