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OPTIMIZING LOCALLY RESONANT ELEMENTS DISTRIBUTION ON ISOTROPIC PLATE FOR INCREASED SOUND INSULATION AND MASS REDUCTION

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ABSTRACT

Locally resonant metamaterials enhance sound and vibration reduction within specific frequency ranges when attached to a base plate. Typically, these metamaterials are designed with periodically distributed elements across the surface, maintaining subwavelength spacing. Using periodic boundary conditions simplifies calculations and reduces the computational power needed compared to finite-sized models. However, no other practical background states that the elements should be distributed evenly in the whole available space for maximum metamaterial effectiveness. The effectiveness of each resonant element depends on the displacement amplitude of the base plate, making elements placed in areas with lower vibration amplitude less effective than those in higher vibration regions. This work presents a topological optimization of the distribution of locally resonant elements. The Method of Moving Asymptotes is employed to minimize the number of resonant elements while preserving the initial sound reduction achieved by periodic metamaterials. Calculations were conducted in COMSOL Multiphysics with MATLAB, using a combined analytical and numerical approach for sound insulation simulation. The results confirm that the number of resonant elements, and therefore the additional mass of the metamaterial on the base plate, can be reduced while maintaining the initial effectiveness of the solution.

Keywords: locally resonant structures, noise and vibration reduction, topological optimization, mass reduction

1. INTRODUCTION

Locally resonant structures (LRS) are widely exploited in the literature in noise and vibration suppression problems. Resonant elements are based on a mass-spring-damper system and when attached to a base plate create a stopband effect. In the stopband frequency range, flexural waves cannot propagate in the base plate, causing vibration and noise mitigation. Typically, LRS consists of a periodical mesh of resonant elements distributed equally over the plate surface at sub-wavelength distances [1]. The periodic distribution is beneficial due to the simplification of manufacturing and the lower computational costs. In the literature, most of the LRS simulation examples are based on unit cell simulations with periodic boundary conditions, as a simplified model that gives information about the whole structure [2]. However, the periodicity is a proper solution in model simplification, and gives a good approximation of how the finite system will behave, it also narrows the possibility of creating quasi-periodic or non-periodic solution. It can be stated that the effectiveness of the resonant elements distributed on the plate depends on the displacement of the base plate at the attachment point [3, 4]. Therefore, in places where the base plate has lower displacement amplitude, resonant elements may have lower effectiveness.

In this paper a topological optimization procedure is pre-

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sented, which allows for resonant element number minimization, while preserving the metamaterial effectiveness in noise and vibration reduction at specified level. This gives the possibility to create quasi-periodic element distribution on the base plate, by removing unnecessary mesh elements, which leads to mass reduction.

2. SOUND INSULATION FOR PLATE WITH RESONANT ELEMENTS

Sound reduction index is a logarithmic variable dependent on transmission coefficient τ_d and is given as

$$R = 10\log_{10}\frac{1}{\tau_d}. \tag{1}$$

Coefficient τ_d for a diffuse field can be described in approximate form as

$$\tau_d \approx 8\rho_0 c_0 \int_0^{\pi/2} \frac{\Re(\overline{z}_{f,\text{fin}})}{|Z_{\text{eff}} + 2\rho_0 c_0 \overline{z}_{f,\text{fin}}|^2}\sin\theta \mathrm{d}\theta, \tag{2}$$

where ρ_0 is air density, c_0 is wave speed in the air, θ stands for elevation incident wave angle, $\overline{z}_{\text{fin}}$ is radiation impedance, and Z_{eff} is mechanical impedance of the structure [3]. In the analyzed case, the radiation impedance is calculated for a finite-size rectangular plate according to the equations given in [5] and [3]. The effectiveness of resonant elements can be determined based on the effective modal mass [6], calculated for the natural frequencies of the single resonant element. The effective modal mass can be used to calculate the effective parameters of the plate with LRS. One of the parameters is effective mechanical impedance Z_{eff} which can be calculated according to equation

$$Z_{\text{eff}} = \frac{B'}{i\omega}\left(k_x^2 + k_y^2\right)^2 + i\omega\left(m_p'' + \frac{m_z'' s_z''(1+i\eta_z)}{i m_z''(\omega_0^2-\omega^2) + i\eta_z s_z''}\right). \tag{3}$$

In Eq. (3) B' stands for plate bending stiffness, m_p'' is the surface mass of the plate, m_z'', s_z'' and η_z are the surface mass, stiffness and damping of the resonant element with a resonant frequency ω_0. The variables k_x, k_y are wavenumber components, and ω is frequency. Based on these equations the effectiveness of the plate with LRS can be determined.

3. OPTIMIZATION OF THE RESONANT ELEMENTS DISTRIBUTION

The Method of Moving Asymptotes algorithm was selected for the topological optimization of the LRS distribution [7]. The problem solved in this chapter was defined as a constrained cost function. The cost function minimizes the number of resonant elements while constraining the mechanical impedance of the base plate. In this case, the constraint was defined as the smallest possible difference between the effective mechanical impedance of the optimized solution and the effective mechanical impedance of the reference solution with uniformly distributed resonant elements.

The contribution of each resonator in the model is determined by the variable ζ^e, which is assigned individually to each resonator. The variable ζ^e can be defined by the following equation

$$\zeta^e = \begin{cases} 0, \text{ dla } e \in P_r \\ 1, \text{ dla } e \notin P_r, \end{cases} \tag{4}$$

where P_r is a vector of resonant elements included in the model. When a resonant element is absent in the P_r vector, $\zeta^e = 0$. The optimization problem is meant to find the values of ζ^e through cost function f_c minimization, while fulfilling the constraint g at the same time, which can be described as [8]

$$\begin{aligned} \min_{\zeta} \quad & f_c(\zeta) \\ \text{s.t.} \quad & g \le 0, \\ & 0 \le \zeta_{min} \le \zeta^e \le 1. \end{aligned} \tag{5}$$

To minimize the number of resonant elements, the cost function was defined as follows

$$f_c(\zeta) = \frac{1}{N_{\text{rezo}}}\left(\sum_{\zeta=0}^{N_{\text{rezo}}} \zeta^e + \alpha \sum_{\zeta=0}^{N_{\text{rezo}}} \zeta^e(1-\zeta^e)\right), \tag{6}$$

where N_{rezo} is the overall number of resonant elements, and α is weighting factor forcing ζ^e to achieve values 0 or 1. The constraint function can be written as

$$g(\zeta, \Delta, P, f) = \frac{1}{\Delta}||\delta_Z||_P - 1, \tag{7}$$

where Δ is the target difference between the mechanical impedance level of the basic and optimized case, and δ_Z is the level difference in the current iteration.

4. HYBRID SIMULATION MODEL

In order to perform topological optimization of resonant elements distribution, a hybrid analytical-numerical model was created. Hybrid modeling allowed for a finite-size plate with resonant elements implementation with reduced time needed for a single calculation. Simulations were conducted using COMSOL Multiphysics with

MATLAB LiveLink. The numerical model created in COMSOL is shown in Fig. 1. 3D model consisted of a shell plate with finite dimensions $x_{\text{plate}} = 0.55$ [m], $y_{\text{plate}} = 0.45$ [m], and thickness 0.001 [m]. The plate material was steel with the material parameters: density $\rho = 7850$ [kg/m^3], Young's modulus $E = 205$ [GPa], and Poisson ratio $\nu = 0.33$ [$-$]. For the base plate, an excitation was defined in the form of a force per unit area, expressed in [N/m^2].

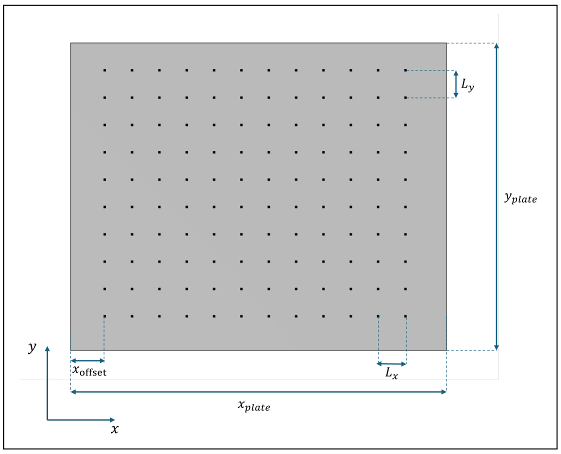

Figure 1. Base plate model with periodically distributed resonant elements, with dimensions.

On the surface of the base plate, a grid of points was created, in which resonant elements modeled as mass-spring-damper systems were placed. The distance between consecutive points is defined along the y axis as L_y and $L_y = L_x$. To model the resonant element, the *Lumped Parameter Modelling* module was used, which allows the parameters of a mass-spring-damper system to be expressed through mechanical parameters, without the need to model the geometry of the element. The effective parameters of the resonant element were determined using a hybrid model based on modal mass calculations [6].

For the developed base plate model, it was possible to determine the vibration velocity of the base plate and, on this basis, the mechanical impedance Z_{eff} of the plate with metamaterial. The mechanical impedance of the plate can subsequently be used to calculate the acoustic insulation of a plate with LRS according to equations presented in Sec. 2.

5. RESULTS

To determine the frequency range of interest, calculations were performed for frequencies in the range $100-300$ Hz. Resonant elements were tuned to one of the base plate resonant frequencies $f = 153$ Hz. The calculations were limited to the implementation of a resonant element with a natural frequency of $\omega_0 = 153$ Hz and effective mass in the z axis. For these elements, the effective mass was $m_r = 0.00051$. The loss factor was assumed to be $\eta = 0.05$ (as for polycarbonate).

For the optimization of the resonator distribution, the frequency range was narrowed to $f = \langle 140, 170 \rangle$ Hz with a step of 3 Hz to reduce the computation time per iteration. In optimization, a target error $\Delta = 3$ dB was assumed. The optimization was run for 600 iterations and the solution was obtained after 225 iterations. The error δ_Z achieved after optimization was 2.8 dB, which met the constraints defined by the constraint function.

In Fig. 2 optimized distribution of resonant elements is presented. In color, the base plate displacement amplitude for 153 Hz is presented. The positions of the resonant elements resulting from the optimization procedure are correlated with areas with larger displacement amplitudes. The acoustic insulation characteristics obtained from the simulation for an empty steel plate, a plate with a uniform distribution of resonant elements, and a plate with an optimized distribution of elements are shown in Fig.3. Despite a 35 % reduction in the number of resonant elements, the differences in the acoustic insulation values for individual frequencies do not exceed 1 dB.

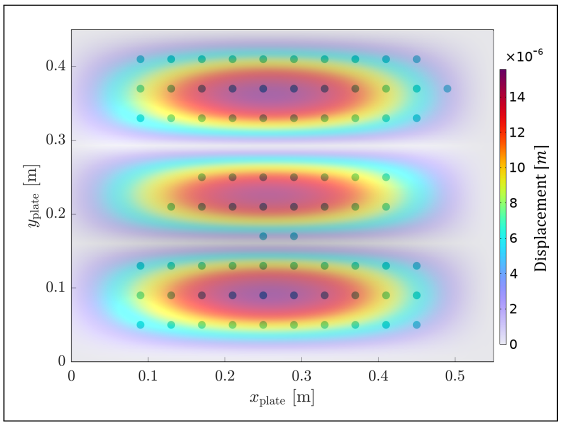

Figure 2. Displacement of the base plate at the frequency $f = 153$ Hz with the optimized distribution of resonant elements on the surface of the base plate.

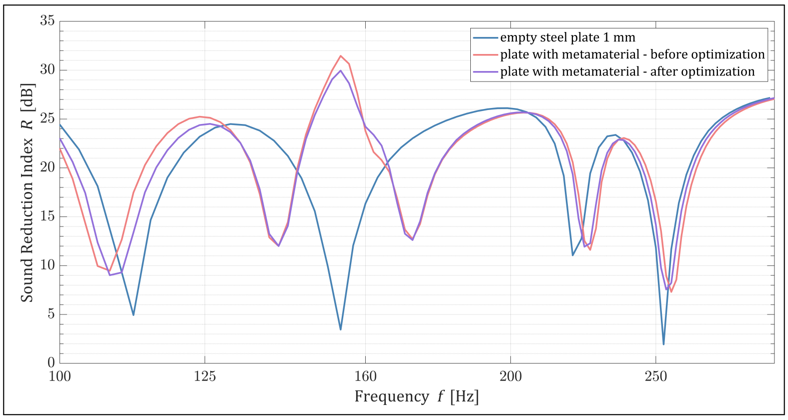

Figure 3. Comparison of the acoustic insulation simulation results for the empty steel plate, the plate with uniformly distributed resonant elements over the entire available surface, and the plate with the optimized distribution of resonant elements.

6. CONCLUSION

In this paper, an optimization procedure is presented, which allows for resonant elements number minimization while preserving the effectiveness of the solution in noise and vibration mitigation in specified frequency range. Thanks to the conducted optimization, the possibility of optimizing the distribution of resonant elements on the base plate in order to minimize the number of resonant elements was confirmed. The effectiveness of the optimization algorithm was validated for the case of a rectangular plate with fixed edges and excitation by a plane wave. The method of optimizing the distribution of elements can be extended, for example, by optimizing the distribution of elements in a three-dimensional machine casing. The developed optimization procedure, combined with algorithms based on hybrid models for LRS simulation, constitutes a solution that enables the improvement and extension of computational capabilities for vibroacoustic metamaterials.

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