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PRELIMINARY STUDY ON THEORETICAL CORRECTIONS TO THE EQUATION OF STATE IN SINGLE-BUBBLE SONOLUMINESCENCE AND EXTENSION OF THE MINIMUM RADIUS STAGE

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ABSTRACT

Single bubble sonoluminescence (SBSL) involves the emission of very short flashes of light by bubbles subjected to a periodic ultrasonic acoustic field in a liquid medium, which slowly expand and rapidly contract, emitting light at the minimum radius stage. Several theories have been proposed to model the behavior and thermodynamic conditions inside the bubble, and most of them treat the gas inside as an ideal gas. Nonetheless, the experimental results obtained to date show conditions that deviate from the range of applicability of the ideal gas equation. This paper proposes a first approach to the study of the SBSL phenomenon through the application of an equation of state corresponding to real gases, and even to plasmas, in order to obtain theoretical corrections to the dynamics and thermodynamic conditions (mainly pressure and temperature) inside the bubble. In addition, the possibility of prolonging the minimum radius stage of the bubble by modifying the applied acoustic field will be analyzed, in order to examine whether the thermodynamic conditions inside the bubble vary or not in this case, and whether these conditions are stable in time.

Keywords: *sonoluminescence, equation of state, collapse prolongation*

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1. INTRODUCTION

Single bubble sonoluminescence (SBSL) refers to the phenomenon in which a gas bubble, when exposed to a periodic ultrasonic acoustic field within a liquid, emits brief but intense flashes of light. This occurs as the bubble undergoes a process of gradual expansion followed by rapid contraction, with the emission of light occurring precisely at the point of minimum bubble radius [1]. The role of the acoustic field is paramount, as the rapid oscillations induced by the ultrasonic waves are critical in driving the bubble dynamics that lead to the high-energy light emission observed during the bubble's collapse. Understanding the acoustics involved is essential, as the precise control of the ultrasonic field influences both the intensity and the characteristics of the emitted sonoluminescent light.

The equation of state (EOS) also plays a crucial role in understanding the extreme conditions reached inside a collapsing bubble in SBSL. During the collapse, the gas within the bubble is subject to rapid compression, leading to extreme temperatures and pressures characteristic of the warm dense matter (WDM) regime. Accurately modeling these conditions is essential for predicting key parameters such as density, temperature, and the subsequent light emission [2].

Different EOS models have been proposed to describe the thermodynamic behavior of the gas inside the bubble. Early approaches relied on the ideal gas law [3], but as experimental and numerical studies revealed significant deviations from ideal behavior [4], more sophisticated models, such as that based on the van der Waals equation [5] and high-complexity real gas models [6], have been introduced. These models incorporate non-ideal gas effects, phase transitions, and even plasma formation [7] at the final stage of collapse.

The theoretical sensitivity of SBSL light emission to the choice of EOS suggests that sonoluminescence could serve as a unique experimental platform for testing and refining EOS models relevant to WDM physics [2].

On the other hand, the applied acoustic field can be modified in such a way that there is a point in space where a very high pressure is reached. In this context, if the bubble is located in that point, its behavior is different from that of normal SBSL. The bubble collapse is prolonged in time and it loses its stability.

The first section will be dedicated to the corrections to the equation of state for the gas in a sonoluminescing bubble. The main objective of this part is to perform a review of the current models which aim to apply realistic EOS in SBSL.

The second section will treat the conditions and numerical results for the prolongation of bubble collapse. The objective of this part is to find theoretical conditions for the bubble to keep collapsed more time.

2. EQUATION OF STATE FOR THE GAS IN THE BUBBLE

During the collapse, the gas inside the bubble in SBSL undergoes rapid compression, reaching temperatures and pressures characteristic of the warm dense matter (WDM) regime. The choice of EOS directly influences the predicted density, temperature, and light emission behavior, making it a key factor in SBSL modeling [2, 8].

First, it is necessary to define the theoretical model that will be used in order to describe the time evolution of the bubble radius. Rayleigh-Plesset model will be used, for which the time evolution of the bubble radius reads:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(P - P_\infty - P(t)\right) \tag{1}$$

$$P = p_{\text{gas}} - 4\mu\frac{\dot{R}}{R} - 2\frac{S}{R} \tag{2}$$

where R is the bubble radius; ρ, the liquid density, p_{gas}, the pressure of gas inside the bubble (given by the chosen equation of state); P_∞, the ambient pressure of the liquid; $P(t) = P_A \sin(\omega t)$, the pressure due to the acoustic field of an angular frequency ω and amplitude P_A; μ, the liquid viscosity; S, the surface tension of the liquid; and the dot indicates time derivative. This is an approximation valid up to the order of $\dot{R}/c$ [9].

This section will review the different EOS used to model the gas inside the bubble in SBSL, including considerations of ideal gas behavior, non-ideal gas effects, and plasma formation, which is poorly studied.

2.1 Early Approaches: Ideal Gas Models

The simplest EOS used in SBSL studies is the ideal gas law, which assumes non-interacting particles [10]. It provides a useful first approximation, particularly in non-collapse conditions when the bubble follows an adiabatic process. The mean pressure of the gas is given by:

$$p = \frac{NkT}{V} = \frac{nRT}{V} \tag{3}$$

where N is the number of molecules; k, is the Boltzmann constant; T, the mean temperature of the gas; and V, the total volume occupied by the gas. For the second equation, n is the number of moles of the substance and R is the ideal gas constant.

The collapse of the bubble is very short-lived (thus, the collapse is considered to be adiabatic). Therefore, the bubble is most of the time (almost the whole cycle) outside the collapse range [11]. Thus, in non-collapse conditions, it can be assumed that the gas inside the bubble obeys the ideal gas law, for simplicity and because through this approach it is possible to extract a lot of information, making it a good first approximation for the study of this phenomenon.

The adiabatic equation applied to SBSL in this case reads [12]:

$$P = P_0\left(\frac{R_0}{R}\right)^{3\gamma} \tag{4}$$

where γ is the adiabatic index.

However, as the bubble collapses, pressure and temperature increase significantly [13], leading to ionization and strong deviations from ideal gas behavior [14].

2.2 Moderate Complexity Models: Van der Waals Equation

Real gases deviate from ideal behavior due to intermolecular interactions [15]. The van der Waals equation introduces correction terms:

$$p + \frac{a}{v^2} = \frac{RT}{v - b} \tag{5}$$

where $v = V/n$ is the molar volume of the gas (n is the number of moles of the substance). In this equation,

a and b are two constants, which are characteristic of the particular type of molecules that it describes [5].

This model predicts higher temperatures and pressures than the ideal gas law and provides a better approximation for SBSL conditions [16]. The adiabatic equation for this case is as follows [17]:

$$p = \left(P_0 + \frac{2S}{R_0}\right)\left(\frac{R_0^3 - h^3}{R^3 - h^3}\right)^{\gamma} \tag{6}$$

Where h is the characteristic van der Waals hard-core radius of the gas inside the bubble.

Despite its improvements, this model still falls short under extreme SBSL conditions.

2.3 Higher complexity models for real gases

Despite improving the ideal gas model, the Van der Waals model is also unsuccessful because of the extreme conditions reached in the collapse. Thus, more advanced EOS models, such as the Peng–Robinson equation, have been developed to improve accuracy under high pressures and temperatures [6]. These models refine SBSL predictions but do not fully account for plasma formation.

2.4 Warm Dense Matter regime

Warm dense matter (WDM) is a branch of plasma physics. Its importance lies in its influence in the structure and formation of giant planets and other astrophysical objects, such as brown dwarfs, as well as in the implosion trajectories of proposed laser fusion capsules [18, 19]. It is characterized by pressures of above a million atmospheres (100 GPa), and with temperatures between 1 and 100 eV (where 1 eV = 11600 K) [20], and its density is sufficiently high to render the ions strongly coupled [21].

In the laboratory, WDM can be created through lasers [22, 23], using a magnetic anvil cell [24], employing extreme ultraviolet radiation [25], or even utilizing a pulsed-power driver [26]. It is hard to obtain, so it is difficult to obtain a theoretical model to describe it [27–29]. Many attempts have been done to obtain an equation of state (EOS) for WDM [30–33].

In this context, SBSL is presented as a phenomenon in which it could be possible to generate WDM in a relatively simple way, given the extreme conditions of pressure, density and temperature inside the bubble in the collapse. Therefore, SBSL can be used to test EOS in WDM regime [2].

2.5 Plasma Formation and Ionization: The Role of Plasma in the Collapse

At extreme temperatures, the gas transitions into plasma, requiring a plasma-specific EOS. While early models relied on the Saha equation, experimental data suggest much higher opacity, pointing to strong Coulomb interactions [34]. Some models incorporate magnetohydrodynamic effects to explain plasma behavior during collapse [35].

Despite significant advances, a universally accurate EOS for all SBSL stages remains elusive, highlighting the need for hybrid approaches that combine different models.

3. COLLAPSE PROLONGATION IN SBSL

In SBSL, the final stage of the collapse (the time the bubble remains contracted at its minimum radius) lasts picoseconds [36], as the pulse width of the emitted light. In this context, some questions may arise: is the emitted light directly related to the minimum radius stage? Can the bubble stay contracted for longer? In that case, would the light pulse widths be longer? Would the light emitted have different characteristics?

In this section, the prolongation of the time the bubble remains collapsed will be numerically studied. In order to achieve that, modifications in the applied acoustic field will be analyzed. Specifically, the application of two excitation sources will be numerically studied.

The numerical model is as follows. A parallelepiped container full of water will be considered, with dimensions a = 10 cm, b = 20 cm and c = 30 cm (referring to the x, y and z coordinates respectively). The two sound sources are located on the same face of the container (one centered and one eccentric source) radiating at 25 kHz in phase, as can be seen in Fig. 1 . This simulation has been performed with ANSYS software. The sound pressure level inside the container can be seen in Fig. 2, where the point of maximum pressure is indicated in red, and the point of minimum pressure is indicated in blue.

In this context, a numerical study will be performed to analyze what is the time evolution of the bubble radius, as well as of the pressure and temperature inside the bubble if it is located at the point of maximum pressure level. The numerical model will consist of solving the Raylegih-Plesset equation using Python. In order to achieve that, a change of variable

$$u = \dot{R} \tag{7}$$

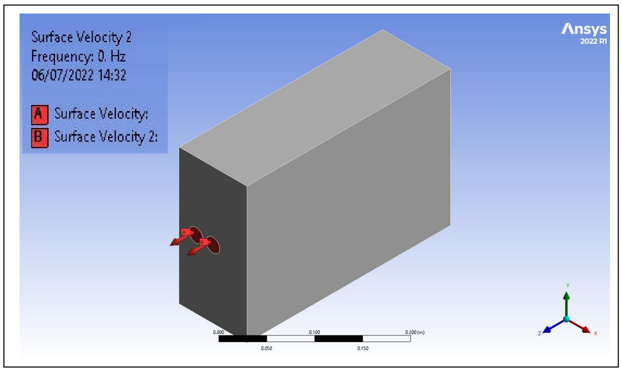

Figure 1. Problem statement of a parallelepiped with two excitation sources.

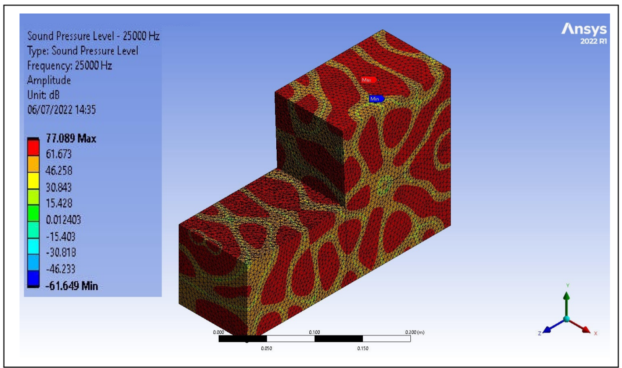

Figure 2. Sound pressure level in a parallelepiped with two excitation sources.

will be used. In this way, it is possible to obtain the following system of equations for the bubble:

$$
\left.
\begin{array}{r}
u = \dot{R} \\
\dot{u} = \frac{1}{R\rho}\left[\left(P_0 + \frac{2\sigma}{R_0} - P_v\right)\left(\frac{R_0}{R}\right)^{3\gamma}\left(1 - \frac{3\gamma u}{c}\right) - \right. \\
- \frac{2\sigma}{R} - \frac{4\mu u}{R} - P_A \sin(\omega t) + \\
\left. + P_v - P_0 - \frac{2\sigma}{R_0}\right] - \frac{3u^2}{2R}
\end{array}
\right\} \tag{8}
$$

where P_v is the vapor pressure; and σ, the surface (interfacial) tension of water. The pressure and temperature will be modeled as:

$$P(t) = \left(P_0 + \frac{2\sigma}{R_0} - P_v\right)\left(\frac{R_0}{R}\right)^{3\gamma} \tag{9}$$

$$T(t) = T_0 \left(\frac{R_0}{R}\right)^{3(\gamma-1)} \tag{10}$$

Numerical results using Python suggest that the radius increases to a disproportionate and meaningless value, as shown in Fig. 3. This means that the movement of the bubble is not stable, i.e. the bubble disappears (explodes). This can also be deduced from the pressure and temperature graphs (Fig. 4), in which their value cancels out after a certain instant. These plots allow us to know the moment when the bubble explodes (from the moment when its value is zero), which is at approximately 20 microseconds. In addition, it should be noted that the pressure and temperature peaks are smoother (the changes are more progressive) than in the case of SBSL, where they are more similar to a Dirac delta, as shown in Figure 5a of [37]. This could mean that, due to the high pressure that the bubble undergoes, it stays longer in the minimum radius stage during the first 20 microseconds.

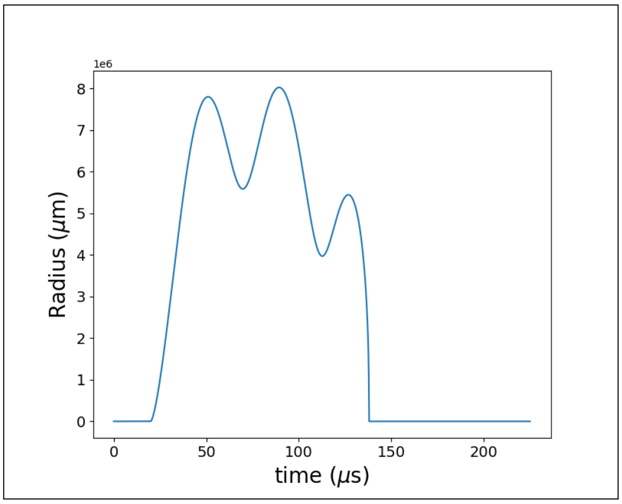

Figure 3. Time evolution of the bubble radius with two excitation sources.

If Fig. 3 is enlarged to see the initial instants, Fig. 5 is obtained, where it can be seen that the bubble remains collapsed almost 20 microseconds. Normal conditions in SBSL experiments show that the bubble collapse lasts a few picoseconds. That means a prolongation of six orders of magnitude for the time in which the bubble remains collapsed.

4. CONCLUSION

Numerical simulations suggest that modifying the acoustic field can prolong the collapse stage six orders of magnitude in time, though this may lead to instability. Addi-

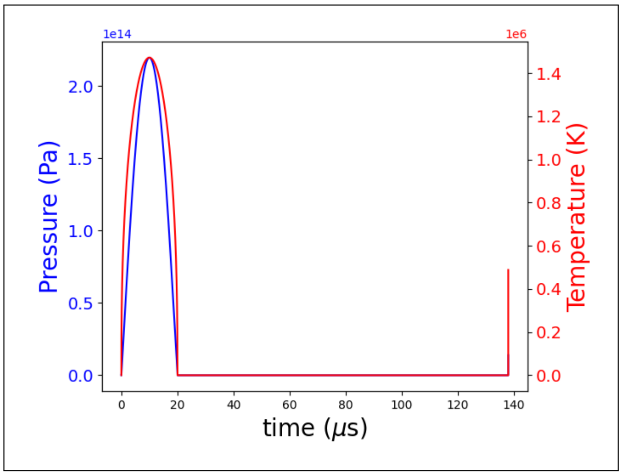

Figure 4. Time evolution of the pressure and temperature in the bubble with two excitation sources.

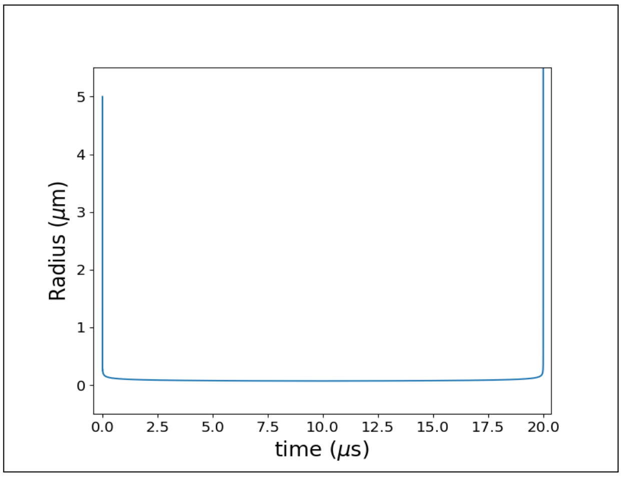

Figure 5. Time evolution of the bubble radius at the first instants of the motion.

tionally, none of the current theoretical models can explain exactly the experimental observations of SBSL, and one of the properties to be corrected is the considered EOS. The application of an EOS for real gases could more accurately model the thermodynamic properties inside a sonoluminescing bubble. This would lead to improved accuracy in fitting theoretical simulations with experimental data, as well as to a more complete explanation of the phenomenon. Although ideal gas models provide a useful starting point, deviations at high pressures and temperatures necessitate more complex approaches, including van der Waals corrections and plasma-based models. The application of these models, or even a combination of them, should give more accurate results that better fit the experimental data. Thus, the need for hybrid EOS models tailored to different SBSL phases is suggested, and also the possibility of collapse prolongation. The use of an appropriate EOS in SBSL, as well as finding conditions for stable prolongation of bubble collapse, is left for future research.

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