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RECIPROCITY DRIVEN OMNI-DIRECTIONAL ABSORPTION

Juan Pablo Escudero[1*] **Jean-Philippe Groby**[1] **Vincent Pagneux**[1]
[1] Laboratoire d'Acoustique de l'Université du Mans (LAUM), UMR 6613,
Institut d'Acoustique - Graduate School (IA-GS), CNRS,
Le Mans Université, France

ABSTRACT

In order to obtain a strong absorption over a large range of incidence angle, the reflection of an incident acoustic plane wave by a rigidly backed lossy architectured layer is studied numerically and theoretically. This layer is composed of a periodic arrangement of inclined resistive wiremeshes. Impedance matching is achieved when the angle of the incident wave corresponds to the angle of inclination of the wiremesh as well as its exact opposite angle, thanks to the reciprocity principle. The structure exhibits either large absorption over a broad frequency band for medium inclination angles or almost omnidirectional large absorption over a subwavelength frequency band for large inclination angles. This peculiar acoustic behavior provides another perspective for the design of perfect omnidirectional subwavelength sound absorbing devices.

Keywords: *wiremesh, effective medium, broadband absorption, omni-directional absorption.*

1. INTRODUCTION

Low frequency noise absorption is a challenging task which has been addressed in numerous studies this last decade [1–3]. The main difficulties to confront this problem are related with the large acoustic wavelength and the required space and amount of material for passive noise control solutions [4]. As an alternative, acoustic metamaterials developed under the slow sound propagation concept have been used to design broadband acoustic perfect absorbers leading to deep-subwavelength structure thicknesses [5]. The performance of these solutions is limited when the acoustic wave arrives from angles that are different from normal incidence or from grazing angles.

Among other metamaterials, layered metasurfaces based on the extraordinary reciprocity property [6, 7] exhibit a symmetric reflection coefficient providing very interesting absorption characteristics. From this property, the reflection coefficient of a structure with ultra thin resistive sheets, in a transmission problem, is zero when the incident field arrives with the same angle as the orientation of the layers. On the other hand, when the incident field arrives with the exact opposite angle, the reflection is again zero and the transmission is different to one. In consequence, impedance matching is produced at this exact opposite angle and the absorption coefficient is close to the unity.

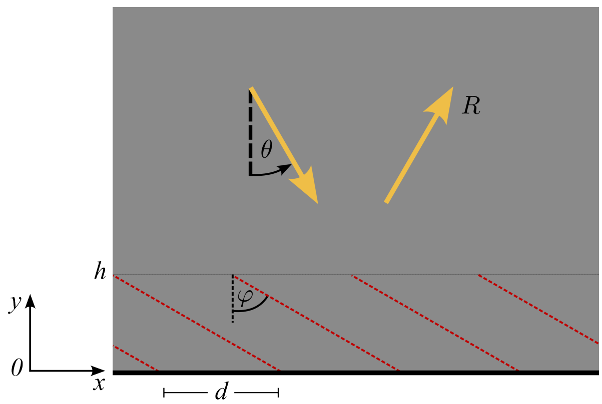

Figure 1. Reflection problem of a wiremesh (red dashed) grating with rigid backing.

Inspired by this reciprocity principle, we explore the reflection of an incident acoustic plane wave by a rigidly

**Corresponding author*: *juan_pablo.escudero_lopez@univ-lemans.fr.*

backed lossy architectured layer composed of resistive wiremesh material. In this study, we compute the absorption coefficient numerically and theoretically. In addition, we show that the structure exhibits either large absorption over a broad frequency band for medium inclination angles or almost omni-directional large absorption over a subwavelength frequency band for large inclination angles. This document is organized as follows. Sec. 2 describes the wiremesh grating structure in a reflection problem. Sec. 3 presents the structure effective medium formulation and an explicit expression for the reflection coefficient. Finally, comparisons between the numerical computations with analytical results are shown in Sec. 4.

2. WIREMESH GRATING

Fig. 1 depicts a reflection problem of a rigid backing wiremesh grating. This problem is described by the Helmholtz equation, $(\Delta + k^2)p = 0$, using the time-harmonic convention $e^{-i\omega t}$. The structure is composed of inclined thin wiremesh units horizontally arranged with a periodicity d, a layer inclination angle φ and a height h. The boundary conditions that define the wiremesh, assuming that the layers does not vibrate, are a normal particle velocity continuous through the wiremesh and a pressure jump around the layer [8, 9], as

$$[\![\partial_n p]\!] = 0 \;\; and \;\; [\![p]\!] = \frac{i\zeta}{k}\partial_n p. \tag{1}$$

In Eqn. (1), ζ is the purely resistive wiremesh impedance normalized by the characteristic impedance of the medium ρc. As the structure is d-periodic along the x-axis, the present study is focused only on the propagation of the fundamental mode, i.e. $kh/\pi \leq 1$ [10].

3. EFFECTIVE MEDIUM FORMULATION

According to the effective medium theory ($kd \ll 1$), a lossy wiremesh layered structure can be represented using an artificial homogeneous anisotropic medium slab, with principal directions of anisotropy $(X,\, Y)$ [11], as it is represented in Fig. 2. This problem is described by the wave equation

$$\nabla \cdot [\mathbf{a}\nabla p(x,y)] + bk^2 p(x,y) = 0. \tag{2}$$

In this equation, $\mathbf{a}$ and b are the effective parameters related with the density and bulk modulus of the slab relative to a reference medium (where $a = 1$ and $b = 1$). The boundary conditions for the reflection problem are

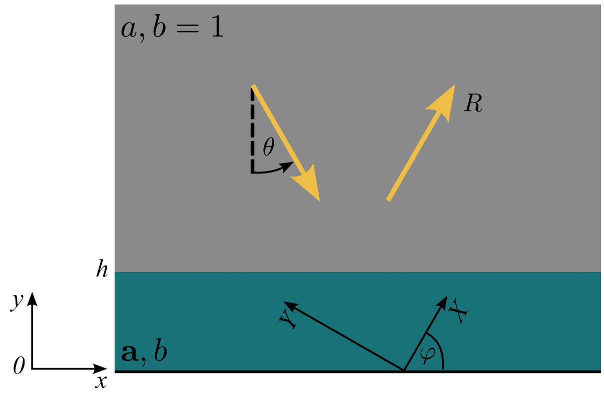

Figure 2. Reflection problem of a rigid backing wiremesh grating.

$$\begin{aligned} [\![p]\!] = 0 \quad and \quad [\![\mathbf{n}\cdot(\mathbf{a}\nabla p)]\!] = 0 \quad at \;\; y = h, \\ \mathbf{n}\cdot(\mathbf{a}\nabla p) = 0 \quad at \;\; y = 0. \end{aligned} \tag{3}$$

Considering that the principal directions of anisotropy (X, Y) coincides with the directions of the unit cell, the effective parameter $\mathbf{a}$ is a diagonal matrix defined as

$$\mathbf{a} = \begin{pmatrix} 1/\rho_{eff} & 0 \\ 0 & 1 \end{pmatrix}, \tag{4}$$

with an effective density [12]

$$\rho_{eff} = 1 + \frac{i}{kh}\left(\frac{\zeta h}{d\cos\varphi}\right). \tag{5}$$

Applying a rotation of the homogenized medium by an angle φ, the $\mathbf{a}$ matrix is mapped to the global components (x, y) in the form

$$\mathbf{a} = \begin{pmatrix} a_x & a_{xy} \\ a_{xy} & a_y \end{pmatrix}. \tag{6}$$

The reflection problem is solved using the wave fields solutions

$$\begin{aligned} p(x, y \geq h) = e^{ik_x x}\left(e^{-ik_y(y-h)} + Re^{ik_y(y-h)}\right), \\ p(x, 0 \leq y \leq h) = e^{ik_x x}\left(Ae^{i\kappa^+ y} + Be^{i\kappa^- y}\right), \end{aligned} \tag{7}$$

with $k_x = k\sin\theta$, $k_y = k\cos\theta$ and k the wave number in the reference medium. In Eqn. (7), $\kappa^\pm$ is the wave number of the anisotropic slab in the normal direction

$$\kappa^{\pm} = -\frac{a_{xy}k_x}{a_y} \pm \frac{k}{a_y}m', \tag{8}$$

with

$$m' = \sqrt{ba_y - (a_x a_y - a_{xy}^2)\sin^2\theta}.$$

Applying the boundary conditions of Eqn. (3), the reflection coefficient of the homogenized slab at $y = h$ is

$$R = \frac{\frac{\cos\theta}{m'} + i\tan\frac{kh}{a_y}m'}{\frac{\cos\theta}{m'} - i\tan\frac{kh}{a_y}m'}, \tag{9}$$

and the absorption coefficient is computed as

$$\alpha = 1 - |R|^2. \tag{10}$$

4. RESULTS

In order to compare the effective medium formulation, the absorption coefficient is calculated numerically using COMSOL Multiphysics and used as reference. In Fig. 3, the absorption coefficient is depicted. On the left-hand side numerical results using input parameters $\zeta = 2$ and $h = 2$ and on the right-hand side the effective medium results for dimensionless parameters $\varphi = 60$ and $\zeta h = 4$. In this figure, the effective medium results exhibit absorption deviations compared with numerical computation when kh/π increases. Furthermore, large absorption is obtained in both cases when the incident angle is exact the inclination angle of the layered structure ($\theta = \pm 60$), for frequencies above 0.4.

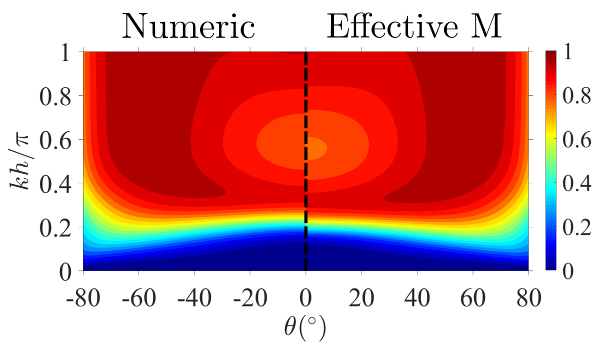

Figure 3. Absorption coefficient of the metasurface for $\varphi = 60$ and $\zeta h = 4$. Left side, numerical result using $\zeta = 2$ and $h = 2$, and at the right side the effective medium formulation.

The effect of varying φ for a fixed ζh value is presented in Fig. 4, for numeric ($\zeta = 2$ and $h = 2$) and effective medium computations. These results show that increasing the inclination angle of the layer structure larger absorption is achieved. A remarkable result is noticed for $\varphi = 75$, where impedance matching is achieved at the exact the inclination angle of the layered structure and broadband absorption is appreciable starting from low frequency ($kh/\pi \leq 0.2$). Furthermore, almost omnidirectional large absorption over a subwavelength frequency band is obtained for $\varphi = 75$ at a $kh/\pi = 0.14$. From the effective medium results, deviations appear as the frequency increase. Despite this, effective medium retains the subwavelength absorption and broadband at the same inclination angle of the structure.

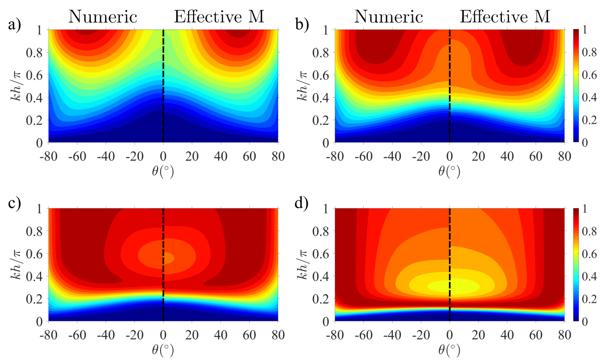

Figure 4. Effect of varying φ for $\zeta h = 4$ value. *a)* $\varphi = 30$, *b)* $\varphi = 45$, *c)* $\varphi = 60$ and *d)* $\varphi = 75$.

5. CONCLUSIONS

In this work, we studied the reflection problem of a rigidly backed lossy architectured layer composed of resistive wiremesh material. The understanding of the structure was performed using the effective medium formulation. From this procedure, we observed the dependency of the reflection coefficient on the dimensionless parameters kh and ζh. To compare the effective medium formulation, we used numerical computations as reference. We showed good correlation between numerical and effective medium results, and deviations where addressed at high frequencies. We observe good agreement between effective medium theory and full numerical solutions for a given ζh, as long as ζ is not large and h not too small. Additionally, we showed that the structure exhibits either large absorption over a broad frequency band for medium inclination angles or almost omni-directional large absorption over a subwavelength frequency band for large inclination angles. This peculiar acoustic behavior provides another perspective for the design of perfect omni-directional subwavelength sound absorbing devices.

6. ACKNOWLEDGMENTS

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7. REFERENCES

[1] V. Romero-García, G. Theocharis, O. Richoux, A. Merkel, V. Tournat, and V. Pagneux, "Perfect and broadband acoustic absorption by critically coupled sub-wavelength resonators," *Scientific reports*, vol. 6, no. 1, p. 19519, 2016.

[2] C. Fu, M. Yang, and Z. H. Hang, "Ultra-broadband symmetrical acoustic coherent perfect absorbers designed by the causality principle," *Physical Review Research*, vol. 5, no. 4, p. L042020, 2023.

[3] Y. Duan, J. Luo, G. Wang, Z. H. Hang, B. Hou, J. Li, P. Sheng, and Y. Lai, "Theoretical requirements for broadband perfect absorption of acoustic waves by ultra-thin elastic meta-films," *Scientific Reports*, vol. 5, no. 1, p. 12139, 2015.

[4] J. Carbajo, S. Ghaffari Mosanenzadeh, S. Kim, and N. X. Fang, "Sound absorption of acoustic resonators with oblique perforations," *Applied Physics Letters*, vol. 116, no. 5, 2020.

[5] J.-P. Groby, N. Jiménez, and V. Romero-García, "Acoustic metamaterial absorbers," *Acoustic Waves in Periodic Structures, Metamaterials, and Porous Media: From Fundamentals to Industrial Applications*, pp. 167–204, 2021.

[6] J. Luo, H. Chu, R. Peng, M. Wang, J. Li, and Y. Lai, "Ultra-broadband reflectionless brewster absorber protected by reciprocity," *Light: Science & Applications*, vol. 10, no. 1, p. 89, 2021.

[7] H. Fan, J. Li, Y. Lai, and J. Luo, "Optical brewster metasurfaces exhibiting ultrabroadband reflectionless absorption and extreme angular asymmetry," *Physical Review Applied*, vol. 16, no. 4, p. 044064, 2021.

[8] M. Farooqui, Y. Aurégan, and V. Pagneux, "Ultrathin resistive sheets for broadband coherent absorption and symmetrization of acoustic waves," *Physical Review Applied*, vol. 18, no. 1, p. 014007, 2022.

[9] S. Kuznetsova, Y. Aurégan, and V. Pagneux, "Loss-induced modal selection by a resistive wiremesh," *The Journal of the Acoustical Society of America*, vol. 156, no. 1, pp. 369–377, 2024.

[10] N. Bonod and J. Neauport, "Diffraction gratings: from principles to applications in high-intensity lasers," *Advances in Optics and Photonics*, vol. 8, no. 1, pp. 156–199, 2016.

[11] O. A. Oleĭnik, A. Shamaev, and G. Yosifian, *Mathematical problems in elasticity and homogenization*, vol. 26. Elsevier, 1992.

[12] J.-P. Groby, M. Mallejac, A. Merkel, V. Romero-García, V. Tournat, D. Torrent, and J. Li, "Analytical modeling of one-dimensional resonant asymmetric and reciprocal acoustic structures as willis materials," *New Journal of Physics*, vol. 23, no. 5, p. 053020, 2021.