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STABILITY ANALYSIS OF STEPWISE SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION FOR ACTIVE NOISE CONTROL

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ABSTRACT

Active noise control (ANC) is particularly effective in mitigating low-frequency noise, with its primary objective being to achieve maximum noise reduction while minimizing both measurement and computational costs. The filtered-x least mean square (FxLMS) algorithm has been widely adopted due to its effective balance between performance, ease of implementation, and computational efficiency. However, FxLMS relies on accurate secondary path estimation, which can be computationally intensive and time consuming. Recently, as a model-free method, the simultaneous perturbation stochastic approximation (SPSA) algorithm has been successfully applied in broadband, multi-channel, and time-varying environments. Despite its demonstrated effectiveness in noise reduction, the stability of SPSA has received limited attention. Following a recently proposed stepwise SPSA extension, which significantly enhances the stability of classical SPSA, this work investigates its stability characteristics with respect to key algorithmic parameters. The analysis provides insights into the parameter selection process, enabling efficient noise reduction while ensuring stability.

Keywords: *active noise control, stepwise SPSA, stability analysis, parameter optimization*

1. INTRODUCTION

The filtered-x least mean square (FxLMS) algorithm has been widely adopted in industrial active noise control (ANC) applications due to its effectiveness and implementation simplicity [1–3]. However, its performance is highly dependent on an accurate model of the secondary path, making it less suitable for time-varying environments.

As an alternative, simultaneous perturbation stochastic approximation (SPSA) provides a model-free approach to ANC. It updates the controller by estimating the gradient of the loss function using only function evaluations, thereby eliminating the need for explicit secondary-path modeling [4]. This characteristic enables SPSA to better adapt to changes in the acoustic environment.

SPSA has been applied in various ANC scenarios, including early adaptive systems [5], feedback control for periodic disturbances [6], time-varying environments [7], cross-correlation-based enhancements [8,9], second-order variants for faster convergence [10], and multi-channel extensions [11]. Despite these developments, SPSA's performance remains sensitive to parameter tuning, particularly the step size a and the perturbation coefficient c, where inappropriate settings may result in instability.

While prior research has primarily focused on the noise reduction performance of SPSA-based ANC, limited attention has been paid to its stability characteristics. Recently, stepwise SPSA (S-SPSA) was introduced to improve the robustness of the algorithm with respect to the selection of a and c [12]. This paper builds upon that foundation by further analyzing the influence of key algorithmic parameters on system stability and performance.

2. ALGORITHM DESCRIPTION

As illustrated in Fig. 1, the reference signal $x(n)$ passes through the primary path $P(n)$, generating the primary noise $d(n)$, where n denotes the discrete time index. To attenuate $d(n)$, the adaptive filter with weights $\boldsymbol{w}(n)$ gen-

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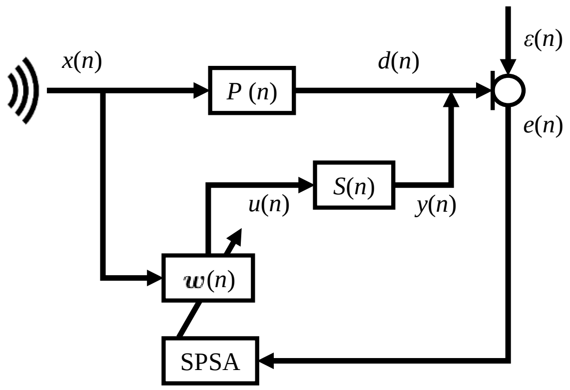


Figure 1: Block diagram of the ANC system using SPSA.

erates a control signal $u(n)$, which passes through the secondary path $S(n)$ to produce the anti-noise signal $y(n)$. The residual error $e(n)$, which also includes background noise $\varepsilon(n)$, is given by

$$e(n) = d(n) - y(n) + \varepsilon(n). \quad (1)$$

SPSA:

To minimize the energy of the residual error, SPSA iteratively updates the filter coefficients using

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) - a\hat{\boldsymbol{g}}(\boldsymbol{w}(k)), \quad (2)$$

where k is the block index, a is the step size, and $\hat{\boldsymbol{g}}(\boldsymbol{w}(k))$ denotes the estimated gradient of the loss function.

The gradient is estimated by applying the perturbation $c\boldsymbol{\Delta}(k)$ to the weights $\boldsymbol{w}(k)$ and evaluating the loss function over two consecutive segments within a data block of length 2λ:

$$J^{+} = J(\boldsymbol{w}(k) + c\boldsymbol{\Delta}(k)) = \frac{1}{2}\sum_{n=1}^{\lambda} e^2(n), \quad (3)$$

$$J^{-} = J(\boldsymbol{w}(k) - c\boldsymbol{\Delta}(k)) = \frac{1}{2}\sum_{n=\lambda+1}^{2\lambda} e^2(n), \quad (4)$$

where J is the loss function defined as the sum of squared error signals, c is the perturbation coefficient, $\boldsymbol{\Delta}(k)$ is a random perturbation vector, and 2λ is the block length. This structure ensures that the gradient approximation is based on distinct error responses to positive and negative perturbations, allowing for an efficient stochastic update.

The gradient is then estimated as:

$$\hat{\boldsymbol{g}}(\boldsymbol{w}(k)) = \frac{J^{+} - J^{-}}{2c\boldsymbol{\Delta}(k)}. \quad (5)$$

S-SPSA:

To improve stability and convergence, the stepwise SPSA (S-SPSA) divides each block into $2m$ equal-length segments. Each segment has a length of λ/m. The loss functions for the i-th segment are defined as:

$$J_i^{s+} = \frac{1}{2}\sum_{n=(i-1)\frac{\lambda}{m}+1}^{i\frac{\lambda}{m}} e^2(n), \quad (6)$$

$$J_i^{s-} = \frac{1}{2}\sum_{n=(i-1)\frac{\lambda}{m}+\lambda+1}^{i\frac{\lambda}{m}+\lambda} e^2(n). \quad (7)$$

Each segment computes its own gradient independently:

$$\hat{\boldsymbol{g}}_i^{s} = \frac{J_i^{s+} - J_i^{s-}}{2c\boldsymbol{\Delta}_i(k)}, \quad \hat{\boldsymbol{g}}_i^{s} \in R^{\frac{L}{m}}, \quad (8)$$

where $\boldsymbol{\Delta}_i(k)$ is the i-th subvector of the perturbation vector $\boldsymbol{\Delta}(k)$, such that:

$$\boldsymbol{\Delta}(k) = \left[\boldsymbol{\Delta}_1^T(k), \boldsymbol{\Delta}_2^T(k), \ldots, \boldsymbol{\Delta}_m^T(k)\right]^T.$$

Finally, each sub-filter $\boldsymbol{w}_i(k)$ is updated independently as:

$$\boldsymbol{w}_i(k+1) = \boldsymbol{w}_i(k) - a\hat{\boldsymbol{g}}_i^{s}(\boldsymbol{w}(k)). \quad (9)$$

3. NUMERICAL SIMULATION AND RESULT DISCUSSION

To investigate the influence of parameters on the stability of the algorithms, numerical simulations are conducted using the following default configuration. The sampling rate is set to 16 kHz. The adaptive filter $\boldsymbol{w}(n)$ has a length of $L = 1024$, and the block length is set to $2\lambda = 4000$. For the S-SPSA, each block is divided into $2m$ segments, where $m = 2$.

The reference signal $x(n)$ consists of a sum of tonal components with equal amplitude $A = 1$, at frequencies of 150 Hz, 200 Hz, ..., up to 700 Hz. Gaussian white noise $\varepsilon(n)$ is added as background interference, yielding a signal-to-noise ratio (SNR) of 10 dB between the original primary noise $d(n)$ and the background noise. The primary and secondary paths are measured experimentally using a Kundt's tube and their frequency responses cover the range from 100 Hz to 750 Hz.

To evaluate the performance of active noise control, the attenuation of noise is quantified using the average noise reduction (ANR), defined as:

$$\mathrm{ANR}(n) = 10\log_{10}\frac{\sum_{i=n-l+1}^{n} e^2(i)}{\sum_{i=n-l+1}^{n}(d(i)+\varepsilon(i))^2}, \quad (10)$$

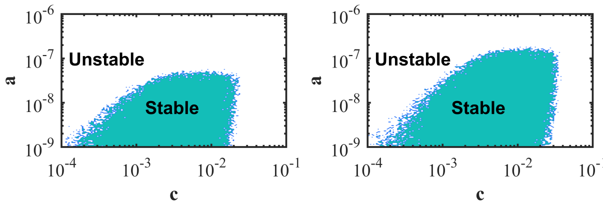
where the window length is set to $l = 2000$ samples.

3.1 Stability Comparison

To evaluate algorithmic stability, each simulation runs for 60 seconds. The system is considered *unstable* if one of the following conditions is met:

$$\begin{cases} \sum_{n=1}^{2\lambda} e^2(n) > 2\sum_{n=1}^{2\lambda} d^2(n), \\ \mathrm{ANR}(t = 60\,\mathrm{s}) \geq 0, \end{cases} \quad (11)$$

and is defined as *stable* otherwise.



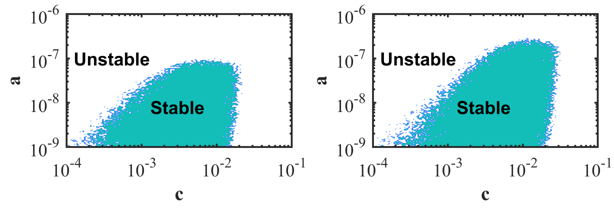
(a) SPSA (b) S-SPSA($m = 2$)

(c) S-SPSA($m = 4$) (d) S-SPSA($m = 8$)

Figure 2: Stability of the algorithms with respect to the parameter m.

As shown in Fig. 2, green regions indicate parameter combinations that lead to stable operation. Compared to the standard SPSA, the stepwise variant S-SPSA with $m = 2$ notably expands the stability region. Further increasing the segment number to $m = 4$ and $m = 8$ raises the upper bounds of stable values for both step size a and perturbation coefficient c. However, S-SPSA with $m = 8$ exhibits a narrower overall stability region, indicating that $m = 2$ and $m = 4$ have better stability.

In Fig. 3, S-SPSA again demonstrates a broader stable region than conventional SPSA. Compared to Fig. 2a and Fig. 2b, it can be observed that reducing either the block length λ or filter length L increases the upper stability boundary for the step size a, implying improved robustness under smaller system dimensions.



(a) SPSA($\lambda = 1$K) (b) S-SPSA($\lambda = 1$K)

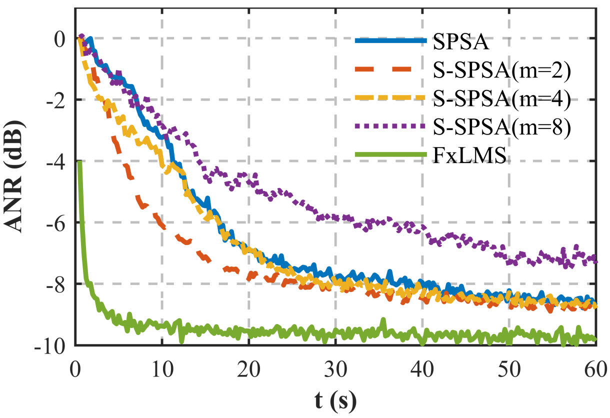
(c) SPSA($L = 512$) (d) S-SPSA($L = 512$)

Figure 3: Stability of the algorithms with respect to the parameters λ and L.

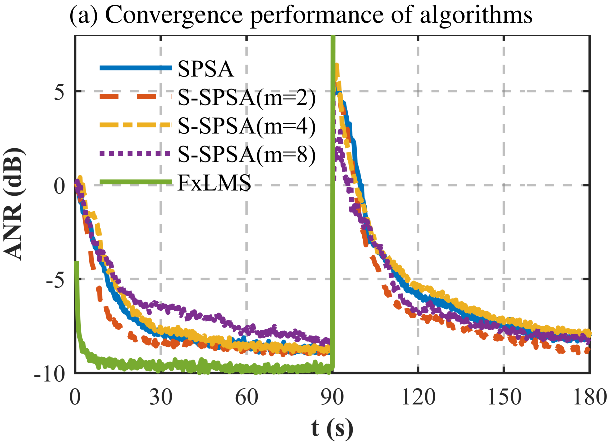
3.2 Performance Comparison

The tracking performance of the algorithms is assessed using the ANR metric, defined in Eq. (10), with optimal parameters a and c. As depicted in Fig. 4a, the convergence behavior of standard SPSA, S-SPSA with $m = 2, 4, 8$, and FxLMS is evaluated over 60 seconds. S-SPSA with $m = 4$ closely approximates the performance of standard SPSA, stabilizing at an ANR of approximately -8.7 dB. The variant with $m = 2$ exhibits the fastest initial convergence among S-SPSA configurations, reaching -6 dB within 10 seconds, while $m = 8$ converges more slowly, stabilizing at a slightly higher ANR of -7.3 dB. In contrast, FxLMS demonstrates superior performance, achieving an ANR below -8 dB within 5 seconds and maintaining a steady-state value near -10 dB.

The robustness of these algorithms to abrupt changes is examined in Fig. 4b, where the secondary path is reversed at $t = 90$ s. After this point, FxLMS diverges

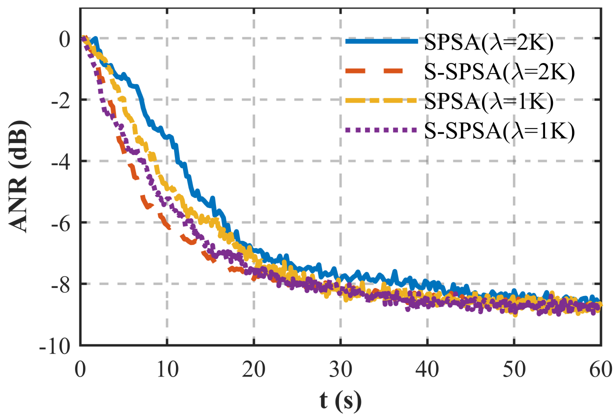


(a) Convergence performance of algorithms

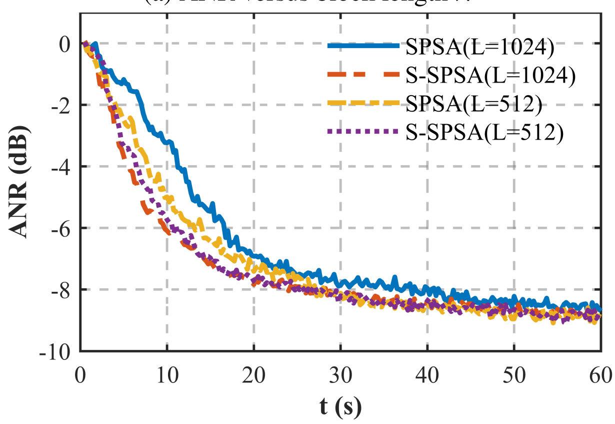


(b) Robustness to secondary path reversal at $t = 90$ s

Figure 4: ANR performance with the parameter m



(a) ANR versus block length λ



(b) ANR versus filter length L

Figure 5: ANR performance with respect to the parameters λ and L

sharply to instability. Conversely, SPSA and S-SPSA variants exhibit adaptivity; their ANR values increase temporarily to over 0 dB but recover gradually over the subsequent 90 seconds.

The influence of block length λ and filter length L on performance is shown in Fig. 5. At $\lambda = 1$K and $L = 512$, the ANR differences across SPSA and S-SPSA diminish, suggesting that shorter block and filter lengths lessen the impact of m on noise reduction performance.

4. CONCLUSION

This paper investigates the impact of the segment number m, half block length λ, and filter length L on the stability and noise reduction performance of the SPSA and S-SPSA algorithms. The results demonstrate that increasing m generally raises the upper stability bound for the step size a, while configurations with $m = 2$ or $m = 4$ are recommended due to their stability and noise attenuation performance.

Additionally, smaller values of λ and L not only enhance algorithmic stability by allowing for larger values of a, but also reduce the performance gap between SPSA and S-SPSA variants. These findings suggest that appropriate segmentation and parameter selection are crucial for robust and efficient adaptive noise control using perturbation-based optimization.

5. ACKNOWLEDGMENTS

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