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STOCHASTIC APPROACH FOR ESTIMATING THE SOUND POWER OF TONAL MODE COMPONENTS IN FLOW DUCTS BASED ON AZIMUTHAL MODE ANALYSIS

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ABSTRACT

The accurate experimental determination of the sound power in the ducts of aircraft engines, engine demonstrators and turbomachinery test rigs is challenging due to limited installation space for sensor arrays. Particularly in the inlet, a high number of propagating modes requires sensor arrays with multiple axial positions in order to decompose the measured sound field into its radial mode constituents. As an alternative, a sensor ring array requires a minimum of axial installation space and provides insight into the mode distribution using an azimuthal mode analysis (AMA). In this case, it is necessary to use radial mode distribution models, such as "equal mode energy" in order to estimate the sound power. However, the difficulty with tonal components, e.g. those generated by rotor-stator interaction, is that the radial modes are mutually coherent. This leads to a complex relation between the azimuthal mode amplitudes and the underlying mode distribution, which varies with the individual case, e.g. operating parameters of the rotor-stator configuration. To account for this variability, a stochastic approach is proposed in which, in particular, the phase relationships of the modes are varied and a distribution of sound power is determined. Its validity is evaluated by comparison with the results of a radial mode analysis obtained at a fan test rig.

Keywords: tonal sound power, azimuthal mode analysis, flow ducts

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1. INTRODUCTION

The next generation of UHBR aircraft engines will be equipped with shorter nacelles in order to reduce the overall engine weight. This development has an impact on the fan inflow and the resulting fan noise excitation, which needs to be assessed experimentally under realistic conditions. Engine demonstrators, just like the real aircraft engines, provide a very limited installation space for microphone arrays, particularly in the inlet. Therefore, microphone ring arrays are applied in many cases enabling the decomposition of the measured sound field into azimuthal modes. In contrast to radial modes, which are determined by radial mode analysis, azimuthal modes do not allow a direct computation of the propagating sound power. Based on the original work of Joseph et al. [1], a sound power estimation technique had been developed on the basis of azimuthal mode spectra and radial mode distribution models and applied to experimental data from small and large-scale test rigs [2–4]. One of the fundamental assumptions of this estimation technique is mutual incoherence of all radial mode constituents, which is an appropriate assumption for broadband sound field components. Regarding tonal mode components, this approach neglects the fact, that all radial modes are fully coherent resulting in interference of the modes and strongly fluctuating sound pressure levels depending on the axial position of the microphone ring array. Despite the inappropriateness of the assumption of mutual mode incoherence, the approach for the broadband case was applied to the analysis of tonal mode components on a LPT rig [2] and on a radial compressor rig [5].

This article explores a stochastic approach that allows to take coherent modes into account and to evaluate the impact of varying phase relations between the modes. Just like in the broadband case, the magnitudes of the radial mode amplitudes are prescribed according to radial mode distribution models. Additionally, the individual radial modes are assigned with randomly selected phases so that the resulting interference can be evaluated stochastically in a Monte Carlo study. The result of this approach is a distribution of sound power, of which the spread and the usefulness of the median value as a possible sound power estimation value are evaluated using measured data from a fan test rig and compared with reference results from radial mode analysis.

2. SOUND POWER ESTIMATION BASED ON AZIMUTHAL MODE ANALYSIS

Sound fields in cylindrical and annular flow ducts with a uniform background flow with Mach number M_x can be decomposed into radial modes of the form

$$p\left(x,r,\phi\right)=\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\left(A_{mn}^{+}e^{ik_{mn}^{+}x}+A_{mn}^{-}e^{ik_{mn}^{-}x}\right)\cdot e^{im\phi}f_{mn}(r) \tag{1}$$

with azimuthal mode order m, radial mode order n, mode amplitudes $A_{mn}^{\pm}$, axial wave number $k_{mn}^{\pm}$, and radial mode shape function $f_{mn}(r)$ [6]. On the basis of the mode amplitude, the mode sound power is calculated as:

$$P_{mn}^{\pm}=\frac{\pi R^2}{\rho c}\frac{\alpha_{mn}(1-M_x^2)^2}{(1\mp\alpha_{mn}M_x)^2}|A_{mn}^{\pm}|^2, \tag{2}$$

with the outer duct radius R, static density ρ and speed of sound c. The term α_{mn} is called the mode propagation factor and plays a particular role in the estimation of the mode sound power. It is defined as:

$$\alpha_{mn}=\sqrt{1-(1-M_x^2)\,\frac{\sigma_{mn}^2}{(kR)^2}}, \tag{3}$$

where σ_{mn} is the radial-transversal eigenvalue. For propagating modes, α_{mn} ranges between 0 at the mode cut-on frequency and 1. It is imaginary for cut-off modes and determines the rate of decay in axial direction through its relation to the axial wave number $k_{mn}^{\pm}=\frac{k}{1-M_x^2}\left(-M_x\pm\alpha_{mn}\right)$.

The determination of the radial mode amplitudes in Eq. (1) via Radial Mode Analysis (RMA) requires a considerable effort in equipment (microphones, traversing devices) and measurement duration. The microphone arrays, that are typically installed at the duct walls, extend over an axial installation space, which increases depending on the highest frequency of interest and is usually limited due to spatial constraints of the test environment [6]. An alternative analysis technique is the azimuthal mode analysis (AMA) (see e.g. [7]), which requires a ring array of wall-flush mounted microphones and can be realised in spatially limited test environments such as the inlets of aero engines or engine demonstrators. Mathematically, AMA corresponds to the discrete form of the spatial Fourier transform of the in-duct sound field in circumferential direction conducted e.g. with microphones installed flush at the outer duct wall:

$$A_m(x)=\frac{1}{N_{\text{mic}}}\sum_{j=1}^{N_{\text{mic}}}p(x,r=R,\phi_j)\cdot e^{-im\phi_j}. \tag{4}$$

The relation of the azimuthal mode amplitude A_m and the amplitudes of the underlying radial modes is obtained from eq. (4):

$$A_m(x)=\sum_{n=0}^{n_{\max}}\left(A_{mn}^{+}e^{ik_{mn}^{+}x}+A_{mn}^{-}e^{ik_{mn}^{-}x}\right)\cdot f_{mn}(R) \tag{5}$$

as a summation over all cut-on radial mode orders. In contrast to the radial mode amplitudes, the azimuthal mode amplitude can vary strongly depending on the axial position of the ring array. In order to estimate the mode sound power from azimuthal mode amplitudes, Eqs. (2) and (5) need to be combined. Since the axial wave numbers are determined from the known duct geometry, frequency and flow conditions, the main difficulty in estimating the mode sound power from azimuthal mode amplitudes lies in the estimation of the unknown amplitudes and phases of the radial modes. In previous studies, the mode sound power in broadband sound fields was estimated using mode distribution models such as "‘equal mode energy"' or "‘equal mode energy density"' to relate the magnitude of the radial mode amplitudes to single parameters of the corresponding mode distribution model [1,3,4] with the assumption of mutual incoherence of the radial modes, which makes it possible to neglect the mode phases and the axial propagation terms $e^{ik_{mn}^{-}x}$. In combination with the assumption that only modes propagating in a single direction, e.g. upstream, are present, the square of Eq. (5) yields for the broadband case:

$$|A_m|^2 = \sum_n |A^-_{mn}|^2 f^2_{mn}(R). \tag{6}$$

Equation (6) can be solved for the radial mode amplitudes $|A^-_{mn}|^2$ using a mode distribution model.

Tonal sound field components comprise coherent modes whose phase relations have to be considered. Because the mode phases are unknown and can not be retrieved via AMA, a stochastic approach is sought. The initial equation, which relates A_m to A^-_{mn}, then yields:

$$A_m = \sum_n |A^-_{mn}| f_{mn}(R) e^{i2\pi\mathcal{U}(1,0)}. \tag{7}$$

In Equation (7), the mode phases are randomly selected from a uniform distribution with unit standard deviation and zero mean. As a consequence, the radial mode amplitudes and the resulting mode sound powers are random variables. An efficient evaluation of these random variables can be realised by Monte Carlo simulations.

3. MODE SOUND POWER DISTRIBUTION MODELS

Currently, generalised mode distribution models are lacking for the tonal mode components from turbomachinery stages. Therefore, two mode distribution models (see [1]), "‘equal mode energy"’ (EME) and "‘equal mode energy density"’ (EMED), that are widely applied for the analysis of fan broadband noise, and the sound power distribution according to point source models are introduced in the following to demonstrate the stochastic approach for mode sound power estimation. However, the expressions can easily be adapted to other mode distribution models in the future.

3.1 "‘Equal mode energy"’ model

The EME model assumes that all propagating modes with the same azimuthal mode order have the same sound energy and as a result, the same mode sound power, i.e. $P'_m := P^-_{mn}$. Now P'_m is inserted in Eq. (2), which is then solved for $|A^-_{mn}|^2$. From Eq. (7) the following expression for the azimuthal mode amplitude is obtained:

$$|A_m|^2 = \frac{\rho c}{\pi R^2}\frac{1}{\left(1-M_x^2\right)^2} P'_m \cdot \left|\sum_n \frac{1+\alpha_{mn}M_x}{\sqrt{\alpha_{mn}}} f_{mn}(R) e^{i2\pi\mathcal{U}(1,0)}\right|^2. \tag{8}$$

Then, the summed sound power $P^{\text{EME,-}}_m = N_n \cdot P'_m$ yields:

$$P^{\text{EME,-}}_m = N_n \frac{\pi R^2}{\rho c}\left(1-M_x^2\right)^2 |A_m|^2 \cdot \left|\sum_n \frac{1+\alpha_{mn}M_x}{\sqrt{\alpha_{mn}}} f_{mn}(R) e^{i2\pi\mathcal{U}(1,0)}\right|^{-2}. \tag{9}$$

The EME model is sensitive to modes that are close to their cut-on frequencies and whose propagation factor α_{mn} is close to 0. In this case, the summation over all radial mode orders is dominated by the contribution of the highest cut-on radial mode order, which effectively is closest to its cut-on frequency. Because the $1/\sqrt{\alpha_{mn}}$-term is unbounded at the cut-on frequency, the estimated sound power is zero directly at this frequency. Hence, the EME model is prone to underestimate the sound power.

3.2 "‘Equal mode energy density"’ model

For the EMED model, the sound power is expressed in terms of the energy density $\Pi^\pm_{mn}$ [1]:

$$P^\pm_{mn} = \Pi^\pm_{mn} S c_{\text{g,mn}} = \Pi^\pm_{mn} S c \frac{\alpha_{mn}\left(1-M_x^2\right)}{1 \mp \alpha_{mn}M_x}, \tag{10}$$

with the duct cross-section S and the mode group velocity $c_{\text{g,mn}}$. It is assumed that all upstream propagating modes with the same azimuthal mode order have the same energy density, i.e. $\Pi'_m := \Pi^-_{mn}$. Modes propagating downstream are neglected at this point. Setting Eq. (10) equal to Eq. (2) and solving for $|A^-_{mn}|^2$ yields:

$$|A^-_{mn}|^2 = S\frac{\rho c^2}{\pi R^2}\frac{1+\alpha_{mn}M_x}{1-M_x^2}\Pi'_m. \tag{11}$$

From Eq. (7) the relation between azimuthal mode amplitude and mode energy density reads:

$$|A_m|^2 = S\frac{\rho c^2}{\pi R^2}\frac{1}{1-M_x^2}\Pi'_m \cdot \left|\sum_n \sqrt{1+\alpha_{mn}M_x} f_{mn}(R) e^{i2\pi\mathcal{U}(1,0)}\right|^2. \tag{12}$$

The summed sound power is computed as follows:

$$P_m^{\text{EMED,-}} = Sc\left(1 - M_x^2\right)\Pi'_m \sum_n \frac{\alpha_{mn}}{1 + \alpha_{mn} M_x}. \quad (13)$$

Combining Eqs. (12) and (13) provides the estimation equation for the EMED model:

$$P_m^{\text{EMED,-}} = \frac{\pi R^2}{\rho c}\left(1 - M_x^2\right)^2 |A_m|^2 \cdot \frac{\sum_n \frac{\alpha_{mn}}{1+\alpha_{mn} M_x}}{\left|\sum_n \sqrt{1+\alpha_{mn} M_x} f_{mn}(R) e^{i2\pi\mathcal{U}(1,0)}\right|^2}. \quad (14)$$

In comparison to the EME model, the EMED model is less sensitive to low values of the mode propagation factor α_{mn}.

3.3 ISI point source model

Point source models such as monopoles and dipoles are suitable to model the sound generation from different interaction mechanisms in turbomachinery stages [8]. For high accuracy of the determined sound power, the blade geometries and aerodynamic properties need to be modelled in detail to calculate the source strengths. Instead, a dense distribution of point sources is assumed across the duct cross-section as a substitute model, which ensures that each mode order can be excited effectively. A distribution of a high number of densely positioned point sources is equivalent to a continuous source distribution as described in [1]. The integration of the contribution of each point source over the duct cross-section yields the total radiated mode field, which depends on mode-specific parameters like the mode propagation factor α_{mn}, but is independent of the radial and azimuthal mode shape. Furthermore, the stochastic sound power estimation approach based on the azimuthal mode analysis is chosen, in which only the distribution of mode amplitudes is predicted by the point source models and the mode phases is selected randomly from a random distribution.

The absolute values of the radial mode amplitudes are predicted on the basis of the mode impedance $\hat{Z}_{mn}^{\pm}$ and the source strength q, which are described for different source types in more detail e.g. in [9]. In this reference, the ISI point source model is introduced, which is characterised by a more realistic excitation of modes close to their cut-on frequencies by virtue of an internal source impedance and applied in the following investigation:

$$|A_{mn}^{\pm}| = |\hat{Z}_{\text{mn,ISI}}^{\pm} q|. \quad (15)$$

The mode impedance of the source distribution with an assumed dipole characteristic is approximated as:

$$\hat{Z}_{\text{mn,ISI}}^{\pm} = -i\frac{\rho c}{4\pi R^2}\frac{1/\alpha_{mn}}{1 + \frac{N_M}{(kR)^2 + \frac{25}{kR}} \cdot \frac{1-\alpha_{mn}}{\alpha_{mn}}} \cdot \left(k_{mn}^{\pm}\cos\alpha_{\text{dipole}} + \frac{\sigma_{mn}}{R}\sin\alpha_{\text{dipole}}\right) \quad (16)$$

with the total number of cut-on modes N_M and the dipole angle α_{dipole}.

From Eq. (7) the azimuthal mode amplitude can be expressed as:

$$A_m = |q|\sum_n |\hat{Z}_{\text{mn,ISI}}^{-}| f_{mn}(R) e^{i2\pi\mathcal{U}(1,0)}. \quad (17)$$

Likewise, the summed mode sound power is directly computed by inserting Eq. (15) in Eq. (2):

$$P_m^{\text{ISI,-}} = \sum_n \frac{\pi R^2}{\rho c}\frac{\alpha_{mn}(1-M_x^2)^2}{(1+\alpha_{mn} M_x)^2}|\hat{Z}_{\text{mn,ISI}}^{-} q|^2 \quad (18)$$

$$= \frac{\pi R^2}{\rho c}|A_m|^2 \sum_n \frac{\alpha_{mn}(1-M_x^2)^2}{(1+\alpha_{mn} M_x)^2} \cdot \frac{|\hat{Z}_{\text{mn,ISI}}^{-}|^2}{|\sum_{\tilde{n}=0}^{n_{\max}} |\hat{Z}_{m\tilde{n},\text{ISI}}^{-}| f_{m\tilde{n}}(R) e^{i2\pi\mathcal{U}(1,0)}|^2}. \quad (19)$$

4. APPLICATION TO FAN TEST RIG DATA

The stochastic approach for sound power estimation based on azimuthal mode amplitudes is demonstrated on both, synthesized and experimental data from DLR's fan test rig CRAFT (Co-/Counter-Rotating Acoustic Fan Test Rig). It is equipped with a fan stage whose rotor has 18 blades and stator has 21 vanes. The duct diameter at the inlet test section is 453.6 mm. At the fan design point, the flow Mach number in the inlet is $M_x = 0.11$. Typically, the upstream radiated sound power of the tonal mode components is determined by radial mode analysis on the basis of traverse measurements with an axial microphone array, which is depicted in Fig. 1. The RMA results serve as reference data for the comparison with the estimated sound powers using the measured data from the ring array in combination with the stochastic estimation approaches.

For further information on the CRAFT test rig itself as well as on the measurements and data analysis of the data used here, the interested reader is referred to Ref. [10] and Ref. [11].

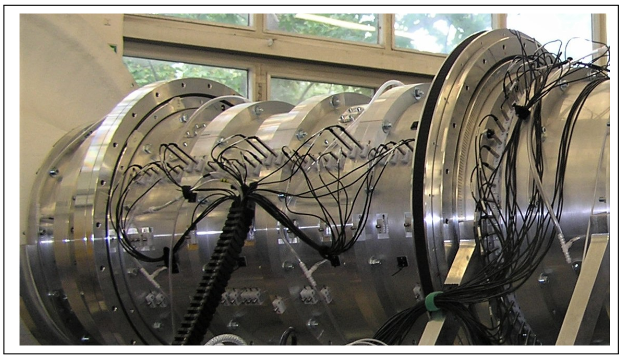

Figure 1. Microphone arrays in the inlet of the CRAFT fan test rig [11]: a traversable, axial line array (left) and ring array (right).

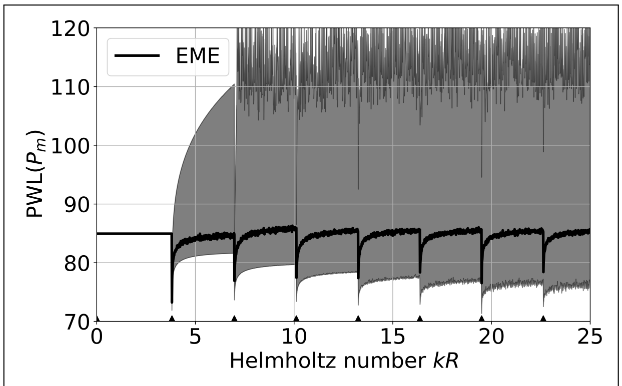

Figure 2. Sound power estimation results for constant azimuthal mode amplitude $A_0 = 1$ assuming "'equal mode energy"'.

First, the stochastic sound power estimation procedure is demonstrated for a synthetic case with constant azimuthal mode amplitude $A_0 = 1$. The results in Figs. 2, 3, and 4 show the estimation results for $m = 0$ using the mode distribution models EME and EMED and the ISI point source model. For the Monte Carlo study, a total of 1000 iterations were performed. The shaded areas indicate the complete dynamic range from the minimum to the maximum of the distribution of

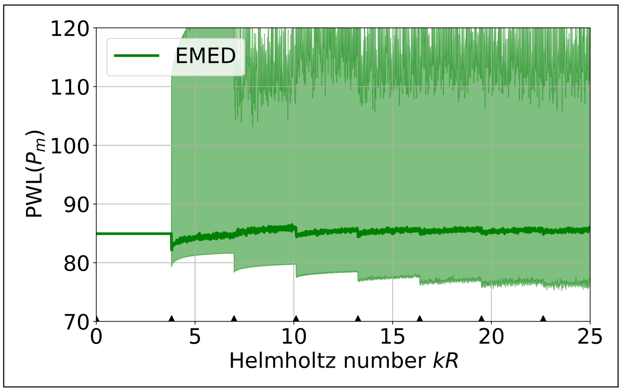

Figure 3. Sound power estimation results for constant azimuthal mode amplitude $A_0 = 1$ assuming "'equal mode energy density"'.

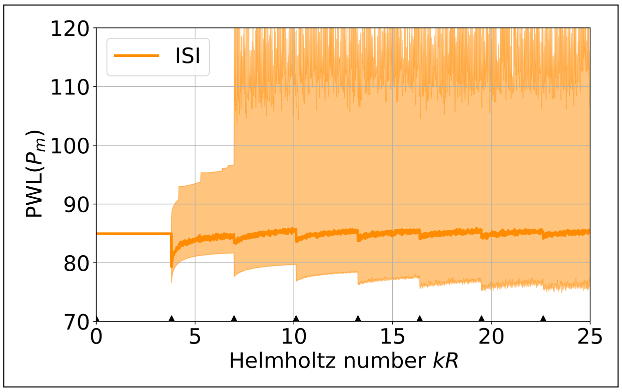

Figure 4. Sound power estimation results for constant azimuthal mode amplitude $A_0 = 1$ using the ISI point source model.

estimated sound powers. The median value of the sound power distribution is shown by the darker lines. Below $kR \approx 4.5$, only the radial mode order $n = 0$ is cut-on, which leads to the same sound power estimation for all models. Furthermore, in this case the sound power distribution is concentrated at a constant value, despite the variation of the mode phase terms. Hence, the stochastic variation, which is introduced in Eq. (7), only has an impact if several radial mode orders are cut-on for the same azimuthal mode order. Also, although no particular sound power distribution is assigned in the comparison of the different models, that is supposed to be reconstructed, it seems plausible that if the azimuthal mode amplitude is constant, the sound power lies in a rather narrow dynamic

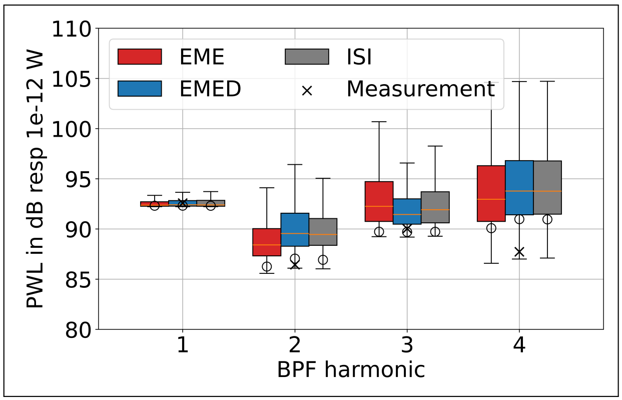

Figure 5. Total estimated sound power for the tonal mode components at 1BPF to 4BPF in comparison to reference measurement data (indicated by crosses) and estimation results assuming mutual mode incoherence (cp. (6), indicated by circles).

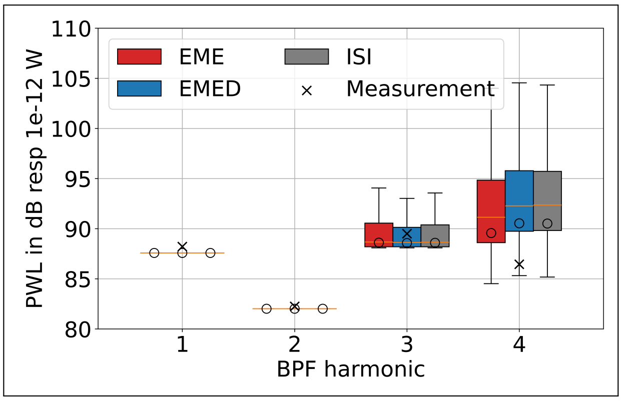

Figure 6. Estimated sound power of the rotor-stator interaction modes at 1BPF to 4BPF in comparison to reference measurement data (indicated by crosses) and estimation results assuming mutual mode incoherence (indicated by circles).

range.

Above the cut-on frequency of $n = 1$, which is indicated by a small triangle at $kR \approx 4.5$ at the bottom of the figure, the dynamic range of the sound power distribution increases rapidly and asymmetrically. The results for the EMED model exhibit the steepest increase reaching a maximum dynamic range of more than 40 dB just below the cut-on frequency of the next higher radial mode order $n = 2$. For the EME model, the estimated sound power drops directly at the mode cut-on frequencies and then increases monotonically up to a dynamic range of about 30 dB, at least up to the cut-on frequency of $n = 2$. The sound power distribution for the ISI model extends over a smaller dynamic range of about 15 dB below the cut-on frequency of $n = 2$. However, above approximately $kR = 7.5$ the dynamic range of the sound power distributions fluctuates between 30 and 45 dB independent of the underlying model.

The median proves to be a robust descriptor for the estimated sound power. For the EME model, the median sound power drops at each mode cut-on frequency and then increases approaching a sound power level of approximately 85 dB. In contrast, the median sound power for the EMED model is almost constant, except for small drop at the cut-on frequency of the mode order $n = 1$. The ISI model leads to results that combine the characteristics of the EME and the EMED model. At relatively low frequencies, the median curve is similar to the one of the EME model, particularly at the cut-on frequency of mode order $n = 1$. With increasing frequency, the median curve of the ISI model becomes more similar to the median curve of the EMED model with very small fluctuations close to the mode cut-on frequencies.

In the following, the stochastic approach is applied to measured data from the inlet of the CRAFT fan test rig in order to estimate the sound power of the tonal components radiated upstream at the blade passing frequency (BPF) and its harmonics. Figure 5 shows the distributions of the estimated total sound power for the three different models in comparison with the reference results, which were obtained by RMA. Additionally, the estimation results under the assumption of mutual mode incoherence, which is a common assumption for the analysis of broadband sound fields, are indicated with circles. Except for 1BPF, the median of the estimated distributions overestimates the radiated sound power in comparison with the RMA results by 2 to 3 dB at the second and third harmonic, and approximately 9 to 10 dB at 4BPF. The results show that none of the three applied models leads to a significantly better agreement with the experimental data than the others.

In Fig. 6, the sound power levels are shown only for

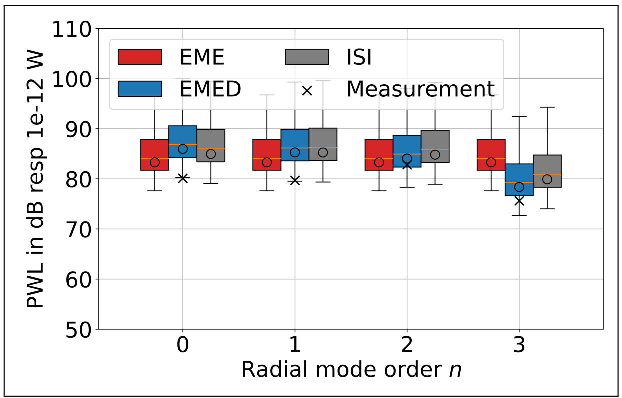

Figure 7. Estimated sound power of the rotor-stator interaction mode with mode order $m = -9$ at 4BPF in comparison to reference measurement data (indicated by crosses) and estimation results assuming mutual mode incoherence (indicated by circles).

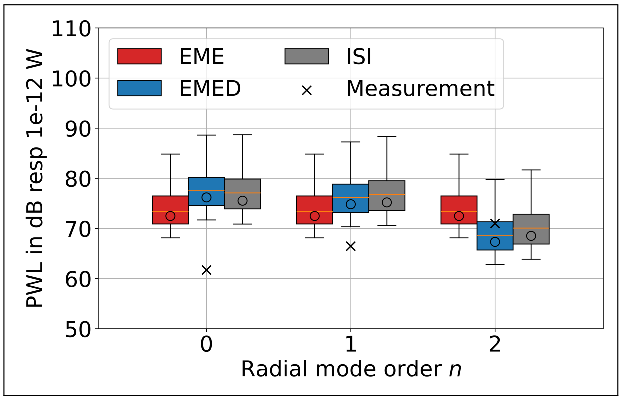

Figure 8. Estimated sound power of the rotor-stator interaction mode with mode order $m = 12$ at 4BPF in comparison to reference measurement data (indicated by crosses) and estimation results assuming mutual mode incoherence (indicated by circles).

the rotor-stator interaction modes, which are obtained from the Tyler-Sofrin rule [12]. For 1BPF and 2BPF, only radial mode order $n = 0$ is cut-on for the respective azimuthal mode orders, so that all models yield the same result for the tonal and broadband analysis. The corresponding sound power estimation results are in very good agreement with the reference results. At 3BPF, the median values of the three estimated distributions are nearly equal and lie within 1 dB of the measurement result. Similar to the results of the total sound power, for 4BPF the median values of all three distributions overestimate the radiated sound power by 9 to 10 dB in comparison with the reference results, which is further investigated next.

The first rotor-stator interaction mode at 4BPF has the azimuthal mode order $m = -9$. Figure 7 shows the sound power levels and distributions for each propagating radial mode order. It is evident, that the relative distribution of sound power over the radial mode orders matches relatively well with the EMED and ISI model. However, for the low radial mode orders both models overestimate the sound power stronger than the EME model.

The mismatch between the actual sound power levels of the propagating radial mode orders and the mode distribution models is even more pronounced for the second rotor-stator interaction mode at 4BPF with the azimuthal mode order $m = 12$. As shown in Fig. 8, the sound power levels of the reference results increase from low to higher radial mode order, which is in contrast to the mode distribution models. This mismatch leads to a strong overestimation of 10 to 12 dB for mode order $n = 0$ and 4 to 6 dB for mode order $n = 1$. This example illustrates an important deficiency of the available mode distribution models: their inability to represent the complex source distribution of turbomachine blade tones, which vary significantly with the rotor-stator stage geometry and the operating-point-dependent rotor wake characteristics (see e.g. [8]). These complex relations are not considered in any of the available mode distribution models.

5. CONCLUSION

A stochastic approach for sound power estimation based on azimuthal mode analysis is proposed, which takes into account the mutual coherence of tonal mode components radiated from turbomachinery stages. The approach combines prescribed magnitudes of the radial modes according to mode distribution models and randomly selected mode phases. Evaluating the sound power estimation in a Monte Carlo study yields a sound power distribution. It is shown that the spread of the distribution depends strongly on the number of propagating radial mode orders, whereas the median value of the distribution is a robust indicator of the estimated sound power. Besides two commonly used mode distribution models, "‘equal mode energy"' and "'equal mode energy density"', a third distribution model is developed based on a dense distribution of point sources, which is equivalent to constant source distribution over the duct cross-section. Here, the internal source impedance model for dipole sources from [9] is applied. However, the results for the estimated sound power from the three distribution models do not show a clear superiority of any distribution model. In the investigated test case from a fan test rig, the estimated sound power is in good agreement with the reference results obtained by radial mode analysis for the first three BPF harmonics. At 4BPF, the sound power is overestimated by 9 to 10 dB, which is likely to be linked to an insufficient match between the mode distribution models and the actual mode sound power distribution with respect to the radial mode order. Due to complex relations between the geometric and aerodynamic design of the rotor-stator configuration and the generated distribution of the radiated mode sound power, it is difficult to determine a generally valid mode distribution model for tonal fan noise. Therefore, the sound power estimation for tonal components based on azimuthal mode analysis is challenging, but it can be useful to determine trends of the radiated sound power in test environments with limited installation space.

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CLEAN AVIATION

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