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CONTROL OF A NONLINEAR ELECTROACOUSTIC RESONATOR: REAL-TIME SELECTION OF THE EQUILIBRIUM POINT

Maxime Morell[1*] **Emanuele De Bono**[3] **Emmanuel Gourdon**[1]
Manuel Collet[2] **Alireza Ture Savadkoohi**[1] **Claude-Henri Lamarque**[1]
[1] ENTPE, Ecole Centrale de Lyon, CNRS, LTDS, UMR5513, 69518 Vaulx-en-Velin, France
[2] CNRS, Ecole Centrale de Lyon, ENTPE, LTDS, UMR5513, 69130 Ecully, France
[3] Ecole Centrale de Lyon, CNRS, ENTPE, LTDS, UMR5513, 69130 Ecully, France

ABSTRACT

To tackle the issue of adaptability to in-situ conditions while providing efficient acoustic absorption in a large frequency bandwidth, impedance control of loudspeakers has been brought as a solution. To control the impedance of a loudspeaker, one should place microphones collocated to the loudspeaker, supplied with a processor that calculates the needed electrical current to send to the loudspeaker's coil to target a specific impedance. This concept has long been considered in the framework of linear dynamics, however the extension to nonlinear dynamics is a current subject of interest. In this study, nonlinear dynamics are enforced through a feedforward control tracking a setpoint defined by a numeric exosystem. The use of the exosystem allows to synthesize uncommon behaviors such as nonpolynomial nonlinear restoring forces. However, nonlinear resonators may present many stable equilibrium points for a single frequency, which may be targeted via the choice of the initial conditions. The choice of the initial conditions in electroacoustics is challenging and would permit better sound absorption. In this study, a method for targeting the equilibrium point of a nonlinear dynamical system in the framework of electroacoustic resonators is presented, and its experimental implementation is demonstrated, leading to many potential acoustic applications.

**Corresponding author*: *maxime.morell@entpe.fr.*

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1. INTRODUCTION

An electroacoustic resonator (ER) is a loudspeaker collocated to microphones and equipped with a processor running a controller that takes into input the measured pressure, and provides as an output the electrical current to enforce to the loudspeaker [1, 2]. The benefits of an ER lies in its efficiency in a bandwidth around its resonance frequency, leading to the possibility of efficient acoustic absorption at low frequencies. The resonance frequency can be tuned through digital programming of the controller. As this concept is widely studied for linear behaviors enforced to the loudspeaker [3], enforcing nonlinear behavior using a digital control at low excitation amplitudes stands as a current subject of interest [4–7]. A nonlinear resonator offers larger frequency bandwidth of efficiency and features nonlinear phenomena useful of the mitigation of vibrations. However, a nonlinear system may present multiple co-existing equilibrium points at a single frequency, which do not have similar efficiency results in acoustic absorption [7, 8]. indeed, the system converges towards a specific equilibrium point depending on the initial conditions. As a result, the acoustic absorption capabilities of a nonlinear resonator, compared to a linear resonator, lies in the initial conditions. In this study, a nonlinear ER created by a digitally programmed controller is used for the mitigation of acoustic vibrations. As the choice of the initial conditions proves to be difficult, the choice of the equilibrium point might be done in the digital programming. The method for targeting a specific equilibrium point is presented.

2. THE DIGITAL CONTROL ENFORCING NONLINEAR BEHAVIOR

The linear dynamics are enforced through the infinite input response method, which can not be used to enforce nonlinear dynamics efficiently [4]. As a result, the nonlinear dynamics are enforced through a real-time based feed-forward controller tracking a setpoint defined by an Exosystem (ES).

Considering a loudspeaker collocated with microphones, the ER is created by assuming that the loudspeaker's response is dominated by its first mode, along with the hypothesis of low amplitudes (to keep the resonator in its linear regime) and low frequencies (to keep the excitation frequencies close to the ER's first mode).
The ES simulates the targeted behavior of the loudspeaker under the measured pressure. The ES model contains the dynamic to enforce, and is supplied with the measured pressure. Its numerical resolution can be realized using classical numerical schemes for solving ordinary differential equations. Once the targeted displacement, speed and acceleration are predicted, the electrical current is calculated using the feed-forward controller containing the loudspeaker's model driven by the predictions. The equations of the control are presented in Eqn. (1):

$$\dot{X}(t) = A(X(t), t, f_{ext}(t), u_d(t)) \tag{1a}$$

$$u_d(t) = \varphi(\dot{X}_d(t), X_d(t), t, f^m_{ext}(t)) \tag{1b}$$

$$\dot{X}_d(t) = F(X_d(t), t, f^m_{ext}(t)) \tag{1c}$$

where Eqn. (1a) is the equation of the system to control, here the loudspeaker. The variable X stands for the loudspeaker's modal coordinate of its first mode, t is the time, f_{ext} represents the external forces applied on the loudspeaker's membrane (the pressure), and u_d is the control variable allowing to apply the control, here the electrical current. The Eqn. (1b) is the equation that permits to determine the electrical current to send to the loudspeaker, which is obtained by inversion of the system Eqn. (1a). The variable f^m_{ext} represents the measured external forces. The calculation is done using the variable X_d standing for the desired modal coordinate provided by solving the ES Eqn. (1c). The ES Eqn. (1c) provides the nonlinear desired behavior by solving it using numerical schemes. These control equations are evaluated at each time step.

3. TARGETING SPECIFIC EQUILIBRIUM POINTS

By forcing the ES to target specific equilibrium points, the feed-forward controller then tracks the setpoint to enforce the ES behavior to the loudspeaker. As a result, modifying the basin of attraction of the ES to raise the size of the set of initial conditions to all the set of initial conditions leads to enforcing the targeted equilibrium point.

This goal can be achieved by adding synthetic variables to the ES equation to control the basin of attraction of the solution. Three methods are considered: adding a synthetic desired displacement X^s_d, or a desired speed $\dot{X}^s_d$, or an additional external force f^s_{ext}.
The form of the synthetic added variable should be selected to force the system onto an equilibrium point, and then set to 0 in order to maintain a passive behavior of the ER.

4. NUMERICAL SIMULATION OF THE CONTROL

To demonstrate that the method allows to choose a specific equilibrium point, numerical simulations are conducted. The control is simulated with and without the added synthetic variables, and solved using a fourth order Runge-Kutta with the following control equations:

$$M_0\ddot{x}(t) + R_0\dot{x}(t) + K_0x(t) = f_{ext}(t) - Blu_d(t) \tag{2a}$$

$$u_d(t) = \frac{1}{Bl}(-M_0\ddot{x}_d(t) - R_0\dot{x}_d(t) - K_0x_d(t) + f^m_{ext}(t)) \tag{2b}$$

$$M_d\ddot{x}_d(t) + R_d\dot{x}_d(t) + K_dx_d(t) + K_d\beta x_d(t)^3 = f^m_{ext}(t) \tag{2c}$$

with $\dot{\bullet}$ corresponds to a time derivative. The parameters M_0, R_0, K_0 stand for the modal mass, damping and stiffness of the loudspeaker without electrical current, and M_d, R_d and K_d represent the desired modal mass, damping and stiffness of the ES model. The parameter β is the nonlinear constant coefficient of the cubic nonlinearity. The simulations are conducted with the parameters $M_d = M_0 = 3.89 \times 10^{-4}$ kg, $R_d = R_0/8 = 3.29 \times 10^{-2}$ kg.s^{-1}, $K_d = K_0 = 4.34 \times 10^3$ kg.s^{-2} and $\beta = 10^{10}$ m^{-2}. The excitation is chosen to be a mono-harmonic sine excitation of amplitude 1.5 Pa. The initial conditions are taken as $(x_d(0), \dot{x}_d(0)) = (0, 0)$. The sampling frequency is chosen to be 50 kHz, with a 20 seconds duration of the

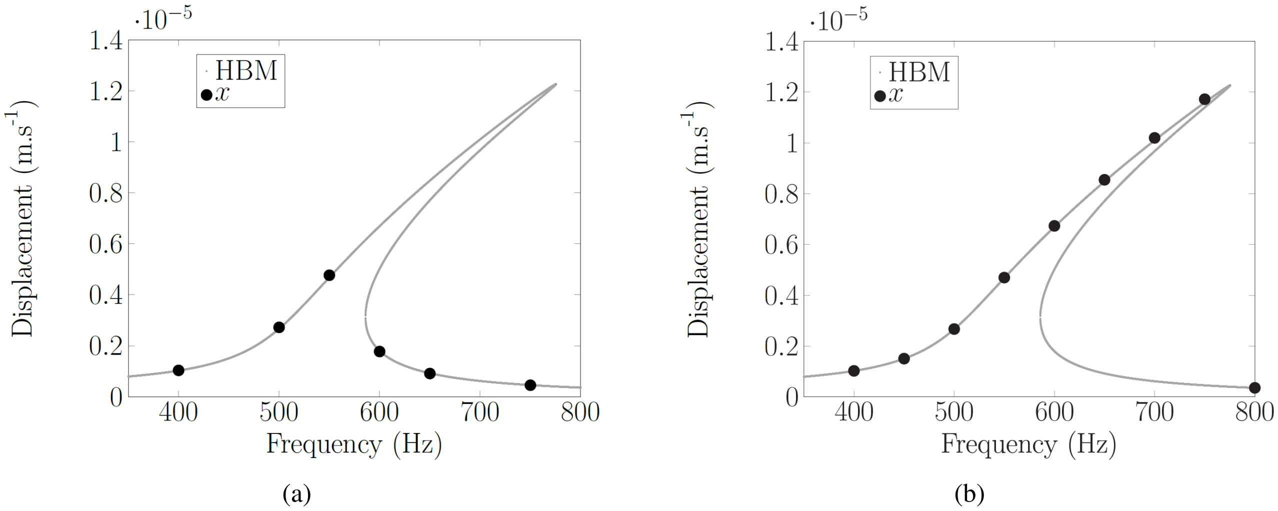

Figure 1: Comparison of the analytical solution of Eq. (1c) obtained with a Harmonic Balance Method (HBM) and the obtained equilibrium point with the numerical simulation of the control with (a) the classical control; (b) the added synthetic variables.

mono-harmonic sine excitation.
The synthetic added variable is chosen as an added external force of the form:

$$f_{ext}^{s} = 200\big(1 - \tanh(t-2)\big) \tag{3}$$

The results of the simulations are presented in Fig. 2. The obtained equilibrium point after 15 seconds of simulation is represented as a black dot. The numerical solutions are compared with a Harmonic Balance Solution (HBM) of the ES equation. One can observe in Fig. 2a that the control without the added synthetic variables targets the lower energy equilibrium point. However, with initial conditions at (0,0), and with the added synthetic variable in Fig. 2b, the control enforces the higher energy equilibrium point. It is due to the control of the basin of attraction of the ES equation with the added variable.
Numerical simulations validates the method, demonstrating its ability to drive the system towards a higher energy equilibrium point, which offers improved acoustic absorption capabilities. The results showed that without the synthetic control variable, the system naturally settled at a lower energy equilibrium point, while the added external synthetic force successfully guided the system toward the desired state.

5. CONCLUSION

In this study, we introduced a novel digital control strategy to enforce nonlinear behaviors in electroacoustic resonators for enhanced acoustic absorption. By leveraging an exosystem to generate targeted nonlinear dynamics, the feed-forward controller successfully tracks and enforces the desired equilibrium state of the loudspeaker. A key challenge of nonlinear systems, the coexistence of multiple equilibrium points with varying efficiency, was addressed by incorporating synthetic variables into the ES equation. This approach effectively reshaped the system's basin of attraction, enabling the selection of a specific equilibrium point through digital programming rather than relying on initial conditions. The proposed approach paves the way for digitally controlled nonlinear resonators with tunable properties, opening new possibilities for vibration and noise mitigation in advanced engineering systems.

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7. REFERENCES

[1] E. Rivet, S. Karkar, and H. Lissek, "Broadband low-frequency electroacoustic absorbers through hybrid sensor-/shunt-based impedance control," *IEEE Trans. Control Syst. Technol.*, vol. 25, pp. 63–72, 1 2017.

[2] E. D. Bono, M. Collet, G. Matten, S. Karkar, H. Lissek, M. Ouisse, K. Billon, T. Laurence, and M. Volery, "Effect of time delay on the impedance control of a pressure-based, current-driven electroacoustic absorber," *J. Sound. Vib.*, vol. 537, p. 117201, 10 2022.

[3] S. Karkar, E. D. Bono, M. Collet, G. Matten, M. Ouisse, and E. Rivet, "Broadband nonreciprocal acoustic propagation using programmable boundary conditions: From analytical modeling to experimental implementation," *Phys.Rev. Appl.*, vol. 12, 11 2019.

[4] X. Guo, H. Lissek, and R. Fleury, "Improving sound absorption through nonlinear active electroacoustic resonators," *Phys.Rev. Appl.*, vol. 13, 1 2020.

[5] X. Guo, H. Lissek, and R. Fleury, "Observation of non-reciprocal harmonic conversion in real sounds," *Commun. Phys.*, vol. 6, p. 93, 5 2023.

[6] E. De Bono, M. Collet, and M. Ouisse, "The advection boundary law in absence of mean flow: Passivity, nonreciprocity and enhanced noise transmission attenuation," *J. Sound. Vib.*, vol. 590, p. 118603, 2024.

[7] M. Morell, E. Gourdon, M. Collet, A. Ture Savadkoohi, E. De Bono, and C.-H. Lamarque, "Towards digitally programmed nonlinear electroacoustic resonators for low amplitude sound pressure levels: Modeling and experiments," *J. Sound. Vib.*, vol. 584, p. 118437, 2024.

[8] M. Morell, M. Collet, E. Gourdon, A. T. Savadkoohi, and E. De Bono, "Experimental validation of the analytical modelling of a digitally created duffing acoustic nonlinear oscillator at low amplitudes," *Mech. Ind.*, vol. 26, p. 2, 2025.