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TURBULENCE EFFECT ON THE ACOUSTIC BOUNDARY LAYER OF PERFORATED ACOUSTIC LINERS

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ABSTRACT

In this work the shear stress effect on the acoustic boundary condition of liners with grazing turbulent mean flow is investigated. The lined wall is supposed to be rigid and homogeneously permeable. We have derived two semi-analytical models supplemented by DNS data which provide insight in the qualitative dependencies and the governing parameters of the turbulence effect. We found a critical frequency $\omega_c^+ = 10^{-2}$, which separates two regimes: For high frequencies the sound-synchronous shear stress does not reach beyond the viscous sublayer and the turbulence has no impact on the acoustic boundary condition. However, for low frequencies the shear wave penetrates into the turbulent flow layer and deforms the turbulent vorticity field. This causes an increased magnitude of sound-coherent wall shear stress as well as a significant change of the wall shear stress impedance and of the effective acoustic boundary condition. It was found that the latter depends particularly strongly on the dynamic processes which take place in the near-wall region of the turbulent wall boundary layer. In order to get more insight into the course of these processes impulse response functions of the sound-coherent shear stress have been calculated from existing DNS data. The results confirm a previous hypothesis that a shear wave propagates from the wall into the viscous sublayer, triggers a reaction of the turbulent shear stress in the turbulent region of the boundary layer, which in turn generates a viscous shear stress field that extends to the wall. This latter reaction manifests itself in a distinct 'turbulent portion' of the sound-coherent wall shear stress. It was found also that this dynamic process takes a characteristic time of $t^+ \approx 160$.

Keywords: *liner, acoustic boundary condition, turbulent flow*

1. INTRODUCTION

Passive acoustic linings are widely used to reduce noise emission for instance in the inlet or bypass duct of aircraft engines or in the duct walls of gas turbines. In most of these applications, the liner is naturally exposed to a highly turbulent grazing flow. It is well known from experiments that the acoustic properties of liners, especially the impedance and the sound absorption, can change significantly due to the grazing mean flow. The mean grazing flow is to be expected to interact with the acoustic particle velocity through the perforations of the rigid face-sheet of the liner. This induces a sound-coherent ('acoustic') shear stress at the wall which can exceed the well-known shear stress of the no-flow case by orders of magnitude. This shear stress plays an essential role in the acoustic boundary condition and its correct consideration is crucial for the design of liners if precise predictions of the sound attenuation of the liner should be achieved. The propagation of shear stress in the so-called acoustic boundary layer is strongly affected by the Reynolds stresses of the turbulence, which in turn depends on the sound-induced temporal change of the acoustic boundary condition. However, little is known about the involved physical effects and its governing parameters and in most models, the acoustic shear stress and the turbulence effect are ignored completely.

This report presents the current status of an ongoing study on the turbulent shear stress effect. The long-term goal from an engineering point of view is to derive an 'effective' acoustic boundary condition which includes the

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distribution of shear stress caused by turbulence together with the effect of the wall impedance. However, for this purpose we must first investigate how sound-coherent shear stress is generated and propagates in the turbulent boundary layer, how the Reynolds stress and other turbulence characteristics react spatially and dynamically to the acoustic perturbation of the mean flow, and what the governing parameters of these processes are.
It has to be noted that in the course of the study and as a first step we have limited ourselves to a particular and somewhat artificial type of liner wall: the rigid homogeneously permeable wall with infinitesimally small pores. For this wall, the acoustic effects due to the macroscopic openings of realistic liners (e.g. propagation and scattering of sound waves and hydrodynamic waves, local production of shear stress, shear layer instabilities) can be neglected and the no-slip condition applies. [1, 2] This is beneficial since it allows a simplified theoretical description and precisely defines the generation of a shear wave and the wall-normal gradient of acoustic shear stress at the wall. Moreover, we assume that the solutions for the homogeneously permeable wall will be at least part of the solution of the full problem (when the effects of the macroscopic openings are included).

2. INFLUENCE OF THE TURBULENCE ON THE EFFECTIVE BOUNDARY CONDITION

In our previous work [2, 3], the influence of turbulence on the effect of the acoustic boundary condition of a rigid, but homogeneously permeable liner has already been qualitatively investigated and some of the governing factors have been determined. These findings are summarized here briefly in order to build on this with further conclusions and analysis on the generation of sound-coherent shear stress:

1. The distribution of sound-coherent shear stress in the acoustic boundary layer is modified by two individual effects or properties of the turbulence:

- Shear deformation of the turbulent vorticity field: The turbulence in the acoustic boundary layer is sheared and deformed by the shear wave which is exited at the wall. This results in a modulated or sound-synchronous part of the Reynolds shear stress which must be modeled somehow. In the few available models [4–8] the classic Boussinesq hypothesis associated with a fictive ‘eddy viscosity’ was used, which is a well-established concept for mean flow turbulence. However, the transfer of this concept to the acoustic case is questionable and the results of a direct numerical simulation (DNS) [9] have supported this concern.
- Relaxation due to wall-normal displacement of the turbulent vorticity field: In a flow in equilibrium, also the statistical characteristics of the turbulent tangle of vortex filaments is in a state of equilibrium depending on the distance from the wall. If the vortex filaments are displaced due to the acoustical flow through the wall, turbulent relaxation processes will set in which seek to restore the previous state of equilibrium. This turbulent relaxation has never been investigated before in case of turbulent flow with an acoustical displacement at the boundary.

2. We have developed two semi-analytical models to include the above mentioned turbulence effects. Both models are based on extremely different physical assumptions:

- The first model (‘Model 1’) follows the traditional concept of an eddy viscosity, i.e. it assumes a strictly local relationship between turbulent shear stress and strain rate. The parameter of the eddy viscosity was estimated by DNS data of Hartmann [9].
- The second model (‘Model 2’) is based on the rather speculative hypothesis, suggested by the results of the DNS, that the sound-synchronous Reynolds stress, at least near the wall, is controlled solely by the acoustic wall shear stress and is independent of any local properties of the flow field. The wall shear stress parameter in this model was also estimated by DNS data.

It has to be emphasized that both models present two very hypothetical and simplified physical concepts and, in fact, we have no idea which approach is closer to reality. Moreover, the models represent two extreme cases for the interaction between the turbulent shear stress and the flow field: Model 1 assumes a strictly local interaction, whereas Model 2 is based on an extreme remote effect. The models can be used to show how strong the influence of the previously neglected effects can be in general and what the qualitative dependencies and the governing parameters of the turbulence effect are.

3. A representative result for the two models is

shown in Fig. 1 for the wall shear stress impedance

$$z_\tau^+ = \frac{1}{\rho_0 u_\tau} \frac{\widetilde{\tau}_w}{\widetilde{u}_0}. \quad (1)$$

We consider z_τ^+ [9, 10] as an appropriate quantity to discuss the turbulence effect since, in first instance, it can be regarded as measure for the acoustic wall shear stress $\widetilde{\tau}_w$ reacting to the streamwise acoustic velocity $\widetilde{u}_0$ at the wall. Also, it has been found that z_τ^+ determines directly the effective admittance of the liner if the Doppler-shift of the frequency is disregarded (not shown here). Note that we use the common normalization by turbulent boundary layer quantities, i.e. the kinematic viscosity $\nu = \mu/\rho$ and the friction velocity u_τ in Eq. 1. Figure 1 depicts the magnitude and phase of the wall shear stress impedance as a function of the normalized frequency $\omega^+ = \omega\nu/u_\tau^2$. The colors of the curves mark the solutions for the two extreme cases of turbulent relaxation: non-relaxed flow (red) or fully-relaxed flow (black). By 'non-relaxed flow' we refer to the initial state of flow before relaxation processes set in after a sudden wall-normal displacement $\widetilde{\xi}$ of the fluid at the boundary. In this case, the displacement would lead to an immediate shift of the flow profile, the turbulent vortex filaments, and their inherent velocity correlations as a whole to a new distance from the wall. This is because the time scales of the acoustic displacement are small compared to the time scales of the turbulent boundary layer. The subsequent start of turbulent relaxation rebalances the flow, i.e. eventually adapt the flow profile and the velocity correlations to the new wall distance. We refer to this second state of flow as the 'fully-relaxed' flow. Note that the solution for a real relaxing flow, which relaxes with a finite relaxation time, is supposed to be somewhere between the solutions of the two relaxation limits. Previous models [4–8, 11, 12] have only considered implicitly the limit of a fully-relaxed flow which implies that the turbulence always adjusts without the slightest delay to the wall-normal displacement.

The results, which are shown in Fig. 1 among others, can be interpreted as follows.

- We found a **'critical frequency'** $\omega_c^+ \approx 10^{-2}$, which separates two regimes of impact of turbulent shear stress on the acoustic boundary condition. This frequency is highlighted in Fig. 1. For 'high' frequencies $\omega > \omega_c$ the results become more and more independent of the applied turbulence model and of the state of relaxation effect. This is because at high frequencies the sound-synchronous shear stress decays close to the wall due to the short wavelength of the according diffusion wave and does not reach beyond the viscous sublayer. Thus, in this regime the turbulence has no impact on the effective boundary condition and can be ignored in practical situations.

- However, for 'low' frequencies $\omega < \omega_c$ the acoustic shear stress wave penetrates into the turbulent flow (at least into the buffer layer) which causes a great change in the spatial distribution of sound-coherent shear stress. This is more pronounced and covers a larger distance from the wall the lower the frequency is. On the one hand, this results in a strongly increased magnitude of acoustic wall shear stress and accordingly on a change of the wall shear stress impedance z_τ^+. On the other hand, it leads to a major change of the effective impedance relative to the liner impedance (not shown here, see [3]) which can reach orders of magnitude depending on the Mach number and the streamwise wavenumber of the sound wave. Also, completely different trends have been found for wave propagation in or against the flow direction. Even nega-

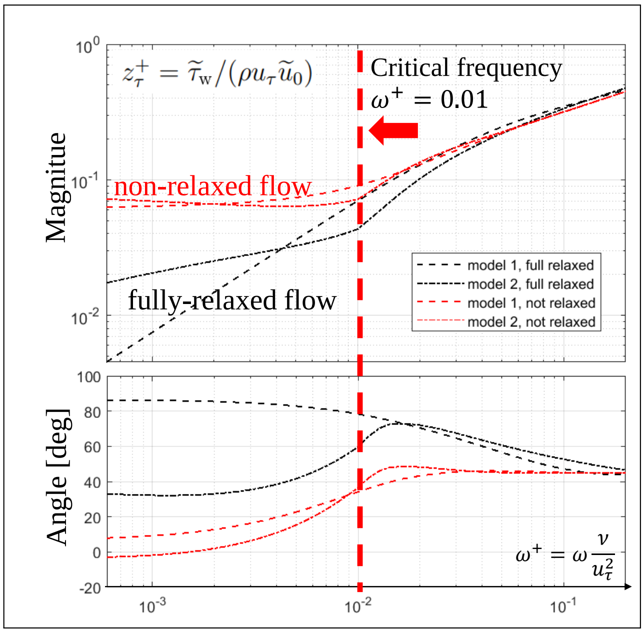

Figure 1. Results for the wall shear stress impedance $z_\tau^+ = \frac{1}{\rho_0 u_\tau} \frac{\widetilde{\tau}_w}{\widetilde{u}_0}$ normalized with wall parameters as a function of the normalized angular frequency.

tive real parts of the effective impedance have been found for cases of wave propagation against the flow and it is still unclear if this indicates production of sound or just an energy transport in the opposite direction.

- Unfortunately, in the low-frequency regime not only the deviation between the solutions for the two limiting flow states, non-relaxed and fully-relaxed flow, are increasing, but also the discrepancies between the solutions of the two models. Fig. 1 gives us a rough estimate of the range of possible solutions, but the uncertainty between the models is considerably increased at low frequencies.
- It is noteworthy, that the low frequency regime is actually achieved in practical liner applications, so the effect cannot simply be neglected. To give a practical example: For a Mach number of 0.17 the critical frequency would be at $f = 1$ kHz. For $M = 0.4$, however, the affected frequency range would be already stretched up to 5 kHz. Moreover, the effect is supposed to be even more pronounced at realistic liner surfaces where the wall is usually not hydraulically smooth but somewhat rough. At a rough wall the viscous sublayer is thinner compared to a smooth wall. Hence, a shear wave that has decayed in the viscous sublayer for a smooth wall would interact with the turbulence (i.e. the turbulent buffer layer) at a rough wall. Thus, the critical frequency is shifted towards higher frequencies and the turbulence effect applies at a wider frequency range.

3. DYNAMIC REACTION OF THE TURBULENT FLOW TO THE ACOUSTIC DISTURBANCE AT THE WALL

3.1 Frequency response and impulse response function of z_τ^+

To overcome the great uncertainty in choosing the right turbulence model ('model 1/2'? or something else?) at low frequencies $\omega < \omega_c$, it is crucial to obtain more insight in the mechanisms of generation and propagation of sound-coherent shear stress in the boundary layer. This includes the question of how the system of the turbulent flow (with its inherent Reynolds shear stresses) reacts dynamically to the sound-induced temporal variation of the boundary condition and how this ultimately results in a certain distribution of the total sound-coherent shear stress in the acoustic boundary layer. To this end an understanding of the temporal sequence of the (local or global) physical mechanisms involved would be beneficial.

A useful base for this could be provided by the im-

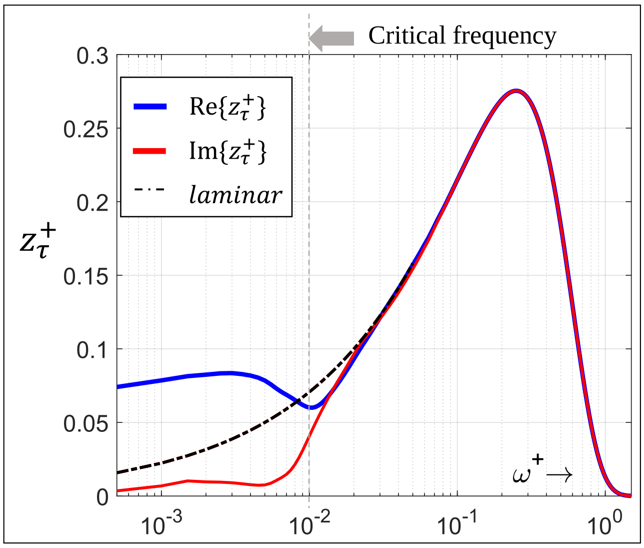

Figure 2. Real and imaginary part of the frequency response (multiplied with a Gaussian function) of the wall shear stress impedance z_τ^+ which was evaluated from the DNS data [9]. The normalization is the same as in Fig. 1. The black dash-dotted curve shows both the real part and the imaginary part of z_τ^+ in a laminar flow.

pulse response function (IRF) of the sound-coherent shear stress. We have started this investigation by evaluating the impulse response function of the shear stress impedance z_τ^+ (Eq. 1). This corresponds to the temporal response of the wall shear stress to a dirac pulse of the streamwise velocity at the wall (which would be observed at the wall without the no-slip condition). Note that the latter velocity can be also regarded as the difference between the streamwise velocities of the bulk flow and the fluid which sticks to the wall. The impulse response function is obtained by an inverse Fourier transformation (ifft) of the frequency response of z_τ^+. The latter was evaluated from the DNS data of Hartmann [9] and from corresponding measurements [10]. Note that the original frequency response was multiplied with a Gaussian function first since the ifft cannot be applied to a monotonically increasing function (as $z_\tau^+ \propto \sqrt{i\omega^+}$). The real and imaginary part of the frequency response for the DNS data (multiplied with the

Gaussian function) is shown in Fig. 2 together with the solution for a laminar flow (black dash-dotted line), i.e. the shear stress is governed only by the molecular viscosity and the shear rate.
Note that the calculation of the impulse response was prone to considerable uncertainties since the frequency response is given only for a certain number of frequencies in both data sets. Thus, some effort has been put in an appropriate data processing which conditions the shear stress impedance by a cost function first (details not shown here).

3.2 Results

Fig. 3 shows the results for the evaluated impulse response function as a black, solid line. Note that the impulse response is causal (there are no contributions at negative times except of a small portion due to the convolution with the Gauss function) which confirms that the streamwise change in the position of the wall relative to the bulk flow can indeed been considered to be the cause of the shear stress. Fig. 3 a) shows that mainly a high, somewhat deformed peak is obtained at $t^+ = 0$, which - physically speaking - reflects the infinitely high wall shear stress at the infinitely high wall velocity during the delta pulse. The deformation of the peak is caused by the dynamics of the radiated viscous shear wave. The corresponding impulse response, which accounts for the purely laminar flow case, is shown as a red dashed line. Since it can hardly be distinguished from the actual impulse response in Fig. 3 b) the same information is shown with an enlarged scaling of the ordinate and a broader time scale. The difference between the total and the laminar impulse response is caused by the reaction of the turbulent shear stress on the wall shear stress. This difference (total solution minus laminar solution) is specifically shown in Fig. 3 c) and compared between the DNS data and the experimental data.

4. CONCLUSIONS AND OUTLOOK

The turbulent portion of the impulse response in Fig. 3 c) clearly reveals a distinct peak after a time $t^+ \approx 160$ (note that t is made dimensionless by ν/u_τ^2). This fits into our previously expected view of the physical processes: Since the wall is smooth, it is directly exposed to the viscous sublayer of the turbulent boundary layer which means that the sound-coherent shear stress at the wall has to be generated exclusively by viscous forces (viscosity times shear rate). Thus, the information of the turbulent shear stress (which originates from e.g. the buffer layer of the turbulent boundary) cannot pass directly to the wall, but is instead (indirectly) transmitted by shear waves. These shear waves propagate from the wall through the viscous sublayer, then trigger a reaction of the turbulent shear stress in the turbulent region of the boundary layer, which in turn generates a viscous shear stress field that extends back to the wall. The result of this latter portion manifests itself in a distinct sound-coherent wall shear stress in Fig. 3 c). This idea is further confirmed by the position of the maximum of the impulse response at a characteristic time $t^+ \approx 160$, which interestingly corresponds to the reciprocal of the characteristic frequency $\omega_c^+ \approx 10^{-2}$. It is assumed that this time span contains the sum of the shear wave transit times and the time required by the turbulence to respond to the shear rate.
Recent results indicate that the impulse responses of the turbulent shear stress and the shear rate as a function of wall distance provide further insight into these processes. In fact, the acoustic boundary condition of the homogeneously-permeable wall with grazing turbulent mean flow depends particularly strongly on the processes that take place in the near-wall region of the turbulent wall boundary layer. The estimation of the impulse responses thus proves to be a meaningful addition to the frequency responses that have been considered almost exclusively up to now. However, due to the difficulty of data preparation in obtaining the impulse response from the frequency response, it would be beneficial in the future to obtain the data directly from a numerical or real experiment, which provides the impulse response of the shear stress to a pulse of the streamwise velocity.

5. ACKNOWLEDGMENTS

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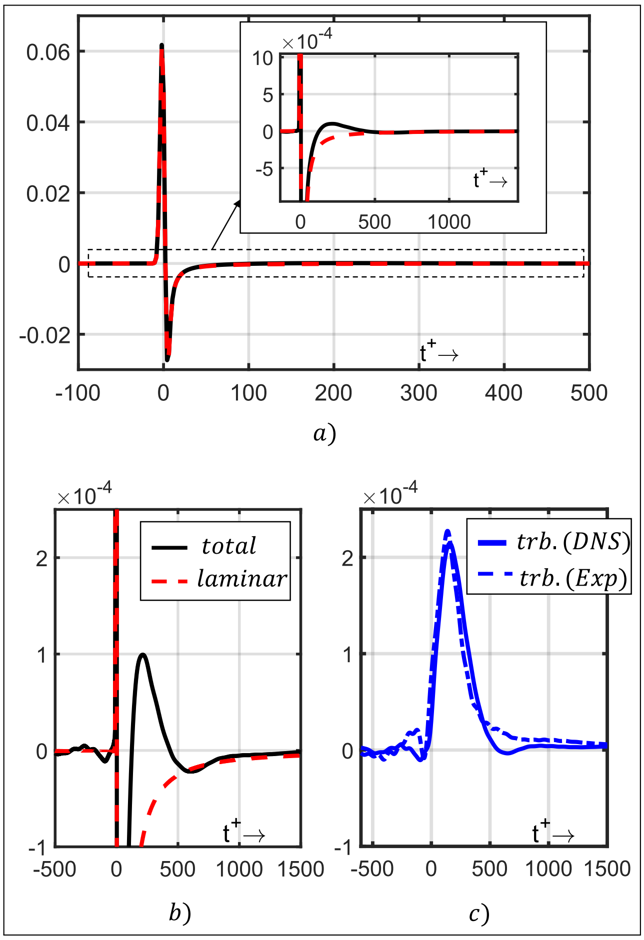

Figure 3. Impulse response of z_τ^+ (which was multiplied with a Gaussian function): a) Black curves represent the actual impulse response, red dashed curves represent the laminar case (no turbulence). b) Enlargement for small values. c) Purely turbulent portion (subtraction of the laminar solution from the total solution), solid line for the results from the DNS data [9], dash-dotted line for results from the experimental data [10].