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UNCERTAINTIES IN PARAMETER ESTIMATION OF POROUS MEDIA IN TUBE MEASUREMENTS

Ziqi Chen[1*] **Ning Xiang**[1]
[1] Graduate Program in Architectural Acoustics, Rensselaer Polytechnic Institute, United States

ABSTRACT

Characteristic impedance is a critical parameter for porous media. This work discusses the performance of porous materials and how uncertainties in acoustic measurements affect porous material models over an extended frequency range. The three-mic method [Salissou & Panneton, J. Acoust. Soc. Am. 128, pp. 2868–2876 (2010)] is employed to measure the characteristic impedance of porous materials. Impulse responses experimentally measured in an empty impedance tube allow acoustic measurement calibration. Bayesian inference is applied to estimate the parameters in characteristic impedance models associated with the measured impulse response at high frequencies. The uncertainties and inaccuracies in the characteristic impedance of porous materials are analyzed. This paper also addresses challenges for an extended frequency range.

Keywords: *Impedance tube, Characteristic impedance, Bayesian inference*

1. INTRODUCTION

Characteristic impedance is one of the most critical parameters commonly used by the acoustic material community to describe the acoustical performance of porous materials. Multiple models have been established to describe the characteristic impedance, such as the Delany-Bazley-Miki model [1] and the Horoshenkov model [2]. This study combines the three-mic method [3] and cylindrical mode decomposition [4] to measure the characteristic impedance in an extended frequency range, and the measured characteristic impedance is compared to the theoretical values to see the performance of the model.

*Corresponding author: chenz33@rpi.edu.

2. METHODOLOGY

2.1 Measurement

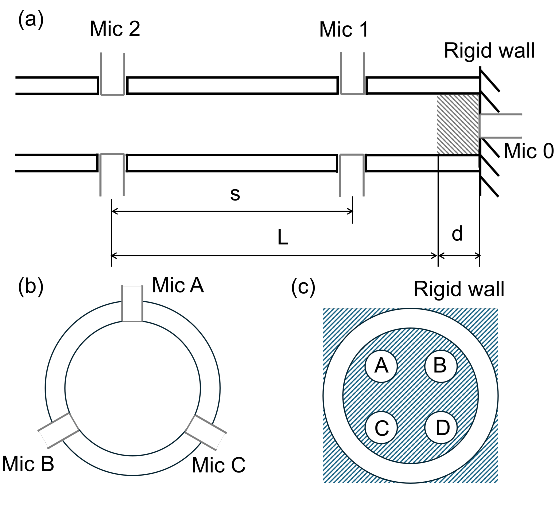

Figure 1. Impedance tube measurement for the characteristic impedance. (a) Arrangement for the measurement. (b) Arrangement of the circumferential position for position 1 and 2. (Arrangement of the Microphone position 0)

Figure 1 shows the impedance tube measurement setup in this work. We measure 3 impulse responses in the same plane for Position 1 and 2 and measure 4 impulse responses at back of the testing material. To extend the frequency range of the impedance tube measurement, we distribute microphone positions evenly along the circumference. The surface reflectance measured is calculated from

$$\underline{R} = \frac{\underline{H}_{12} - \mathrm{e}^{-\underline{\gamma} s}}{\mathrm{e}^{\underline{\gamma} s} - \underline{H}_{12}} \mathrm{e}^{2\underline{\gamma} L}, \tag{1}$$

where γ is the propagation coefficient, s is the separation between position 1 and 2, L is the distance from position 2 to the front surface of the testing materials, and H_{12} is the ratio of the average sound pressures. The transfer function is

$$\underline{H}_{ij} = \frac{\overline{P}_i}{\overline{P}_j} \tag{2}$$

where $\overline{P}$ is defined as the average value of sound pressures measured at different circumferential positions and the same longitudinal position.

$$\overline{P}_i = \frac{1}{N} \sum_{n}^{N} P_{in} \tag{3}$$

$$\tag{4}$$

Therefore the wave number and the characteristic impedance of the material is [3]

$$\underline{k} = \frac{1}{d} \cos^{-1} \left(\frac{1 + \underline{R}}{\mathrm{e}^{\underline{\gamma}(L-s)} + \underline{R} \mathrm{e}^{-\underline{\gamma}(L-s)}} H_{10} \right) \tag{5}$$

$$\underline{Z}_D = \mathrm{j} \frac{1 + \underline{R}}{1 - \underline{R}} \tan(\underline{k} d) \tag{6}$$

where d is the thickness of the material.

The cut-off frequency of the extended impedance tube measurement is [4]

$$f_{mn} = \frac{\alpha_{mn} c}{\pi d}, \tag{7}$$

where m is the circumferential modal number of the Bessel function, n is the radial modal number of the Bessel function, α_{mn} is the positive zeros of the Bessel function, c is the sound speed in the tube, and d is the diameter of the tube. With two microphones placed evenly along the circumference, the cutting-off frequency for a tube with a diameter of 1.4 inches (35.6 cm) is 9300 Hz. Note that the valid frequency range for a tube with a diameter of 1.4 inches is only up to 5600 Hz according to ASTM E1050 [5].

2.2 Model

Multiple models have been developed to describe the characteristic impedance of the porous material. In this work, we choose the Miki generalized model for the Bayesian inference to analyze the uncertainties and inaccuracies. According to the Miki generalized model, the characteristic impedance of porous materials can be approximated as a function of frequency and flow resistivity [1]

$$Z_M = \rho_0 c_0 \frac{\sqrt{\alpha_\infty}}{\phi} \left[1 + 0.07 \left(\frac{f}{\sigma_e} \right)^{-0.632} - \mathrm{j}\, 0.107 \left(\frac{f}{\sigma_e} \right)^{-0.632} \right], \tag{8}$$

where ρ_0 is the air density, c_0 is the sound speed in air, α_∞ is the tortuosity, and ϕ is the porosity. the effective flow resistivity σ_e is defined as

$$\sigma_e = \frac{\phi}{\alpha_\infty} \sigma \tag{9}$$

2.3 Bayesian framework

In this work, we estimate critical parameters using Bayesian inference. Bayes' theorem for the estimation is

$$\overbrace{p(\boldsymbol{\theta} | \underline{Z}_{\mathrm{D}}, \underline{Z}_{\mathrm{M}}, I)}^{\text{posterior}} = \frac{\overbrace{p(\underline{Z}_{\mathrm{D}} | \boldsymbol{\theta}, \underline{Z}_{\mathrm{M}}, I)}^{\text{likelihood}} \times \overbrace{p(\boldsymbol{\theta} | \underline{Z}_{\mathrm{M}}, I)}^{\text{prior}}}{p(\underline{Z}_{\mathrm{D}} | \underline{Z}_{\mathrm{M}}, I)}, \tag{10}$$

where θ collectively represents parameters for characteristic impedance models, and I is the background information, including that '*the prediction model is able to describe the data well*'. The likelihood distribution is assigned to

$$p(\underline{Z}_{\mathrm{D}} | \boldsymbol{\theta}, \underline{Z}_{\mathrm{M}}, I) = \frac{\Gamma(K/2)}{2} \left(\pi \sum_{k=1}^{K} \epsilon_k^2 \right)^{-K/2}, \tag{11}$$

where $\Gamma(\dots)$ is the standard Gamma function, and ϵ_k is the residual error. The residual error ϵ_k is defined as,

$$\epsilon_k^2 = \mathrm{Re}(\underline{Z}_{\mathrm{D}} - \underline{Z}_{\mathrm{M}})^2 + \mathrm{Im}(\underline{Z}_{\mathrm{D}} - \underline{Z}_{\mathrm{M}})^2. \tag{12}$$

3. RESULT

3.1 Air Characteristic Impedance

To eliminate the influence of tube dissipation on the accuracy of measurement [6], we measure the empty tube

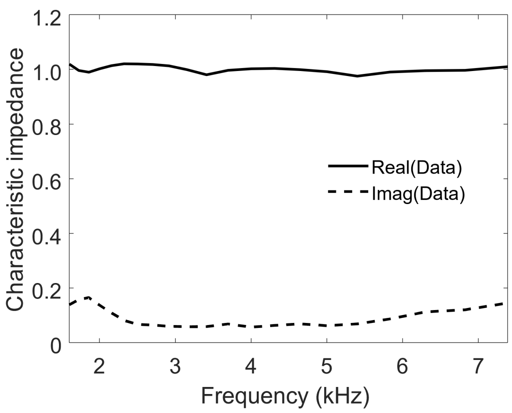

Figure 2. Normalized characteristic impedance of the hypothetical air layer.

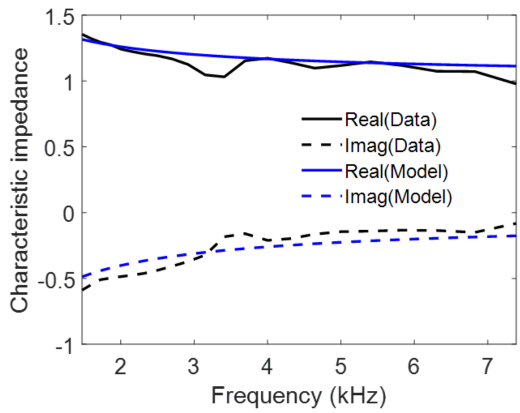

Figure 3. Estimated and measured normalized characteristic impedance.

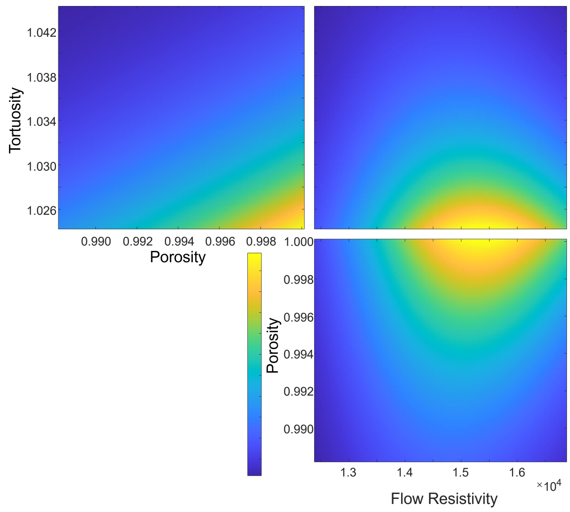

Figure 4. Posterior probability distributions over two-dimensional parameter space for the Miki generalized model.

and assume a hypothetical air layer in front of the rigid termination as the testing material. Figure 2 shows the normalized characteristic impedance of the hypothetical air layer after calibration. Although the calibrated characteristic impedance is not ideal, it is generally equal to the theoretical characteristic impedance of air.

3.2 Porous Material

With the estimated tube dissipation and sound speed after calibration, we are able to estimate the flow resistivity of the porous material. In this work, we used a piece of melamine foam as a test material. Bayesian inference is employed to inversely estimate the critical parameters in the Miki generalized model. Figure 3 demonstrates the estimated characteristic impedance of porous material. The roughness in the measured characteristic impedance may be due to energy leakage on the surface of the rigid termination and the ill-posedness of the testing material [7]. Without microphones evenly distributed along the circumference, the measurement is valid only up to 5600 Hz. Figure 4 shows the posterior probability distributions between parameters.

4. CONCLUSION

In this work, we combine the cylindrical mode decomposition and three-microphone impedance tube measurement to extend the frequency range of characteristic impedance measurements. Bayesian inference is applied to calibrate the impedance tube and estimate the critical parameters in the Miki generalized model based on the measured characteristic impedance. The melamine foam is used to validate the performance of the Miki generalized model at extended frequency.

5. REFERENCES

[1] Y. Miki, "Acoustical properties of porous materials-modifications of Delany-Bazley models," *J. Acoust. Soc. Japan*, vol. 11, no. 1, pp. 19–24, 1990.

[2] K. V. Horoshenkov, A. Hurrell, and J.-P. Groby, "A three-parameter analytical model for the acoustical properties of porous media," *The Journal of the Acoustical Society of America*, vol. 145, no. 4, pp. 2512–2517, 2019.

[3] Y. Salissou and R. Panneton, "Wideband characterization of the complex wave number and characteristic impedance of sound absorbers," *J. Acoust. Soc. Am.*, vol. 128, no. 5, pp. 2868–2876, 2010.

[4] J. Panzer, "Extracting the fundamental mode from sound pressure measurements in an acoustic tube," in *Audio Engineering Society Convention 147*, Audio Engineering Society, 2019.

[5] ASTM E1050-19, "Standard Test Method for Impedance and Absorption of Acoustical Materials Using a Tube, Two Microphones and a Digital Frequency Analysis System," 2019. (American Society for Testing and Materials, Philadelphia).

[6] Z. Chen and N. Xiang, "Bayesian estimation of dissipation and sound speed in tube measurements using a transfer-function model," *J. Acoust. Soc. Am.*, vol. 155, no. 4, pp. 2646–2658, 2024.

[7] R. Roncen, Z. E. A. Fellah, and E. Ogam, "Addressing the ill-posedness of multi-layer porous media characterization in impedance tubes through the addition of air gaps behind the sample: Numerical validation," *J. Sound & Vib.*, vol. 520, p. 116601, 2022.