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UNCERTAINTY RELATIONS FOR BROADBAND ULTRASONIC WAVEFORMS: THE CASE OF ACOUSTO-OPTIC FEMTOSECOND LASER PULSE SHAPING

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ABSTRACT

Broadband acoustic waveforms with phase modulation and complex-valued spectra are widely used for femtosecond laser pulse shaping in acousto-optic dispersive delay lines (AODDLs). Arbitrary transmission functions of quasi-collinear AODDLs can be achieved using the method of dispersive Fourier synthesis (DFS). Being a fast and efficient tool for generation different transmission functions with both phase-and-amplitude spectral modulation, the DFS method also provides the insight on fundamental limitations ultrasound signal performances. In particular, we demonstrate that there is an uncertainty relation between spectral resolution and modulation contrast for chirped ultrasonic waveforms with finite time window. More precisely, the product of spectral resolution by contrast is maximized when total frequency bandwidth equals twofold carrier frequency modulation range of the chirped ultrasonic signal. Experimental validation of the uncertainty principle is demonstrated with a high-resolution AODDL and 12 fs laser pulses at 800 nm. The applications of the DFS method with optimized parameters include programmable generation of arbitrary ultrashort pulse trains.

Keywords: *acousto-optics, femtosecond laser pulse, waveform synthesis, Fresnel transform*

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1. INTRODUCTION

Ultrasonic digital waveform synthesis has become a powerful tool for providing arbitrary transmission functions of acousto-optic tunable filters [1]. It enables creating the complex-valued transmission functions with predefined dispersion that is necessary for compression of ultrashort laser pulses [2]. Optimal parameters for synthesis of acoustic waveforms were investigated in this research.

2. ANALYSIS

To compute a chirped ultrasound waveform with given spectrum $S_{\rm ac}(\omega)$, we use the Fresnel transform:

$$S(t) = \mathrm{FT}_{(\omega,t)}\left\{S_{\rm ac}(\omega)\exp\left[iD_2(\omega-\omega_0)^2\right]\right\}, \quad (1)$$

where D_2 in the second order dispersion coefficient, and FT is the Fourier transform operator in (ω, t) domain. The resulting waveform duration $T_{\rm eff}$ is proportional to the frequency modulation band $\Delta\omega$:

$$T_{\rm eff} = D_2\Delta\omega. \quad (2)$$

A chirped waveform (1) can be decomposed as a sum of elementary signals each having a duration of τ. According to Gabor's uncertainty relation [3], the required bandwidth $\delta\omega$ is reciprocal to τ. Thus, we can define $\delta\omega = 4\pi\rho/\tau$, where the dimensionless parameter ρ that determines the signal contrast. Full bandwidth that is required for a chirped modulated signal is

$$B_\omega = \Delta\omega + 4\pi\rho/\tau, \quad (3)$$

the first term being the frequency modulation band.

The number of resolvable elements in the waveform defined as

$$N = T_{\rm eff}/\tau = D_2\Delta\omega/\tau. \quad (4)$$

Thus we can derive the information capacity of the chirped ultrasonic waveform as

$$N\rho = \frac{D_2}{4\pi}\Delta\omega(B_\omega - \Delta\omega) \le \frac{D_2\Delta\omega^2}{4\pi} \quad (5)$$

The quantity $N\rho$ is maximized when $B_\omega = 2\Delta\omega$ that corresponds to the optimal signal bandwidth. This inequality represents the uncertainty principle for chirped pulses with arbitrary spectral modulation [4], which is illustrated in Fig. 1.

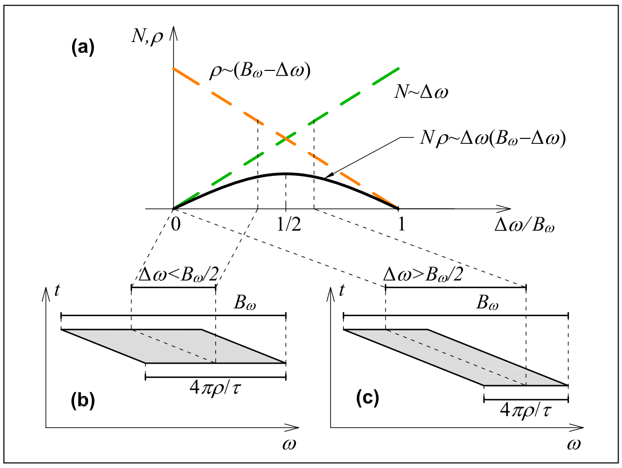

Figure 1. Graphical representation of the uncertainty relations for chirped pulses: (a) number of channels N and available channel bandwidth ρ; (b,c) spectrograms of a chirped pulse.

In the case of acousto-optic pulse shaping and synthesis of arbitrary transmission functions, the frequency modulation band $\Delta\omega$ is defined by the instantaneous bandwidth of processed light, and the full waveform duration is limited by the time aperture of the acousto-optic filter T_a. This allows one to find the optimal acoustic second order dispersion coefficient used in the Fresnel transform (1):

$$D_2 = T_a/(4\Delta\omega). \quad (6)$$

3. RESULTS AND DISCUSSION

The effect of optimal bandwidth selection was verified experimentally. The spectrum $S_{ac}(\omega)$ was defined as a meander with the full width $\Delta\omega/2\pi = 20$ MHz and modulation period of 0.2 MHz; the waveforms were calculated with the signal's time aperture $T_a = 50$ μs. The resulting spectra in Fig. 2 illustrate two cases: (a) in the case of minimum bandwidth $B_\omega = \Delta\omega$ the effective waveform duration is $T_{eff} = T_a$; (b) in the case of optimal bandwidth $B_\omega = 2\Delta\omega$ the effective waveform duration is $T_{eff} = 0.5T_a$, and 6 dB enhancement in modulation contrast is observed.

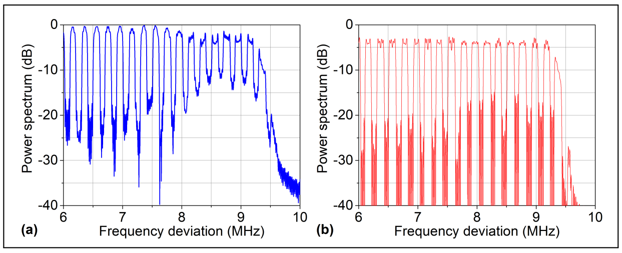

Figure 2. Numerical experiment on binary modulated chirped pulse spectrum: (a) minimal bandwidth $B_\omega = \Delta\omega$ causes contrast degradation near the window boundaries; (b) optimal bandwidth $B_\omega = 2\Delta\omega$ results in maximized contrast.

Similar contrast enhancement was obtained experimentally using broadband femtosecond laser radiation as a probe for arbitrary transmission function of a high-resolution quasi-collinear acousto-optic filter [4]. An original software package based on the Fresnel transform with optimized parameters was developed for controlling programmable acousto-optic filters [2].

4. REFERENCES

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