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USING OPEN SOFTWARE MODELING TOOLS FOR INNOVATIVE TRANSDUCER DESIGNS

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ABSTRACT

The advance in computer modeling techniques observed in the last decades allows for creating so-called computer twins in variety of real-world applications. One of the well-developed technique that turn out to be extremely useful in variety of acoustic areas is based on using Finite Element Method and Boundary Element Method. Both has been already commercialized in the form of advanced software packages that facilitate verification of whole designs.

However, in many research areas a ready to use commercial software are still not prepared for innovative designs where low level scientific verification is a requirement. Hence the idea of using programmable methods that allows for the insight into internal workings of the computer design with a usage of methods dedicated for Finite/Boundary Element approach. Among many of long-developed open-source packages is FreeFEM++ that is characterized by maturity in implementing of sophisticated numerical algorithms and giving higher abstraction level in software programming than modern object oriented languages.

To illustrate the proposed idea, the design process of sonic crystal transducer for underwater applications is presented. The example is mainly oriented for showing readability of the code and for giving example of several test cases oriented for optimizing focusing performance of such transducer.

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1. MODELING PIEZOELECTRIC TRANSDUCER

Modeling piezoelectric transducer is based on defining partial differential equations for each element of the design and setting up boundary conditions for adjacent elements. In proposed design four different kinds of materials will be used: a piezoelectric material, three elastic materials, water and steel. The following subsections presents the equations governing the physics of wave generation by piezoelectric element and its radiation in water in the variational form as used in the implementation of fluid-structure interactions by finite element method.

1.1 Modeling piezoelectric circular disc

When a time-harmonic solution for operated angular frequency ω is considered, the piezoelectric differential equations relating mechanical stress T and strain S with electric flux density D and field E are given as:

$$\begin{aligned} -\omega^2 \rho_p u_i &= T_{ij,j} \\ D_{i,i} &= 0 \\ T_{ij} &= c^E_{ijkl} S_{kl}(u) - e_{kij} E_k(\phi) \\ D_i &= e_{ikl} S_{kl}(u) + \epsilon^S_{ik} E_k(\phi) \end{aligned} \tag{1}$$

along with the appropriate boundary conditions. Unknown variables in such formulated boundary-value problem are displacement vector u and potential ϕ. The coefficients c^E, e and ϵ^S are respectively: elastic stiffness constant tensor evaluated at constant electric field, piezoelectric constant tensor and dielectric constant tensor evaluated at constant strain.

The weak formulation usually used for obtaining numerical solution may be obtained by multiplying the differential equations with weight functions, and integrating over the solution domain Ω_p. It leads to [1]:

$$\begin{gathered} -\omega^2 \int_{\Omega_p} \rho_p v_i u_i \, d\Omega = \int_{\Omega_p} v_i T_{ij,j} \, d\Omega \\ \int_{\Omega_p} \chi D_{i,i} \, d\Omega = 0 \end{gathered} \tag{2}$$

where v_i for $i = 1, 2, 3$ represents weight functions for displacement components and χ - for potential. The weight functions are arbitrary except boundary where they need to be equal to zero.

The axisymmetric case allows for 2D formulation. Adding both Eqs (2) and combining variables to form matrices it gives:

$$\begin{gathered} \int_{\Omega_p} \omega^2 \rho_p [v]^T [u] dS - \\ - \int_{\Omega_p} ([L_{u\phi}] \begin{bmatrix} v \\ \chi \end{bmatrix})^T \; [C] \; ([L_{u\phi}] \begin{bmatrix} u \\ \phi \end{bmatrix}) \; dS = 0 \end{gathered} \tag{3}$$

where $[u]$ matrix contains unknown displacement compontents u_z and u_r, $[L_{u\phi}]$ matrix represents summed operator for derivatives in cylindrical coordinates:

$$[L_{u\phi}] = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \\ \frac{\partial}{\partial r} & 0 \\ 0 & \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{4}$$

and 6 x 6 coefficients' matrix $[C]$ is defined as:

$$[C] = \begin{bmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & e_{31} \\ c_{12}^E & c_{11}^E & c_{13}^E & 0 & 0 & e_{31} \\ c_{13}^E & c_{13}^E & c_{33}^E & 0 & 0 & e_{33} \\ 0 & 0 & 0 & c_{44}^E & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & -\epsilon_{11}^S & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & -\epsilon_{33}^S \end{bmatrix} \tag{5}$$

Boundary conditions on transducer electrodes when it is excited with voltage source represents Dirichlet conditions for voltage potential ϕ.

1.2 Modeling matching layer, backing and housing

Matching layer, backing and housing of a transducer could be modeled by classical elastic linear equations. It means that the piezoelectric equations presented in section 1.1 may be reduced to the form with just displacement variables ensuring that electrical potential variable ϕ is forced to be zero in these non-piezoelectric regions. Additionally, differential operator could be reduced to:

$$[L_u] = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{6}$$

and material constants matrix can be represented by 4 x 4 coefficients' matrix $[A]$:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{12} & 0 \\ a_{12} & a_{11} & a_{12} & 0 \\ a_{12} & a_{12} & a_{11} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \tag{7}$$

1.3 Modeling fluid

Modeling transducer in its fluid operational environment requires adding additional variable to previously defined set. Usually for fluid - due to boundary condition definition for fluid structure interface - it is more convenient to use velocity potential ψ as a independent variable. It is also governed by the Helmholtz equation:

$$\psi_{,ii} = -\frac{\omega^2}{c_f^2} \psi \tag{8}$$

where c_f represents the sound velocity in the fluid medium. To ensure that the energy propagates towards infinity, the Sommerfeld radiation condition must be applied for boundaries representing outward computational domain parts:

$$\psi_{,i} n_i = j \frac{\omega}{c_f} \psi \tag{9}$$

where n_i represents normal to boundary surface with j representing imaginary unit. For water boundaries interfacing elastic and piezoelectric elements the continuity boundary condition must be fulfilled:

$$\psi_{,i} n_i = j \omega u_i n_i \tag{10}$$

Combining equations (8), (9) and (10) and using Green's integration formula with ξ as a weight function gives variational version for fluid domain Ω_f with its inner fluid-structure boundary Γ_f and outer Γ_∞:

$$\int_{\Omega_f} \frac{\omega^2}{c_f^2} \xi\psi d\Omega = \int_{\Omega_f} \xi_{,i}\psi_{,i} d\Omega + \\ + \int_{\Gamma_\infty} j\frac{\omega}{c_f}\xi\psi d\Gamma + \int_{\Gamma_f} j\omega\xi u_i n_i d\Gamma \tag{11}$$

1.4 Modeling sonic crystals

The vanishing of diffraction or the self-collimation of wave beams, was predicted in the field of optics for electromagnetic waves propagating through optically periodic materials, the so-called photonic crystals. The theoretical consideration related to applying the theory of sonic crystals in underwater designs were presented in several papers including [2], [3] and [4] near two decades ago. Its implementation is based on introducing soft or hard impedance contrast in radiating medium. In acoustic modeling, it can be achieved by immersing empty holes or a set of steel balls, although the latter choice is more feasible in real-world application. When modelling, the crystals physics is governed by the same wave equation but introduce another region with different sound velocity and different density.

2. IMPLEMENTING TRANSDUCER MODEL IN FREEFEM

As the case study of using FreeFEM [5] for modeling innovative designs, the transducer example considered in [6] was implemented. In this PhD thesis the author has modelled the transducer with matched layer, backing and housing in COMSOL and FEMP and measured later on. In this paper the same materials and the same geometry were implemented in FreeFEM with PIC255 as the piezoelectric material, EL217C as the matching layer, TD1049 as the backing and ABS as the housing. As the novel step, the sonic crystals set was added with materials having parameters of steel. The schematic design of whole transducer is presented in Fig.1.

Figure 2 presents the excerpt from FreeFEM code with essential lines that implements variational forms of equations presented in section 1 for axisymmetric case. The four unknowns are `ur`, `uz`, `phi` and `psi`. The region labels used in double integral `int2d` directive are as follows: `1` - a piezoelectric material, `2`, `3`, `4` - regions with elastic materials, `5` - water and `6` - sonic crystals. The following boundary labels are used in `on` directive (Dirichlet BC) and line integral `int1d` directive (Neumann BC): `1` - piezoelectric bottom electrode, `3` - piezoelectric top electrode, `4` - piezoelectric axis of symmetry, `5`, `8`, `9` - water-structure interface Γ_f and `15` - water outer boundary Γ_∞. The differential operators for piezoelectric material `L1`, elastic materials `L2` and water `L5` are defined using macro directive.

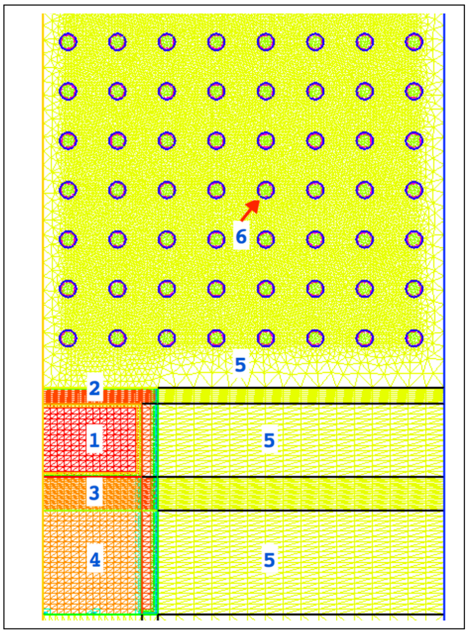

Figure 1. Axisymmetric mesh model of circular disk transducer with sonic crystals - the region numbers are explained in the text of the article.

3. MODELING RESULTS AND CONCLUSION

The prepared FreeFEM code modeling radiation of a transducer can be parametrized in several ways: it could be executed with or without sonic crystals set, for one frequency or for a range of frequencies, calculating electric impedance or acoustic radiation, etc. In case of the option with sonic crystals it can setup configuration of the set i.e. distances, dimensions and number of crystals in space. Everything can be modified in source code having around 200 lines of code. As an example Fig. 3 shows two plots of acoustic radiation obtained when executing code with and without sonic crystals for designed frequency equal to 126 kHz. As the simulation is performed in 2D for axisymmetric forms, the set of 8 x 8 crystals with diameter of 16 mm and separated by half of transducer radius can be

```
macro L1(u,v,p) [dx(u), u/x, dy(v), dy(u)+dx(v),
    dx(p),dy(p)]//EOM
macro L2(u,v) [dx(u), u/x, dy(v), dy(u)+dx(v)]//
    EOM
macro L5(p) [dx(p), dy(p)]//EOM

// problem definition
problem PiezoAx2D([ur,uz,phi,psi],[vur,vuz,vphi,
    vpsi]) =
// mass - everywhere except water:
- int2d(Th,1,2,3,4)( x * rho * w*w  * [ur,uz]'*[
    vur,vuz]  )
// piezoelectric stiffness
+ int2d(Th,1)( x * L1(vur,vuz,vphi)' * C * L1(ur
    ,uz,phi) )
// electric potential on electrodes
+ on(1,phi=1)     // 1V
+ on(3,phi=0)     // 0V
// axis of symmetry
+ on(4,ur=0)
// elastic stiffness
+ int2d(Th,2,3,4)( x * L2(vur,vuz)' * A * L2(ur,
    uz) )
// water
+ int2d(Th,5,6)( x * rho * w*w / (c*c) * vpsi*
    psi )
- int2d(Th,5,6)( x * rho * L5(vpsi)' * L5(psi) )
// water-structure interface
- int1d(Th,5,8,9)(x * rho * 1i*w * (vpsi*(ur*N.x
    +uz*N.y)+(vur*N.x+vuz*N.y)*psi))
// Sommerfeld boundary condition
- int1d(Th,15)( x * rho * 1i * w/cf * vpsi*psi )
// phi=0 everywhere except piezo
+ int2d(Th,2,3,4,5,6)( 1e-30 * vphi*phi )
// psi=0 everywhere except water
+ int2d(Th,1,2,3,4)( 1e-30 * vpsi*psi )
// u=0 in water
+ int2d(Th,5,6)( 1e-30 * (vur*ur+vuz*uz) );
```

Figure 2. The excerpt from FreeFEM code defining differential operators, variational integrals and boundary conditions.

interpreted as a set of donut shaped steel rings. The focusing of the beam is visible from visual inspection of both subplots. However, the optimal configuration requires further intensive research including 3D modeling. Moving to full three-dimensional modeling is straightforward in case of code changes, but it is not so trivial when preparing mesh model of whole design. In such case, the processor and memory requirements would increase and its execution is hardly possible on standalone computers.

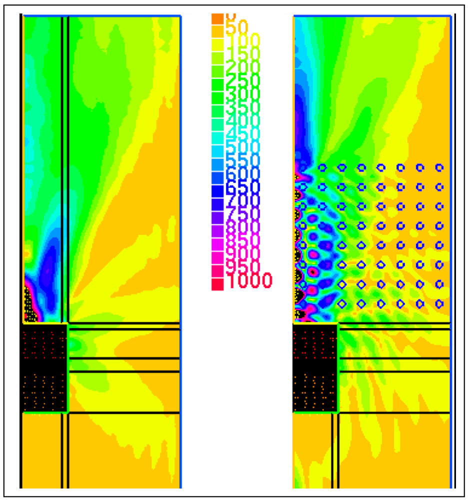

Figure 3. The comparison of acoustic pressure amplitude $p = |j\omega\rho_f\psi|$ in pascals obtained without sonic crystals (left) and with 8 x 8 set of steel sonic crystals (right) at 126 kHz.

4. REFERENCES

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