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VIBRATION CONTROL IN A HONEYCOMB SANDWICH METAPANEL MADE UP OF PERIODIC CELLS WITH AN ABH-LIKE THICKNESS PROFILE.

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ABSTRACT

Acoustic Black Holes (ABHs) have gained significant attention for their potential in vibration control by effectively trapping flexural waves within structures featuring gradually tapered thickness profiles. In this study, we investigate the wave propagation of bending waves in a honeycomb sandwich panel that integrates ABH effects to attenuate vibrations in a high broadband frequency band and also in a low tunable frequency band. By periodically modulating the thickness of the panel with an ABH-like thickness profile, the design leverages both Bragg band gaps and the ABH effect to enhance wave attenuation. The sandwich configuration offers a high stiffness-to-mass ratio, making it particularly attractive for lightweight structural applications. The band structure is analyzed using a plane wave expansion method based on Kirchhoff's assumptions, and finite element (FE) simulations are conducted to validate the theoretical predictions. The results confirm the existence of tunable low-frequency band gaps controlled by key geometric parameters. This approach provides an effective strategy for designing lightweight, stiff, and highly efficient vibration-damping panels, suitable for advanced engineering applications

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1. INTRODUCTION

In recent years, periodic materials and structures have garnered significant attention in physics and engineering due to their unique wave propagation characteristics and their ability to control vibrations [1]. One particularly promising approach is the integration of acoustic black holes (ABHs) into engineered structures. A particularly promising approach involves integrating acoustic black holes (ABHs) into engineered structures. The ABH effect, first introduced by Mironov in 1988 [2] and further developed by Krylov [3], employs a gradual reduction in structural thickness, typically following a power-law decreasing profile, to create a near-zero reflection condition for bending waves. This process results in a progressive reduction in wave velocity and the accumulation of vibrational energy, which, when paired with suitable damping layers, results in effective energy dissipation [4].

Combining ABHs with sandwich panels offers significant advantages in low-frequency vibration suppression. Sandwich structures, widely used in aerospace and automotive industries due to their high stiffness-to-weight ratio and efficient load-bearing capacity, often suffer from poor noise insulation and susceptibility to structural vibrations. To overcome these challenges, periodic thickness modulation has emerged as an effective strategy for enhancing wave attenuation. Recent studies [5] have demon-

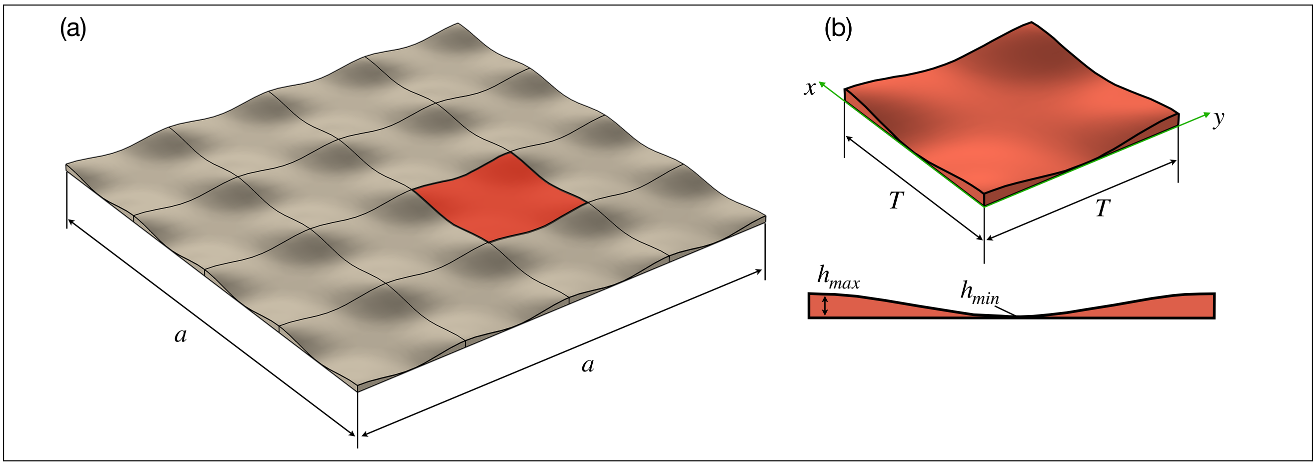

Figure 1. The proposed design of the ABH PC plate: (a) 3D view composed of a 4×4 array of identical rectangular unit cells, and (b) detailed layout of the rectangular unit cell defined in Eqn. (1).

strated that incorporating periodic effects, such as Bragg bandgaps (BGs), into solid panels can substantially reduce vibrations and sound transmission. By integrating ABHs with periodic structures, vibration control can extend beyond the traditional cut-on frequency limitations of ABHs, improving broadband performance. The same undulation periodicity design in plates was explored by G. Trainiti [6], where multiple scattering between the cells of an infinite periodic lattice leads to complete bandgaps. Geometrical undulations have proven to be effective design parameters that can be adjusted to achieve the desired dynamic characteristics.

This study aims to investigate the synergistic effects of ABHs and periodic modulation in sandwich panels for enhanced vibration and noise control. By combining the wave-trapping properties of ABHs with the bandgap effects of periodic structures, we seek to develop a novel approach for lightweight panels with improved acoustic performance. The plate is analyzed based on dispersion relations using the plane wave expansion (PWE) method, alongside the finite element (FE) method, both of which are employed to obtain the band structure. The FE method is also employed to compute the dynamic response of the acoustic black hole phononic crystal plate.

2. DESIGN AND GEOMETRY OF THE MODULATED PANEL

The acoustic black hole phononic crystals (ABH PCs) is shown in Fig.1. The geometry of the panel is characterized by a periodic variation in thickness, h, with a spatial period, T. For simplicity and to ensure an integer number of unit cells in the modulated panel, we consider a rectangular unit cell, as shown in Fig.1b. The curvature undulation is defined as a combination of cosine functions of the spatial coordinates (u, v) for the rectangular unit cell, as follows

$$h(u,v) = \alpha + \beta \left[\cos\left(\frac{2\pi}{T}u\right) + \cos\left(\frac{2\pi}{T}v\right) + \gamma \cos\left(\frac{2\pi}{T}u\right) \cos\left(\frac{2\pi}{T}v\right) \right], \tag{1}$$

where the average thickness of the plate, for the shape parameter $\gamma = -1$, is given by $\alpha = \dfrac{\gamma h_{max} + (\gamma - 2)h_{min}}{2\gamma - 2}$ and the contrast parameter is $\beta = \dfrac{h_{max} - h_{min}}{2\gamma - 2}$, which is defined in terms of the maximum and minimum thickness values. A diverse set of thickness distributions can be achieved by adjusting the shape parameter γ [5].

3. BAND DIAGRAMS AND VIBRATION ANALYSIS

3.1 PWE and FE modeling of a rectangular unit cell

We focus on an inhomogeneous thin plate based on Kirchhoff-Love theory. Using the PWE method, we analyze the band structure of an infinite periodic rectangular unit cell, as shown in Fig.1b. The equation of motion for the PWE method, described by the flexural displacement $w(\mathbf{r})$, is given as follows:

$$\begin{aligned} &-\rho h(\mathbf{r})\omega^2 w(\mathbf{r}) + \Delta(D(\mathbf{r})\Delta w(\mathbf{r})) \\ &= (1-\nu)\Big(\frac{\partial^2 D(\mathbf{r})}{\partial y^2}\frac{\partial^2 w(\mathbf{r})}{\partial x^2} - 2\frac{\partial^2 D(\mathbf{r})}{\partial x \partial y}\frac{\partial^2 w(\mathbf{r})}{\partial x \partial y} \\ &\quad + \frac{\partial^2 D(\mathbf{r})}{\partial x^2}\frac{\partial^2 w(\mathbf{r})}{\partial y^2}\Big). \end{aligned} \quad (2)$$

where ρ is the mass density, ω is the angular frequency, $D(\mathbf{r}) = \dfrac{Eh(\mathbf{r})^3}{12(1-\nu^2)}$ is the bending stiffness of the plate, $\mathbf{r}$ is the spatial position vector, depending on the coordinates x and y, and ν is Poisson's ratio. Due to the periodicity of the ABH PC, these material parameters can be expanded as a Fourier series [5, 7]. According to Bloch's theorem, the Eqn. (2) can be used to compute the eigenvalue problem of $\omega(\mathbf{k})$, and the energy band structure can be obtained by solving this equation. The wave vector $\mathbf{k} = [k_x, k_y]$ can be taken at the boundary of the irreducible Brillouin zone (IBZ) of a two-dimensional rectangular unit cell. We can calculate energy band properties along the IBZ path ΓXM and obtain the corresponding dispersion curves. The Bloch wave vector $\mathbf{k}$ starts from Γ and moves along the x-direction to X, where $k_x \in (0, \pi/T)$ and $k_y = 0$. Next, the wave vector continues from X to M, where $k_x = \pi/T$ and $k_y \in (0, \pi/T)$. Finally, the wave vector returns to Γ from M, following the path where $k_x = k_y \in (0, \pi/T)$.

Since we are interested in the BGs at low frequencies and assume that the panel does not exhibit shear deformation or core compressibility, the total plate stiffness of the sandwich panel $D(\mathbf{r})$ is considered frequency-independent. The ABH PC is modeled using the following equivalent homogenized material properties: Young's modulus $E = 54 \cdot 10^8$Pa, $\nu = 0.3$, and $\rho = 126$kg/m^3.

The numerical simulations based on the finite element (FE) method are implemented using the commercial software COMSOL solid mechanics package to calculate the band structures. A Floquet periodic boundary condition is applied to both sides of the rectangular unit cell, as derived from the Bloch's theorem. A parametric sweep of the reduced wave vector $\mathbf{k}$ is performed over the first irreducible Brillouin zone. The model materials correspond to the equivalent homogenized material as indicated above, with no damping loss.

The frequency dependence on the wave number is plotted in Fig.2. The geometric parameters of the rectangular unit cell from Fig.1b were chosen as follows: the maximum and minimum thicknesses are $h_{max} = 12$mm and $h_{min} = 0.6$mm, with a length of $T = 200$mm. It is clearly observed that the FE (blue circles) and PWE (black line) results are highly consistent. The green-shaded area in the diagram represents the three generated BGs of the rectangular unit cell, where wave transmission is prohibited. The first band gap opens around 615Hz and continues up to 912Hz. The widest band gap spans the frequency range from 912Hz to 1318Hz. The third band gap appears in the frequency range from 1544Hz to 1696Hz.

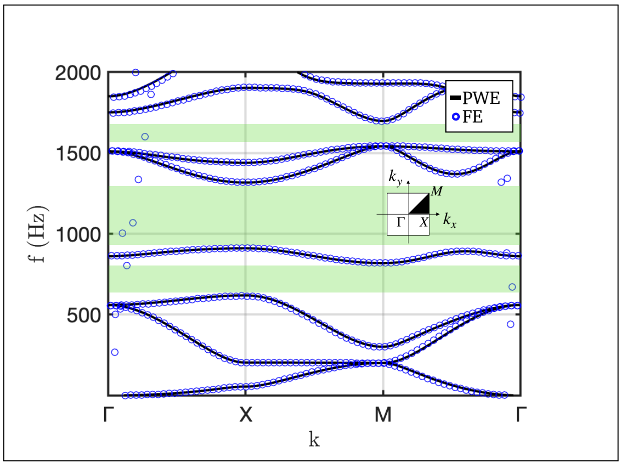

Figure 2. Dispersion curves with green-shaded areas denoting band gap distribution of the rectangular unit cell. The band structure along the IBZ path was computed using FE (blue circles) and PWE (black line) methods.

3.2 Numerical results for the finite panel with multiple ABHs

We consider an ABH PC plate composed of 4×4 array of identical rectangular unit cells, as shown in Fig.1a, and analyze its response when subjected to an external point force of $F = 1$N. The excitation point is located at the center of the finite structure, and the plate is assumed to be clamped along its edges. The width of the ABH PC plate is $a = 800$mm, with other material and geometric characteristics are taken from the previous section. We focus on the spatial average squared mobility of this plate, defined as $10log_{10}(v/F)$, where v is the velocity.

Fig.3a compares the mobility of the ABH PC plate with a 4×4 array of identical rectangular unit cells (red line), an ABH PC plate with an 8×8 array of identical unit cells (black line), and the reference plate (blue line), modeled as a uniform plate with a total thickness corresponding to the maximum thickness of the ABH PC plate, $h_{tot} = 12$mm. As shown in Fig.3a, the mobility of the ABH PC plates is significantly reduced, resulting in noticeable vibration attenuation at low frequencies, which is attributed to the formation of BGs.

Additionally, Figs.3b,c,d show the displacement field at 821Hz (green dashed line) for the different panels. These results demonstrate complete vibration isolation in the ABH PC plate, with the wave progressively attenuated as it passes through each unit cell. Furthermore, the mobility decreases substantially as the number of unit cells increases.

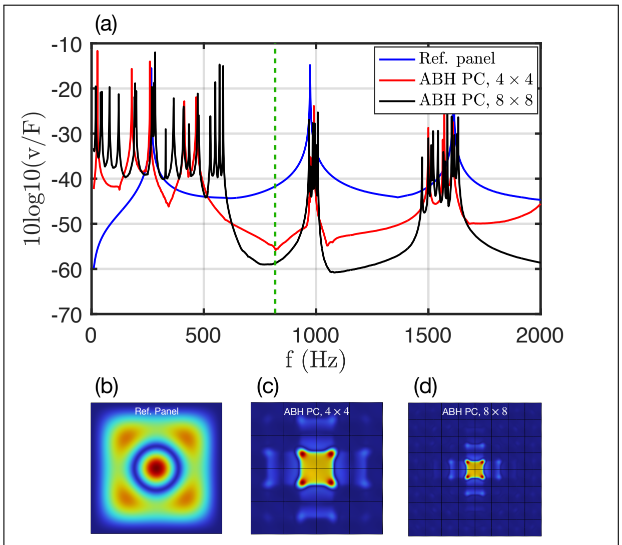

Figure 3. a) The spatial average squared mobility of the ABH PC plate composed of a 4×4 array of identical rectangular unit cells (red line), an 8×8 array of identical unit cells (black line), and the uniform plate (blue line). b,c,d) Corresponding displacement fields at 821Hz (green dashed line).

4. CONCLUSION

This study investigates the synergistic effects of ABHs and periodic modulation in sandwich panels for enhanced vibration and noise control. The band structure analysis using the PWE method and dynamic response evaluated through FE simulations revealed the presence of tunable low-frequency band gaps, which effectively attenuate mechanical waves and significantly reduce vibrational energy. The dynamic response analysis of the ABH PC plate, along with the effect of the number of unit cells, further confirmed a substantial reduction in the spatial average squared mobility compared to the uniform plate, demonstrating the efficiency of the proposed design. This work advances the understanding of acoustic metamaterials and offers a promising approach for developing lightweight materials for noise isolation and vibration suppression in various engineering applications.

5. ACKNOWLEDGMENTS

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